

引力波望远镜的装调误差对 TTL 耦合噪声的影响

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摘要 本文研究了引力波望远镜的装调误差对望远镜的 TTL (tilt-to-length, 角度抖动与光程读出之间的) 耦合噪声的影响。保持其他参量不变, 仅对引力波望远镜中的某项装调公差进行赋值, 仿真分析引力波望远镜中的某项装调公差对出瞳位置变化的影响, 进而计算出激光干涉信号经过全玻璃激光干涉仪最终在四象限光电探测器上进行干涉时, 由于引力波望远镜的装调公差的存在导致的 TTL 耦合噪声的变化情况。通过对比发现, 引力波望远镜的主镜与次镜的距离公差, 对引力波望远镜的 TTL 耦合噪声的影响大于其他光学元件之间的距离公差对 TTL 耦合噪声的影响。主镜和次镜之间的距离公差导致的 TTL 耦合噪声的变化与次镜和三镜之间的距离公差、三镜和四镜之间的距离公差导致的 TTL 耦合噪声的变化符号相反; 光阑和主镜之间的距离装调公差对 TTL 耦合噪声的变化影响很小, 可以忽略不计。各个光学元件的距离装调公差导致的 TTL 耦合噪声的变化量与抖动角度之间呈抛物线规律分布。在装调空间引力波望远镜时, 应着重控制主镜与次镜之间的距离误差。并且可以使用次镜和三镜之间的距离装调误差、三镜和四镜之间的距离装调误差导致的 TTL 耦合噪声来抵消由装调主镜和次镜之间的距离误差导致的 TTL 耦合噪声。

关键词 仪器, 测量与计量; 引力波; 干涉测量; 望远镜; 装调公差

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1 引言

自从 LIGO 地面引力波天文台首次直接探测到引力波后^[1], 人类便发现了探索宇宙奥秘的全新窗口, 证实了爱因斯坦对引力波的预言^[2-3], 开辟了采用引力波研究宇宙天文奥秘的新手段, 对天文学和物理学的发展意义重大。

由于地面引力波探测受到地球曲率半径、环境扰动、重力梯度、地震等各种地面噪声^[4]的影响, 所探测的引力波频段范围内的波源十分有限, 因此需发展空间引力波探测技术。太极计划中, 采用星间激光干涉测量光学系统, 在太空中对引力波进行探测。但是由于太空环境存在各种扰动, 星间激光干涉测量系统的接收端或发射端处于抖动状态, 导致用于星间测距的两束干涉光束存在一定的角度抖动。由两个干涉光束之间的角度抖动耦合到光程信号读出的噪声称之为 TTL (tilt-to-length, 角度抖动与光程读出之间的) 耦合噪声。

要在百万千米的距离^[5]上实现皮米量级的干涉测距, 要求的技术指标精度非常严苛, 精度以皮米量级计算。装调引力波望远镜的过程中, 由于装调误差的存在, 会导致望远镜出瞳位置变化, 致使 TTL 耦合噪声

产生变化, 影响干涉测量的精度。所以研究引力波望远镜的装调误差对 TTL 耦合噪声的影响十分重要。研究引力波望远镜的装调误差对 TTL 耦合噪声影响的规律性, 可以用于指导实际的空间引力波望远镜的装调工作, 可更好地将 TTL 耦合噪声控制在一定的低水平, 以保证实际生产出的空间引力波望远镜能够满足实际的使用需求。

目前, 国外已设计了多种空间引力波望远镜并对其进行了研究, 如 Verlaan 等^[6]“测试了为欧洲航天局 (ESA) 设计的引力波望远镜的热稳定性和抗振动性能。Schuster 等^[7]研究了减小测试质量干涉仪的 TTL 耦合噪声的方法。Gudrun^[8]讨论了减小 TTL 耦合噪声的两镜成像系统和四镜成像系统。Korytov 等^[9]探究了使用碳化硅材料制造的天基引力波望远镜的尺寸结构的稳定性。Bender^[10]研究了 LISA 望远镜的波前畸变和光束指向等问题, 研究了对抖动角度反应最灵敏的波前差 Zernike 拟合分量。Jean-Yves Vinet 等^[11]从光学系统的相位关系分析 LISA 望远镜的指向抖动噪声。Livas 等^[12-13]从光程稳定性和杂散光的角度阐述设计引力波望远镜的依据。Sanz 等^[14]系统阐述了引力波望远镜的工作环境并设计了望远镜。Sonke^[15]介绍了 TTL 耦合噪声的模型。陈胜楠

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等^[16]根据杂散光和波前的技术指标要求设计了离轴四反望远镜。以上研究都未涉及到望远镜的装调公差与 TTL 耦合噪声之间的内在关系。空间引力波望远镜作为星间激光干涉测量光学系统的重要组成部分,决定着星间激光干涉测量的准确性,以及空间引力波探测的成败。

由于 TTL 耦合噪声是引力波望远镜中的第二大噪声源^[17],引力波望远镜的实际工程化过程中,装调又会对 TTL 耦合噪声产生影响,所以对望远镜的装调公差与 TTL 耦合噪声之间联系的研究对引力波望远镜的工程化至关重要,引力波望远镜的装调公差对 TTL 耦合噪声会产生怎样的影响决定着最终的引力波望远镜是否满足使用要求。但是对于望远镜的装调与 TTL 耦合噪声之间关联的研究却鲜有报道。

本文通过仿真设计满足波前差指标要求的引力波望远镜,计算设计的引力波望远镜的 TTL 耦合噪声,分析望远镜的装调公差对 TTL 耦合噪声产生的影响,来判断装调引力波望远镜的过程中对 TTL 耦合噪声敏感的装调误差变量,该研究结果可指导引力波望远镜的装调。

2 装调误差对 TTL 耦合噪声的影响

2.1 空间引力波望远镜的设计

在星间激光干涉测量光学系统中,为了在百万千米距离上实现激光信号的发射与接收^[14],使两束干涉信号满足杂散光指标要求和光程稳定性要求,一般使用离轴四反望远镜作为激光信号的发射和接收装置。根据设计指标要求,设计的空间引力波望远镜的结构参数如表 1 所示。

表 1 离轴四反望远镜结构参数

Table 1 Structural parameters of off-axis quadruple mirror telescope

No.	Name	Radius of curvature / mm	Thickness / mm
1	Primary mirror	-12xx. xxx	-600
2	Secondary mirror	-8x. xxx	660
3	Third mirror	-6xx. xxx	-58. 141
4	Forth mirror	3xx. xxx	118. 013

本次设计的引力波望远镜的光路图如图 1 所示,无穷远处的平面波经过入瞳后,先经过主反射镜与次反射镜的反射成一次像,再经过三镜和四镜的反射后,在出瞳处以平行光射出。

引力波望远镜的设计波前差如图 2 所示,光学系统的设计波前残余差为 0.0085λ ($\lambda = 1064 \text{ nm}$)。

为了方便说明问题,将星间激光干涉测量光学系统简化为图 3 所示。

引力波望远镜的入瞳控制进入整个引力波望远镜

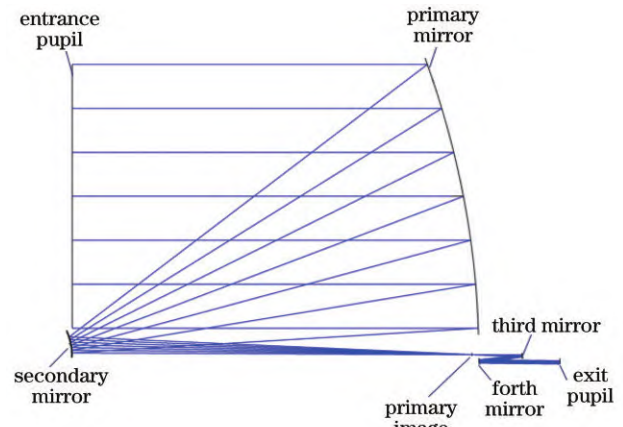


图 1 空间引力波望远镜光路图

Fig. 1 Optical path diagram of space gravitational wave telescope

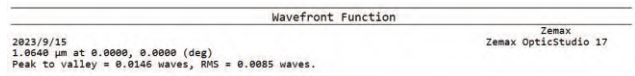
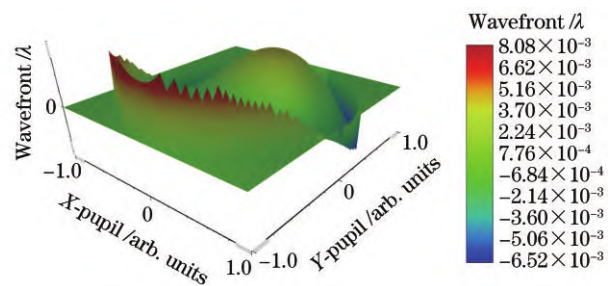


图 2 空间引力波望远镜的波前差

Fig. 2 Wavefront difference of space gravitational wave telescope

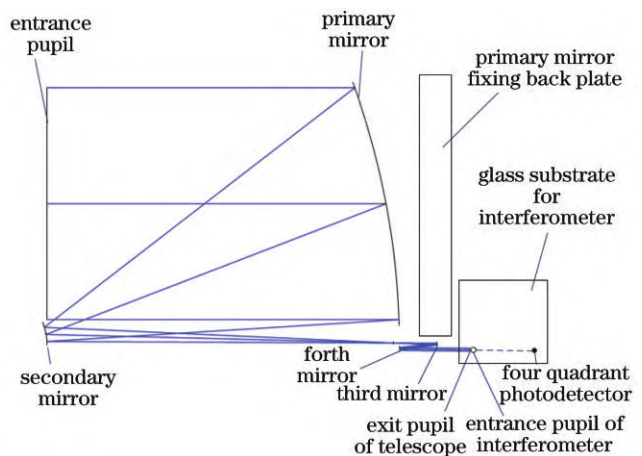


图 3 星间激光干涉仪光路简图

Fig. 3 Optical path diagram of interstellar laser interferometer

的光信号能量的强弱,望远镜的出瞳位置如图 3 所示。望远镜出瞳要与干涉仪的入瞳相衔接,在干涉仪内部的光路用虚线代替,望远镜的出瞳到四象限光电探测器(QPD)之间的距离为激光干涉仪的干涉臂长。对于设计的星间激光干涉测量光学系统而言,理想望远

镜的出瞳与干涉仪基板入瞳重合,并靠刚性结构固定,此刚性结构要满足光程稳定性要求,同时要将引力波望远镜的主镜固定背板、全玻璃激光干涉仪全部固定,同时保证理想状态下设计的引力波望远镜的出瞳与干涉仪的入瞳重合。

假设设计和加工的刚性结构能够使理想的引力波望远镜的出瞳与干涉仪的入瞳重合,但将引力波望远镜用刚性结构固定后,由于装调误差的存在,引力波望远镜的出瞳会偏离激光干涉仪的入瞳,致使引力波望远镜实际引入的 TTL 耦合噪声与理想情况下的 TTL 耦合噪声有差别。TTL 耦合噪声的计算公式^[18]为

$$I_{nt} = \int_s E_{meas} E_{ref}^* dr^2, \quad (1)$$

式中: I_{nt} 代表积分运算; E_{meas} 代表测量光束; E_{ref}^* 代表参考光束 E_{ref} 的共轭; S 代表积分区域,测量光束通常是平面波,参考光束通常是高斯光束。然后推导出 LPF 信号(LISA 探路者所用的光程信号)产生的轴向路径长度信号,计算式(1)并进行泰勒展开。本文仅从应用光学的角度考虑装调公差对 TTL 耦合噪声的影响,所以忽略波动光学的相关项,得到光程差的计算公式为

$$L_{LPF} \approx -\frac{1}{2} d\alpha^2 + \frac{1}{2} (d - z_0)\alpha^2 + o(\alpha^4), \quad (2)$$

式中: α 表示抖动角度; d 表示望远镜的出瞳到 QPD 的距离; z_0 表示高斯光束的束腰位置; $o(\alpha^4)$ 代表与抖动角度 α 无关的高阶量,可不必考虑。

假设平顶光束是理想的平面波,高斯光束是理想的高斯线型分布,并且高斯光束的束腰位置位于 QPD 感光面上。所以忽略由于波前不匹配导致的 TTL 耦合噪声,只关注由于几何光程路径差导致的 TTL 耦合噪声。那么 TTL 耦合噪声的计算公式可简化为

$$L_{LPF} \approx -\frac{1}{2} d\alpha^2. \quad (3)$$

2.2 装调公差对 TTL 耦合噪声的影响

理想情况下,由于望远镜与干涉仪之间的相对位置固定,望远镜的设计出瞳与干涉仪的 QPD 之间的距离为 417 mm。实际情况下,望远镜的出瞳位置受到加工公差的影响,整个望远镜所对应的 TTL 耦合噪声会发生相应变化,但是变化的程度和变化规律目前并不知晓,所以研究引力波望远镜的装调公差对星间激光干涉测量系统的 TTL 耦合噪声的影响十分必要。将星间激光干涉测量系统的 TTL 耦合噪声与望远镜装调公差的敏感性建立联系,并以 TTL 耦合噪声的要求为判据,建立装调公差项与 TTL 耦合噪声变化的模型关系。

根据计算 TTL 耦合噪声的公式,理想位置情况下, TTL 耦合噪声随抖动角度的变化曲线,与存在一定的装调公差条件下, TTL 耦合噪声随抖动角度的变化曲线,二者之差 ΔS_{LPF} 作为与抖动角度的关系曲线图。以相邻两镜组之间的距离增加 0.2 mm 作为装调公差,定量分析装调公差对 TTL 耦合噪声的影响。本文设计的引力波望远镜的装调公差如表 2 所示。

表 2 装调的距离公差值

Table 2 Distance tolerance values of installation and adjustment

No.	Component distance	Design distance /mm	Tolerance value /mm	Actual distance /mm
1	Aperture and primary mirror	600.000	0.200	600.200
2	Primary and secondary mirror	-600.000	-0.200	-600.200
3	Secondary and third mirror	660.000	0.200	660.200
4	Third and fourth mirror	-58.141	-0.200	-58.341

当光阑与主镜之间的距离存在+0.2 mm 的装调误差时,即光阑与主镜的距离为 600.2 mm 时,与理想情况下,无装调公差的引力波望远镜相比较, TTL 耦

合噪声的变化如图 4 所示。图 4(a)中显示 TTL 耦合噪声的变化 ΔS_{LPF} 几乎为一条水平直线,即 TTL 耦合噪声变化几乎为零。图 4(b)中显示的 ΔS_{LPF} 曲线变化

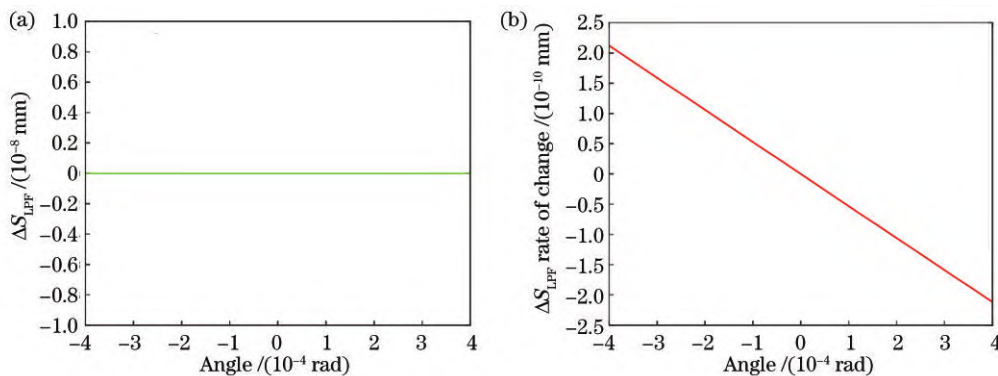


图 4 光阑与主镜之间存在安装误差。(a) ΔS_{LPF} 变化曲线图;(b) ΔS_{LPF} 变化率直线图

Fig. 4 There is an installation error between aperture and primary mirror. (a) ΔS_{LPF} change curve; (b) Line graph of ΔS_{LPF} rate of change

率是一条直线,此直线的斜率为 -5.296×10^{-7} 。

当空间引力波望远镜的主次镜存在装调误差时,例如存在 $+0.2 \text{ mm}$ 的装调误差,即在实际装调过程中,主镜与次镜之间的实际距离为 600.2 mm 时,TTL耦合噪声曲线与不存在装调误差条件下的

TTL耦合噪声的差值如图5所示。图5(a)中显示TTL耦合噪声的变化 $\Delta S_{\text{L,PF}}$ 随倾斜抖动角度 α 之间呈抛物线规律变化,且抛物线开口向上,抛物线的斜率变化如图5(b)所示,该直线的斜率大小为 9.021×10^{-4} 。

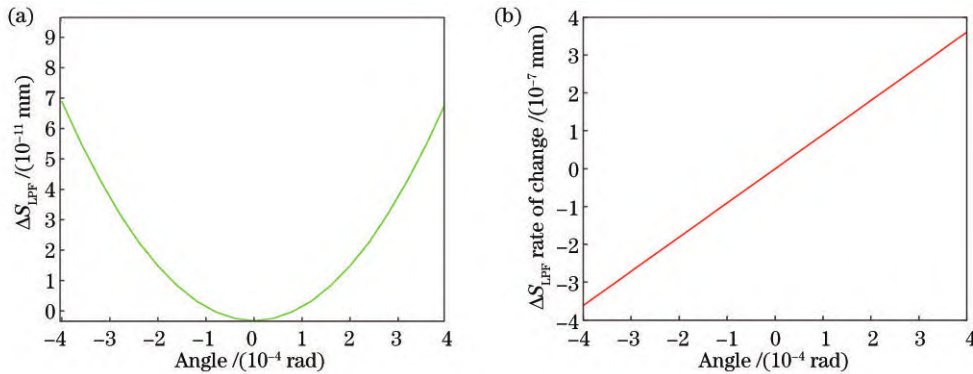


图5 主镜与次镜之间存在安装误差。(a) $\Delta S_{\text{L,PF}}$ 变化曲线图;(b) $\Delta S_{\text{L,PF}}$ 变化率直线图

Fig. 5 There is an installation error between primary mirror and secondary mirror. (a) $\Delta S_{\text{L,PF}}$ change curve; (b) Line graph of $\Delta S_{\text{L,PF}}$ rate of change

当次镜与三镜之间存在装调误差的情况下,如存在 $+0.2 \text{ mm}$ 的装调误差,即次镜与三镜之间的距离由 660 mm 变为 660.2 mm 时,TTL耦合噪声曲线与不存在装调误差条件下的TTL耦合噪声的差值曲线如图6所示。图6(a)中显示TTL耦合噪声曲线的变化 $\Delta S_{\text{L,PF}}$ 与抖动角度 α 呈抛物线规律变化,且抛物线的开口向下,抛物线的斜率变化如图6(b)所示,该

直线的斜率为 -5.824×10^{-5} 。与图5相比,图5的斜率变化是图6斜率变化的 15.489 倍,且方向相反。说明主次镜距离变化引起的TTL耦合噪声变化是次镜三镜距离变化引起的TTL耦合噪声变化的 15.489 倍。说明主次镜的装调误差对TTL耦合噪声的影响程度大于次镜三镜的影响程度,并且变化方向相反。

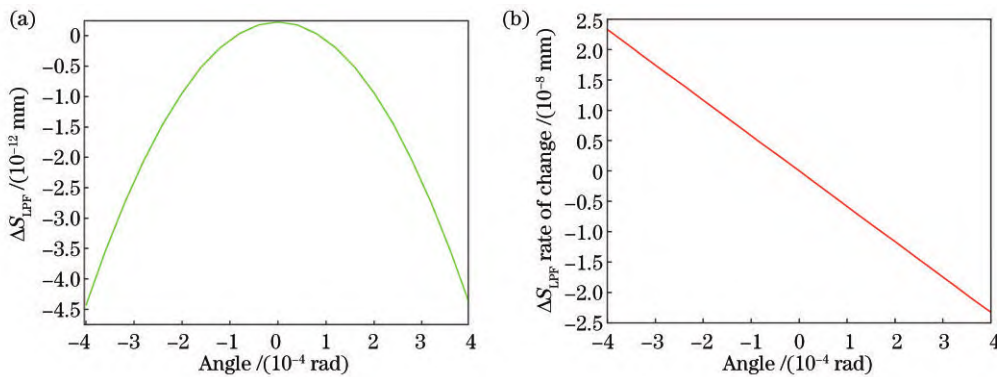


图6 次镜与三镜之间存在安装误差。(a) $\Delta S_{\text{L,PF}}$ 变化曲线图;(b) $\Delta S_{\text{L,PF}}$ 变化率直线图

Fig. 6 There is an installation error between secondary mirror and third mirror. (a) $\Delta S_{\text{L,PF}}$ change curve; (b) Line graph of $\Delta S_{\text{L,PF}}$ rate of change

当三镜与四镜之间存在装调误差的情况下,如存在 0.2 mm 的装调误差,三镜和四镜之间的距离由 -58.1406 mm 变为 -58.3406 mm 时,TTL耦合噪声曲线与不存在装调误差条件下的TTL耦合噪声的差值曲线图如图7所示。图中显示TTL耦合噪声曲线的变化 $\Delta S_{\text{L,PF}}$ 与抖动角度 α 之间呈抛物线变化,抛物线开口向下,抛物线的斜率变化如图7(b)所示,直线的斜率为 -9.689×10^{-5} 。与图5(b)相比,图5(b)斜率是图7(b)斜率的 9.311 倍,且开口向下,说明主次镜距离变化引起的TTL耦合噪声变化是三镜四镜距离变

化引起的TTL耦合噪声变化的 9.311 倍,即主次镜距离误差的敏感性是三镜四镜距离误差的敏感性的 9.311 倍。说明主次镜距离误差对TTL耦合噪声的影响程度大于三镜四镜的影响程度,并且方向相反,而三镜四镜距离误差的敏感程度是次镜三镜距离误差的敏感程度的 1.664 倍,并且方向相同。

综上,主次镜之间的距离公差对TTL耦合噪声的变化影响最大,变化关系呈抛物线分布。次镜与三镜之间的距离公差和三镜与四镜之间的距离公差,导致TTL耦合噪声的变化趋势相同,能够与主次镜之间的

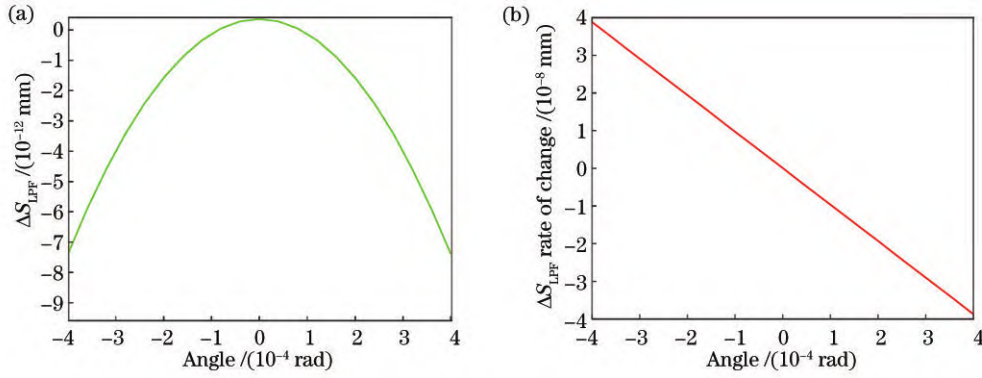


图 7 三镜与四镜之间存在安装误差。(a) ΔS_{LPF} 变化曲线图; (b) ΔS_{LPF} 变化率直线图

Fig. 7 There is an installation error between third mirror and fourth mirror. (a) ΔS_{LPF} change curve; (b) Line graph of ΔS_{LPF} rate of change

距离公差导致的 TTL 耦合噪声相互抵消。光阑与主镜之间的距离公差对 TTL 耦合噪声的变化影响几乎可以忽略。

3 分析与讨论

本文设计的空间引力波望远镜采用离轴四反望远镜的结构形式,将图 1 中的离轴四反望远镜进行等价转换。由于设计的引力波望远镜的主镜、次镜、三镜和

四镜分别是会聚透镜、发散透镜、会聚透镜、会聚透镜,主镜的焦距 f_1 为正,次镜的焦距 f_2 为负,三镜的焦距 f_3 为正,四镜的焦距 f_4 为正。用 d_1 代表入瞳与主镜之间的距离, d_2 代表主镜与次镜之间的距离, d_3 代表次镜与一次像面的距离, d_4 代表一次像面与三镜的距离, d_5 代表三镜与四镜之间的距离, d_6 代表四镜与出瞳之间的距离, d 代表出瞳到 QPD 之间的距离。等效处理后的光路图如图 8 所示。

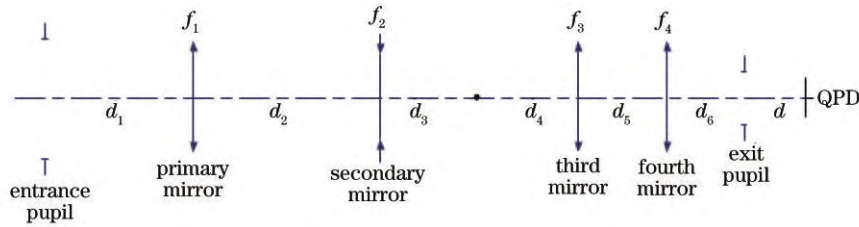


图 8 引力波望远镜的等效光路图

Fig. 8 Equivalent optical path diagram of gravitational wave telescope

计算光阑依次经过主镜、次镜、三镜和四镜组成的光学系统后,在像空间的像距,由高斯公式^[19]

$$\frac{1}{l'} - \frac{1}{l} = \frac{1}{f'} \quad (4)$$

计算,光阑经过主镜、次镜、三镜和四镜后,出瞳位置 l'_4 的大小等于

$$l'_4 = -1 / \{ 1 / \{ d_5 + 1 / (1 / \{ d_3 + d_4 - 1 / \{ 1 / f_2 - 1 / [d_2 - 1 / (1 / d_1 + 1 / f_1)] \} } - 1 / f_3 \} } - 1 / f_4 \} \} \quad (5)$$

当分析图 8 中所示的光路与 TTL 耦合噪声的关系时,为了使整体内容精简,仅以分析距离 d_3 为例,说明整个分析方法及结果。将 l'_4 写成与 d_3 之间的距离关系公式,最终的形式为

$$l'_4 = [(d_1 d_2 f_3 f_4 - d_1 d_2 d_5 f_4 + d_1 d_5 f_1 f_4 + d_1 d_5 f_2 f_4 - d_2 d_5 f_1 f_4 - d_1 f_1 f_3 f_4 - d_1 f_2 f_3 f_4 + d_2 f_1 f_3 f_4 + d_5 f_1 f_2 f_4 - f_1 f_2 f_3 f_4) \times d_3 + d_1 d_2 d_4 f_3 f_4 - d_1 d_2 d_4 d_5 f_4 + d_1 d_2 d_5 f_2 f_4 + d_1 d_2 d_5 f_3 f_4 + d_1 d_4 d_5 f_1 f_4 + d_1 d_4 d_5 f_2 f_4 - d_2 d_4 d_5 f_1 f_4 - d_1 d_2 f_2 f_3 f_4 - d_1 d_4 f_1 f_3 f_4 - d_1 d_5 f_1 f_2 f_4 - d_1 d_4 f_2 f_3 f_4 - d_1 d_5 f_1 f_3 f_4 + d_2 d_4 f_1 f_3 f_4 + d_2 d_5 f_1 f_2 f_4 - d_1 d_5 f_1 f_2 f_4 + d_2 d_5 f_1 f_3 f_4 + d_4 d_5 f_1 f_2 f_4 + d_1 f_1 f_2 f_3 f_4 - d_2 f_1 f_2 f_3 f_4 - d_4 f_1 f_2 f_3 f_4 - d_5 f_1 f_2 f_3 f_4] / [(d_1 d_2 f_3 - d_1 d_2 d_5 + d_1 d_2 f_4 + d_1 d_5 f_2 - d_2 d_5 f_1 - d_1 f_1 f_3 - d_1 f_1 f_4 - d_1 f_2 f_3 + d_2 f_1 f_3 - d_1 f_2 f_4 + d_2 f_1 f_4 + d_5 f_1 f_2 - f_1 f_2 f_3 - f_1 f_2 f_4) \times d_3 + d_1 d_2 d_4 f_3 - d_1 d_2 d_4 d_5 + d_1 d_2 d_5 f_2 + d_1 d_2 d_4 f_4 + d_1 d_2 d_5 f_3 + d_1 d_4 d_5 f_1 + d_1 d_4 d_5 f_2 - d_2 d_4 d_5 f_1 - d_1 d_2 f_2 f_3 - d_1 d_2 f_2 f_4 d_1 d_4 f_1 f_3 - d_1 d_5 f_1 f_2 - d_1 d_2 f_3 f_4 - d_1 d_4 f_1 f_4 - d_1 d_4 f_2 f_3 - d_1 d_5 f_1 f_3 + d_2 d_4 f_1 f_3 + d_2 d_5 f_1 f_2 - d_1 d_4 f_2 f_4 - d_1 d_5 f_2 f_3 + d_2 d_4 f_1 f_4 + d_2 d_5 f_1 f_3 + d_4 d_5 f_1 f_2 + d_1 f_1 f_2 f_3 + d_1 f_1 f_2 f_4 - d_2 f_1 f_2 f_3 + d_1 f_1 f_3 f_4 - d_2 f_1 f_2 f_4 + d_1 f_2 f_3 f_4 - d_2 f_1 f_3 f_4 - d_4 f_1 f_2 f_3 - d_4 f_1 f_2 f_4 - d_5 f_1 f_2 f_3 + f_1 f_2 f_3 f_4], \quad (6)$$

通过提取公因式,对式(6)进行处理,得到的结果为

$$l'_4 = \frac{k_{31}d_3 + b_{31}}{k_{32}d_3 + b_{32}}, \quad (7)$$

式中:

$$k_{31} = d_1d_2f_3f_4 - d_1d_2d_5f_4 + d_1d_5f_1f_4 + d_1d_5f_2f_4 - d_2d_5f_1f_4 - d_1f_1f_3f_4 - d_1f_2f_3f_4 + d_2f_1f_3f_4 + d_5f_1f_2f_4 - f_1f_2f_3f_4; \quad (8)$$

$$b_{31} = d_1d_2d_4f_3f_4 - d_1d_2d_4d_5f_4 + d_1d_2d_5f_2f_4 + d_1d_2d_5f_3f_4 + d_1d_4d_5f_1f_4 + d_1d_4d_5f_2f_4 - d_2d_4d_5f_1f_4 + d_1d_2f_2f_3f_4 - d_1d_4f_1f_3f_4 - d_1d_5f_1f_2f_4 - d_1d_4f_2f_3f_4 - d_1d_5f_1f_3f_4 + d_2d_4f_1f_3f_4 + d_2d_5f_1f_2f_4 - d_1d_5f_1f_2f_4 + d_2d_5f_1f_3f_4 + d_4d_5f_1f_2f_4 + d_1f_1f_2f_3f_4 - d_2f_1f_2f_3f_4 - d_4f_1f_2f_3f_4 - d_5f_1f_2f_3f_4; \quad (9)$$

$$k_{32} = d_1d_2f_3 - d_1d_2d_5 + d_1d_2f_4 + d_1d_5f_2 - d_2d_5f_1 - d_1f_1f_3 - d_1f_1f_4 - d_1f_2f_3 + d_2f_1f_3 - d_1f_2f_4 + d_2f_1f_4 + d_5f_1f_2 - f_1f_2f_3 - f_1f_2f_4; \quad (10)$$

$$b_{32} = d_1d_2d_4f_3 - d_1d_2d_4d_5 + d_1d_2d_5f_2 + d_1d_2d_4f_4 + d_1d_2d_5f_3 + d_1d_4d_5f_1 + d_1d_4d_5f_2 - d_2d_4d_5f_1 - d_1d_2f_2f_3 - d_1d_2f_2f_4 + d_1d_4f_1f_3 - d_1d_5f_1f_2 - d_1d_2f_3f_4 - d_1d_4f_1f_4 - d_1d_4f_2f_3 - d_1d_5f_1f_3 + d_2d_4f_1f_3 + d_2d_5f_1f_2 - d_1d_4f_2f_4 - d_1d_5f_2f_3 + d_2d_4f_1f_4 + d_2d_5f_1f_3 + d_4d_5f_1f_2 + d_1f_1f_2f_3 + d_1f_1f_2f_4 - d_2f_1f_2f_3 + d_1f_1f_3f_4 - d_2f_1f_2f_4 + d_1f_2f_3f_4 - d_2f_1f_3f_4 - d_4f_1f_2f_3 - d_4f_1f_2f_4 - d_5f_1f_2f_3 + f_1f_2f_3f_4. \quad (11)$$

对式(7)进行化简处理得到下式:

$$l'_4 = \frac{k_{31}d_3 + b_{31}}{k_{32}d_3 + b_{32}} = \frac{\frac{k_{31}}{k_{32}}d_3 + \frac{b_{31}}{k_{32}}}{d_3 + \frac{b_{32}}{k_{32}}} = \frac{k_{33}d_3 + b_{33}}{d_3 + b_{34}} = k_{33} + \frac{b_{33} - k_{33}b_{34}}{d_3 + b_{34}} = k_{33} + \frac{b_{35}}{d_3 + b_{34}}. \quad (12)$$

当存在实际的装调公差时, d_3 会变成 d'_3 , 即次镜与三镜之间存在装调公差, 大小为 $d'_3 - d_3$, 那么会导致 l'_4 的变化量为

$$\Delta l'_4 = l''_4 - l'_4 = b_{35} \times \left(\frac{1}{d'_3 + b_{34}} - \frac{1}{d_3 + b_{34}} \right) = b_{35} \times \frac{(d_3 - d'_3)}{(d'_3 + b_{34})(d_3 + b_{34})}, \quad (13)$$

式中, l''_4 是存在装调公差时, 入瞳经过引力波望远镜系统在像空间所成的像, 即出瞳的位置。其中 d_3 与 d'_3 之间存在如下关系:

$$\Delta d_3 = d'_3 - d_3. \quad (14)$$

将式(13)进行整理, 写成等效形式为

$$\Delta l'_4 = b_{35} \times \frac{-\Delta d_3}{(d_3 + \Delta d_3 + b_{34})(d_3 + b_{34})} = \frac{-b_{35} \cdot \Delta d_3}{\Delta d_3(d_3 + b_{34}) + (d_3 + b_{34})^2} = \frac{\frac{-b_{35}}{d_3 + b_{34}} \cdot \Delta d_3}{\Delta d_3 + (d_3 + b_{34})} = \frac{-b_{36} \cdot \Delta d_3}{\Delta d_3 + (d_3 + b_{34})} = \frac{-b_{36} \cdot \Delta d_3}{-b_{36} + \frac{b_{36}(d_3 + b_{34})}{\Delta d_3 + (d_3 + b_{34})}}. \quad (15)$$

式(15)是次镜与三镜之间存在装调公差后, 导致的出瞳位置的变化量。

由于 TTL 耦合噪声的计算公式为

$$T_{TL} = \frac{1}{2} \alpha^2, \quad (16)$$

对 TTL 耦合噪声的变化量进行计算, 得到如下等式:

$$\Delta T_{TL} = \frac{1}{2} \Delta l'_4 \alpha^2 = \frac{1}{2} \alpha^2 \frac{-b_{36} \cdot \Delta d_3}{\Delta d_3 + (d_3 + b_{34})}. \quad (17)$$

由于 d_3 远大于 Δd_3 , 对上述等式进行近似, 最终得到的等式为

$$\Delta T_{TL} = \frac{1}{2} \alpha^2 \frac{-b_{36} \cdot \Delta d_3}{(d_3 + b_{34})}, \quad (18)$$

仍然近似于一个抛物线分布, 与模拟计算得到的 TTL 耦合噪声变化情况相符合。

4 结 论

通过对引力波望远镜的装调公差进行分析, 建立

起引力波望远镜的装调公差与 TTL 耦合噪声变化的联系, 通过对其进行分析与讨论, 得知引力波望远镜的 TTL 耦合噪声的敏感程度, 对主次镜的距离变化敏感程度最高, 是次镜与三镜距离变化敏感程度的 15.489 倍, 是三镜与四镜距离变化敏感程度的 9.311 倍, 并且主次镜之间由于位置误差引起的 TTL 耦合噪声可由次镜与三镜的位置误差、三镜与四镜的位置误差引起的 TTL 耦合噪声局部抵消。所以实际对引力波望远镜进行装调时, 要着重考虑主次镜之间的距离公差, 其次考虑三四镜之间的距离公差, 最后保证次镜和三镜之间的位置公差。对引力波望远镜的装调误差与 TTL 耦合噪声的影响敏感程度进行分析, 可以对实际的引力波望远镜的装调过程进行指导。本文目前仅考虑装调公差对引力波望远镜的 TTL 耦合噪声的影响, 后续将讨论加工公差对 TTL 耦合噪声的影响, 进而指导引力波望远镜的加工和装调。

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Influence of Installation and Adjustment Error of Gravitational Wave Telescope on TTL Noise

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Abstract

Objective The influence of the installation and adjustment error of gravitational wave telescopes on the TTL coupling noise of the telescopes is studied. Since the TTL coupling noise is the second largest noise source, during the actual engineering of gravitational wave telescopes, the installation and adjustment will affect the TTL coupling noise, with little correlation between the telescope's installation and adjustment and TTL coupling noise. Therefore, the research on the relationship between the telescope's installation and adjustment tolerance and TTL coupling noise is significant for the engineering of gravitational wave telescopes, and how the gravitational wave telescope's installation and adjustment tolerance will affect the TTL coupling noise determines whether the final gravitational wave telescope meets the requirements for use. The research results can guide the installation and installation and adjustment of gravitational wave telescopes.

Methods We can judge the installation and adjustment processes of gravitational wave telescopes by simulating and designing a gravitational wave telescope that meets the requirements of the wave-front difference index, calculating the TTL coupling noise of the designed telescope, and analyzing the influence of the telescope's installation and adjustment tolerance on the TTL coupling noise. The variable of installation and adjustment tolerance is sensitive to TTL coupling

noise. By controlling the variable method with other parameters unchanged, only a certain installation and adjustment tolerance is assigned in the gravitational wave telescope, and the influence of the installation and adjustment tolerance in the telescope on the change of the exit pupil position is simulated and analyzed. Then when the laser interference signal passes through the laser interferometer and finally interferes with the four-quadrant detector, the variation of TTL coupling noise due to the installation and adjustment tolerance of the telescope is calculated. The relationship between the TTL coupling noise of the intersatellite laser interferometry system and the sensitivity of the telescope's installation and adjustment tolerance is established. In addition, the requirements of the TTL coupling noise are employed as the criterion to establish the model relationship between the installation and adjustment tolerance and the change of TTL coupling noise.

Results and Discussions Comparison shows that the distance tolerance between the primary mirror and the secondary mirror of the gravitational wave telescope exerts more influence on the TTL coupling noise of the gravitational wave telescope than the distance tolerance between other optical elements exerts on the TTL coupling noise. The change in TTL coupling noise due to the distance tolerance between the primary and secondary mirrors is opposite in sign to that due to the distance tolerance between the secondary and third mirrors and between the third and fourth mirrors. The installation and adjustment tolerance of the distance between the diaphragm and the primary mirror has little effect on the variation of the TTL coupling noise and can be ignored. The variation of the TTL coupling noise caused by the distance installation and adjustment tolerance of each optical element and the jitter angle are distributed in a parabolic law. By analyzing the installation and adjustment tolerance of the gravitational wave telescope, the relationship between the installation and adjustment tolerance of the gravitational wave telescope and the change of TTL coupling noise is established. Via the above analysis and discussion, the sensitivity of the TTL coupling noise of the gravitational wave telescope is known. The primary and secondary distance sensitivity of the mirror is the highest, which is 15.489 times the sensitivity of the secondary and third mirrors, and 9.311 times the sensitivity of the third and fourth mirrors. The TTL coupling noise caused by the position error between the primary and secondary mirrors can be reduced by the secondary and third mirrors, and that caused by the position error between the primary and secondary mirrors can be reduced by the position error between the third and fourth mirrors.

Conclusions When adjusting the space gravitational wave telescope, we should focus on controlling the distance error between the primary and the secondary mirrors. The TTL coupling noise caused by the distance installation and adjustment error between the secondary and the third mirrors, and the distance installation and adjustment error between the third and fourth mirrors can be adopted to partially offset the TTL coupling caused by the distance error between the noise of the primary and secondary mirrors. During actually adjusting the gravitational wave telescope, the distance tolerances between the primary and secondary mirrors and between the third and fourth mirrors should be considered successively, and the position tolerance between the secondary and third mirrors should be guaranteed. Our study analyzes the sensitivity of the installation and adjustment tolerance of the gravitational wave telescope to the influence of the TTL coupling noise, which can guide the actual installation and adjustment of gravitational wave telescopes. At present, we only consider the influence of installation and adjustment tolerance on the TTL coupling noise of gravitational wave telescopes, and the influence of processing tolerance on TTL coupling noises will be discussed later to guide the processing and installation of gravitational wave telescopes.

Key words instruments, measurement, and metrology; gravitational wave; interferometry; telescope; installation and adjustment tolerance