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# A robust fractional-order sliding mode control technique for piezoelectric nanopositioning stages in trajectory-tracking applications

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#### ABSTRACT

High-precision control of the piezoelectric nanopositioning stages is becoming a key technique in some precision manufacturing equipment. However, the piezoelectric nanopositioning stages suffer from the hysteresis effect and load disturbance, which damage its positioning accuracy. This paper proposes a fractional-order sliding mode tracking control method combining the fractional-order sliding mode surface and a new two-power reaching law to achieve high-precision tracking control of the piezoelectric nanopositioning stages under different loads. The fractional-order sliding mode surface effectively alleviates its chattering problem. The new two-power reaching law improves the convergence rate of the proposed controller. We conduct a rigorous theoretical analysis of the convergence interval and stability of the proposed controller. To evaluate the performance of the proposed controller, we carry out comparative trajectory tracking experiments on the piezoelectric nanopositioning stages. Results show that the proposed controller exhibits a satisfactory tracking performance even under different loads. Moreover, comparisons show that the proposed fractional-order sliding mode tracking controller outperforms the conventional PID and sliding mode control methods. The tracking performance of the proposed fractional-order sliding mode control method is improved by 59.17% and 39.42% compared with the PID and sliding mode control methods when tracking a sinusoidal signal with a frequency of 100 Hz.

## 1. Introduction

High-precision positioning control of the piezoelectric nanopositioning stages (PNS) has been attracting increasing attention from researchers, owing to its widespread applications, including the atomic force microscope [1,2], precision mirror alignment [3], and micromanipulation [4]. However, the hysteresis of the piezoceramic material severely damages the positioning accuracy of the micro-nano platform. As shown in Fig. 1, hysteresis curves vary with the frequency of the input signal. Thus, a suitable control strategy is imperative to achieve high-precision tracking positioning of the PNS under different loads.

Recently, several types of control methods have been proposed [5–7], to deal with the hysteresis behavior of the PNS. The hysteresis effect is effectively eliminated by implementing the exact inverse hysteresis model as the feed-forward compensator, such as the inverse Bouc–Wen model [8], inverse Duhem model [9], and Prandtl–Ishlinskii model [10] (including Play or Stop operators), etc. Li et al. adopted an inverse multiplicative structure to construct several feed-forward compensators,

where the adopted hysteresis models include the Duhem, Preisach, and Krasnosel'skii-Pokrovskii models, among others [11,12]. Considering that the residual error includes modeling uncertainties and external disturbances of the platform, the closed-loop control method combined with the feed-forward compensator is designed as a hybrid controller, where the most commonly adopted closed-loop control method includes the PID control [13], sliding mode control (SMC) [14], and iterative learning control [15], etc. Tan et al. proposed an adaptive conditional servo compensator without requiring explicit inversion of the hysteresis to achieve nanopositioning tracking control of stages driven by piezoelectric materials [16]. However, the performance of the compensator is heavily dependent on the modeling accuracy of the inverse hysteresis model. Therefore, several inversion-free control methods were studied in the field of micro-nano positioning control based on piezoelectric actuators [17,18]. Xu et al. proposed a self-adaptive compensation control method without an inverse hysteresis model to

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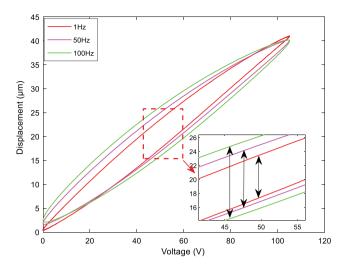


Fig. 1. Hysteresis characteristics of the PNS.

eliminate the hysteresis behavior of the PNS [19]. An inversion-free predictive controller, based on the dynamic linearized neural network model, was designed for the piezoelectric actuators [20]. Further, other control strategies have been studied in [21,22] and likewise yielded excellent performances.

In particular, a robust SMC technique emerged as an attractive control strategy in the nanopositioning field, owing to its inherent merits of highly robust performance to external disturbance or parameter uncertainty, fast response, and better transient performance [23,24]. The purpose of the SMC is to force the system state trajectory onto a defined sliding mode surface, and to maintain it on the surface for the subsequent time. However, it is difficult to design an appropriate sliding mode surface to guarantee its convergence. To address this issue, the fractional-order calculation was introduced into the design of the sliding mode surface. The extra degrees of freedom from the fractionalorder sliding mode surface (FOSMS) improve the performance of the corresponding controller in comparison with the integer-order sliding mode controller [25,26]. The fractional-order sliding mode controller was applied predominantly in servo drive control (such as permanent magnet synchronous motors [27,28] and power electronic buck converters [29]) owing to its high performance.

From a practical perspective, the system states of the traditional SMC method do not remain on the FOSMS due to the discontinuous control action derived from the modeling uncertainties and exterior disturbances. Consequently, the traditional SMC usually generates the chattering phenomenon, which may give rise to the undesired highfrequency dynamic oscillation and even deteriorate the tracking or positioning accuracy of the PNS. To tackle the inherent chattering problem of the traditional SMC method, a notable effort has been studied by numerous scholars, and several methods were proposed in the literature. For example, Xu et al. utilized an uncertainty and disturbance estimator to realize a chattering-free sliding mode controller for the high-precision tracking control of piezo-driven stages [30]. Further, the boundary layer technique [31], high-order sliding mode control method [32,33], adaptive SMC strategy [34], and low-pass filtering based SMC scheme [35] were introduced to alleviate the chattering effect.

This study is mainly bank for the current problems in the design of the SMC method. It also proposes a FOSMC method with a new two-power reaching law to implement high accuracy control of the PNS. The proposed two-power reaching law is highly efficient for addressing modeling uncertainty and load disturbance of the PNS. The chattering issue is effectively alleviated by the FOSMS. The stability and convergence region of the presented FOSMC technique are analyzed in

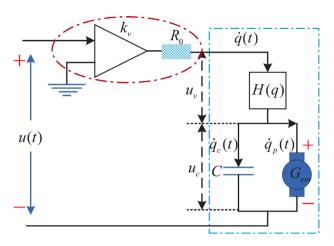


Fig. 2. Electrical model of the PNS.

detail and demonstrated using the finite-time and Lyapunov theories. Finally, some comparative experiments are carried out on the PNS to verify the superiority of proposed FOSMC technique.

The remainder of this paper is organized as follows. System modeling and problem formulation are described in Section 2. Then, a FOSMC approach is designed to achieve tracking control of the PNS, and the convergence and stability of the proposed controller are analyzed in Section 3. Section 4 describes the experimental setup and the identified parameters of the Duhem model. This forms the basis for the tracking experiments of the PNS. Some comparative tracking experiments are carried out to verify the performance of the proposed FOSMC method in Section 5. Section 6 presents the conclusion and future prospects.

## 2. System modeling and problem formulation

The modeling of the PNS requires consideration of piezoelectric ceramic circuits, drive amplifier circuits, and flexible hinge structures. As shown in Fig. 2, it is the circuit model of the PNS. The driver amplifier is described as a series circuit of the gain  $k_v$  and the resistance  $R_0$ , the piezoelectric ceramic actuator is described as a series–parallel circuit of capacitors C and electromechanical converters  $G_{em}$ . C is the total capacitance of piezoelectric ceramics, q is the total charge of the piezoelectric ceramics actuator,  $q_c$  and  $q_p$  are the charge stored in the capacitor C and induced charge from piezoelectric effect. Next, we define  $u_a(t)$  as the voltage from the piezoelectric effect, which is equal to the voltage  $u_c(t)$  of the capacitor C. So, we can obtain the circuit equation as

$$\begin{cases} R_0 \dot{q}(t) + u_v(t) + u_c(t) = k_v u(t) \\ q_p(t) = G_{em} x(t) \\ u_c(t) = q_c(t) / C \\ q(t) = q_c(t) + q_p(t) \\ u_v(t) = H(q), \end{cases}$$
 (1)

where u(t) is the input voltage of the PNS, x(t) is the output displacement. The formula (1) is rewritten as

$$R_0 C \dot{q}(t) + q(t) - G_{em} x(t) = C k_v [u(t) - \frac{1}{k_v} H(q)].$$
 (2)

Next, the mechanical characteristics of the flexible hinge structure is modeled as

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = G_{em}u_a(t), \tag{3}$$

where m, b, and k are the equivalent mass, damping, and stiffness, respectively.

Combining (2) with (3), it can be obtained that

$$\ddot{x}(t) + (\frac{1}{R_0C} + \frac{b}{m})\ddot{x}(t) + (\frac{b}{mR_0C} + \frac{kC + G_{em}^2}{mC})\dot{x}(t) + \frac{k}{mR_0C}x(t) = \frac{k_vG_{em}}{mR_0C}(u(t) - \frac{1}{k_v}H(q)).$$
(4)

According to (4), the PNS is described as a third-order system. In this work, limited by the operating characteristics of the PNS, we do not consider the high frequency characteristics of this stage, established model (4) is reduced to the following Duhem model form:

$$\begin{cases} \dot{x}(t) = a_0 x(t) + a_1 u(t) + a_2 \hbar(t) + \phi(t) \\ \dot{\hbar}(t) = \alpha |\dot{u}(t)| [f(u(t)) - \hbar(t)] + \dot{u}(t) g(u(t)) \\ y(t) = x(t), \end{cases}$$
 (5)

Herein, y(t) is the output of the model;  $\hbar(t)$  is the duhem operator, which represents the hysteresis part H(q) of the piezoelectric ceramic.  $f(\cdot)$  and  $g(\cdot)$  are piecewise continuous functions, which are used to approach the hysteresis part of the Duhem model;  $\dot{x}(t)$ ,  $\dot{u}(t)$ , and  $\dot{\hbar}(t)$  are the first order time derivatives of x(t), u(t), and  $\hbar(t)$  with respect to time, respectively;  $a_0 = -k/(b + kCR_0 + G_{em}^2R_0)$ ,  $a_1 = -(k_vG_{em})/(b + kCR_0 + G_{em}^2R_0)$ ,  $a_2 = -G_{em}/(b + kCR_0 + G_{em}^2R_0)$  represent the stage systems parameters;  $\dot{\phi}(t)$  denotes the system perturbation produced by uncertain parameters and external disturbance.

Followingly, the unknown functions  $f(\cdot)$  and  $g(\cdot)$  are obtained by the Weierstrass approximation theorem. This method has the advantages of strong approximation ability and a low computational complexity. In this study,  $f(\cdot)$  and  $g(\cdot)$  are replaced by two polynomial functions  $\hat{f}(\cdot)$  and  $\hat{g}(\cdot)$ , respectively, which are expressed as:

$$\begin{cases} \hat{f}(u(t)) \cong f_0 + f_1 u(t) + \cdots + f_{\varrho_1} u^{\varrho_1}(t) = \sum_{i=0}^{\varrho_1} f_i u^i(t) \\ \hat{g}(u(t)) \cong g_0 + g_1 u(t) + \cdots + g_{\varrho_2} u^{\varrho_2}(t) = \sum_{j=0}^{\varrho_2} g_j u^j(t), \end{cases}$$
(6)

where  $f_i$  and  $g_i$  are the constants,  $\varrho_1$  and  $\varrho_2$  are the orders of the  $\hat{f}(\cdot)$  and  $\hat{g}(\cdot)$ , respectively. Thus, the Duhem hysteresis model is reformulated as:

$$\begin{cases} \dot{x}(t) = a_0 x(t) + a_1 u(t) + a_2 \hat{h}(t) + \phi(t) \\ \dot{\hat{h}}(t) = \alpha |\dot{u}(t)| \left( \hat{f}(u(t)) - \hat{h}(t) \right) + \dot{u}(t) \, \hat{g}(u(t)) \\ y(t) = x(t). \end{cases}$$
(7)

where  $\hat{h}$  is the estimated value of h based on  $\hat{f}(\cdot)$  and  $\hat{g}(\cdot)$ .

The objective of this study is to achieve a superior positioning performance of the PNS with the lump disturbance  $\phi(t)$ . To suppress the tracking error and achieve a desired locating ability, a novel control technique is devised in the following section.

## 3. Fractional-order sliding mode controller

A FOSMC strategy is proposed to eliminate the hysteresis and suppress the perturbation caused by parameter uncertainties and unknown disturbances (such as the external loads) of the PNS. We introduce the fractional calculus into the design of the proposed FOSMC method, it exhibits low chattering in the sliding motion. A new reaching law forces the system states to reach the sliding manifold in finite time, and improves convergence rate of system states. These improve the performance of the FOSMC and achieve high precision tracking control of the PNS.

## 3.1. Fractional-order sliding mode surface

At first, we design a FOSMS, written as:

$$\sigma(t) = \bar{e}(t) + \lambda({}_{0}D_{t}^{r}\bar{e}(t)), \tag{8}$$

where  $\bar{e}(t)$  is the error between the desired output state  $x_d(t)$  and the actual output state x(t) of the PNS;  $\lambda$  is a constant, which influences the reaching rate of the system states.  ${}_0D_t^r(\cdot)$  is the fractional calculus,

and r is a constant and it satisfies 0 < r < 1. Generally, the fractional calculus  ${}_{0}D_{r}^{r}(\cdot)$  is described as:

$${}_{0}D_{t}^{r}(\cdot) = \begin{cases} \frac{d^{r}}{dt^{r}} & R(r) > 0\\ 1 & R(r) = 0\\ \int_{0}^{t} (d\tau)^{-r} & R(r) < 0, \end{cases}$$
(9)

where R(r) represents the real part of r. There are several definitions for the fractional calculus in the existing literature [36,37]. Riemann–Liouville fractional calculus is an important and commonly definition, which is expressed as follows [38]:

$${}^{RL}_{t_0} D^r_t f(t) = \frac{1}{\Gamma(\zeta - r)} (\frac{d}{dt})^{\zeta} \int_{t_0}^t \frac{f(\tau)}{(t - \tau)^{1 + r - \zeta}},$$
(10)

where  $\zeta - 1 < r < \zeta$ , and  $\zeta \in N$ . The  $\Gamma(\cdot)$  is the Gamma function. In this study, to apply the fractional calculus into the controller, we transform it into Laplace form, and it is given by

$$L\binom{RL}{t_0}D_t^r f(t) = \int_{t_0}^{\infty} e^{-st} {}_{t_0} D_t^r f(t) dt$$

$$= s^r F(s) - \sum_{k=0}^{m-1} s^k {}_{t_0} D_t^{r-k-1} f(t) |_{t=0},$$
(11)

where s is the Laplacian operation; when the initial condition of the f(t) is zero, formula (11) is rewritten as:

$$L\binom{RL}{0}D_t^r f(t) = \int_0^\infty e^{-st} {_0}D_t^r f(t)dt = s^r F(s).$$
 (12)

In general, the fractional order calculus is difficult to implement directly in engineering. Oustaloup method, as an efficient filter, is widely used to approximate the fractional order calculus because of its advantages of simple calculation and easy engineering implementation. In this study, we use the Oustaloup method to approximate the fractional order calculus, and  $s^r$  can be written as:

$$O(s) = \left(\frac{\sqrt{\omega_L \omega_H}}{\omega_H}\right)^r \prod_{k=1}^N \frac{1 + s/\omega_k'}{1 + s/\omega_k},\tag{13}$$

where N is a constant, and 2N+1 represents the number of zeros or poles of the function O(s).  $\omega_L$  and  $\omega_H$  denote the lower and upper frequencies of the fitting curve, respectively.  $\omega_k$  and  $\omega_k'$  are calculated as

$$\omega_k = \omega_L(\frac{\omega_H}{\omega_L})^{\frac{K+N+\frac{1}{2}+\frac{r}{2}}{2N+1}}$$
 (14)

$$\omega_k' = \omega_L \left(\frac{\omega_H}{\omega_L}\right)^{\frac{K+N+\frac{1}{2}-\frac{r}{2}}{2N+1}} \tag{15}$$

In this study,  $\omega_L$  and  $\omega_H$  are selected as 1 and 1000, respectively. N is chosen as 1 to balance the computational complexity and approximate precision. For a more detailed discussion of parameters selection for the Oustaloup method, please refer to [39].

We introduce fractional order calculus into the design of sliding mode controller, and the FOSMS is designed. When the sliding mode controller generates sliding mode motion, the states of the FOSMS converge to balance point fast and stably. To illustrate the stability of the proposed FOSMS (the condition for the existence of sliding mode is satisfied), the following lemma is introduced.

Lemma 1 ([40]). For the fractional-order system:

$$_{0}D_{t}^{r}x(t) = Ax(t), x(0) = x_{0},$$
(16)

where r is the differential order,  $x \in R^n$  and  $A \in R^{n \times n}$ , is asymptotically stable if  $|arg(eig(A))| > \frac{r\pi}{2}$ .

Letting  $\sigma = 0$ , we obtain

$${}_{0}D_{t}^{r}\bar{e}\left(t\right) = -\frac{1}{\lambda}\bar{e}\left(t\right). \tag{17}$$

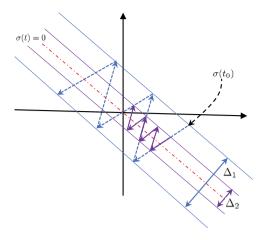


Fig. 3. Converging process comparison between FOSMS and integer order sliding mode surface.

Based on Lemma 1, if  $\left| \arg(\operatorname{eig}(-\frac{1}{\lambda})) \right| = \pi > \frac{r\pi}{2}$  is satisfied, the fractional-order system (16) is stable. Because  $0 < r < 1, \ 0 < \frac{r\pi}{2} < \frac{\pi}{2}$ , the  $\left| \arg(\operatorname{eig}(A)) \right| > \frac{r\pi}{2}$  holds always true. Therefore, the FOSMS is asymptotically stable.

**Remark 1.** The chattering problem of the SMC is caused by the delay of the actual system. As shown in Fig. 3, it notes that the integer-order sliding mode surface converges to the equilibrium point in an exponential law. However, the convergence rate of the FOSMS can be adjusted by the fractional order derivative r. The convergence rate of the FOSMS is less than the convergence rate of the integer-order sliding mode surface by selecting r. The integer-order sliding mode system states are switched in region  $\Delta_1$ , and the fractional-order sliding mode system states are switched in region  $\Delta_2$ , and they satisfy  $\Delta_1 > \Delta_2$ . Therefore, the FOSMS transfers energy more slowly than the integer-order. The FOSMS can effectively reduce the chattering.

## 3.2. Controller design

Before the design of the FOSMC method, the following assumption and Lemma are introduced.

**Assumption 1.** The lump disturbance of the PNS is bound and satisfies that  $|\phi| \le \Phi$ , where  $\Phi$  is a positive constant.

**Lemma 2** ([41]). If a continuous, positive definite function F(t) satisfies

$$\dot{F}(t) \le \zeta F^{\gamma}(t), \forall t \le t_0, F(t_0) \ge 0, \tag{18}$$

where two constants  $\zeta$  and  $\gamma$  satisfy  $\zeta > 0$ ,  $0 < \gamma < 1$ , then the function F(t) is convergent in finite time.

**Theorem 1.** Considering the PNS system is described by (7) with the FOSMS (8), if the control law is (19), the FOSMS  $\sigma(t)$  will converge to  $\psi$ 

$$u(t) = \frac{1}{a_1} [u_1(t) + u_2(t)], \tag{19}$$

with

$$u_1(t) = \dot{x}_d(t) - a_0 x(t) - a_2 \hat{h}(t) + \lambda ({}_0 D_t^{r+1} e(t)).$$
 (20)

$$u_{2}(t) = k_{1} \operatorname{Fal}\left(\sigma\left(t\right), \varepsilon_{1}, \delta_{\Delta_{1}}\right) + k_{2} \operatorname{Fal}\left(\sigma\left(t\right), \varepsilon_{2}, \delta_{\Delta_{2}}\right). \tag{21}$$

where  $k_1$  and  $k_2$  are positive constants;  $\epsilon_1$  and  $\epsilon_2$  are the parameters of the two-power reaching law, and they satisfy  $\epsilon_1 > 1$ ,  $0 < \epsilon_2 < 1$ , and the function Fal  $(\sigma(t), \epsilon, \delta_A)$  is described as follows:

$$\operatorname{Fal}\left(\sigma\left(t\right),\varepsilon,\delta_{\Delta}\right) = \begin{cases} |\sigma\left(t\right)|^{\varepsilon} \operatorname{sgn}\left(\sigma\left(t\right)\right) & |\sigma\left(t\right)| > \delta_{\Delta} \\ \frac{\sigma\left(t\right)}{\delta_{\Delta}^{\left(1-\varepsilon\right)}} & |\sigma\left(t\right)| \leq \delta_{\Delta}, \end{cases} \tag{22}$$

where  $\delta_{\Delta} > 0$  is the boundary layer of the Fal function. It is worth noting that the main role of  $u_1(t)$  is to make the system states converge to the FOSMS, and the  $u_2(t)$  is used to compensate the lump disturbance and keeps the system states on the FOSMS.

**Proof.** First, we define the Lyapunov function as follows:

$$V(t) = \frac{1}{2}\sigma^{2}(t).$$
 (23)

Taking the first-order time derivative of (23),

$$\dot{V}(t) = \sigma(t)\dot{\sigma}(t). \tag{24}$$

Then, taking the first order time derivative of the FOSMS (8),

$$\dot{\sigma}(t) = \dot{\bar{e}}(t) + \lambda(_0 D_t^{r+1} \bar{e}(t)).$$
 (25)

In view of the first formula in (7), the following formula is shown as:

$$\begin{split} \dot{\sigma}(t) &= \dot{x}_d(t) - a_0 x(t) - a_1 u(t) - a_2 \hat{\hbar}(t) - \phi(t) \\ &+ \lambda (_0 D_t^{r+1} \bar{e}(t)). \end{split} \tag{26}$$

Substituting (19) and (26) into (24), leads to

$$\begin{split} \dot{V}\left(t\right) &= \sigma\left(t\right) \left(\dot{x}_{d}(t) - a_{0}x(t) - a_{1}(u_{1}(t) + u_{2}(t))\right) \\ &- a_{2}\hat{h}(t) - \phi(t) + \lambda({}_{0}D_{t}^{r+1}\bar{e}\left(t\right))\right) \\ &= \left[-k_{1}\mathrm{Fal}\left(\sigma\left(t\right), \epsilon_{1}, \delta_{\varDelta_{1}}\right)\right. \\ &- k_{2}\mathrm{Fal}\left(\sigma\left(t\right), \epsilon_{2}, \delta_{\varDelta_{2}}\right)\right] \sigma\left(t\right) - \phi(t)\sigma\left(t\right) \\ &\leq \left[-k_{1}\mathrm{Fal}\left(\sigma\left(t\right), \epsilon_{1}, \delta_{\varDelta_{1}}\right)\right. \\ &\left. - k_{2}\mathrm{Fal}\left(\sigma\left(t\right), \epsilon_{2}, \delta_{\varDelta_{2}}\right)\right] \left|\sigma\left(t\right)\right| + \Phi\left|\sigma\left(t\right)\right|. \end{split} \tag{27}$$

Case 1: when the system state is remote from the FOSMS, i.e., in the distance reaching phase. At this moment, the term  $k_1 \operatorname{Fal}\left(\sigma\left(t\right), \epsilon_1, \delta_{\Delta_1}\right)$  plays a key role due to  $\epsilon_1 > 1$ . As show in Fig. 4(a),  $t_1$  represents the this distance reaching phase, it shows that the reaching law makes the states approach the FOSMS rapidly.

If  $|\sigma(t)| > \delta_{\Delta}$ , formula (28) is based on (22) and (27).

$$\dot{V}(t) \le |\sigma(t)| \left[ -k_1 |\sigma(t)|^{\varepsilon_1} \operatorname{sgn}(\sigma(t)) - k_2 |\sigma(t)|^{\varepsilon_2} \operatorname{sgn}(\sigma(t)) + \Phi \right], \tag{28}$$

and rewritten as follow:

$$\dot{V}(t) \le -k_2 |\sigma(t)|^{\varepsilon_2 + 1} - \left(k_1 |\sigma(t)|^{\varepsilon_1} - \Phi\right) |\sigma(t)|. \tag{29}$$

When  $k_1 |\sigma(t)|^{\varepsilon_1} - \Phi \ge 0$ , in view of (29), it can be obtained that

$$\dot{V}(t) \le -k_2 |\sigma(t)|^{\varepsilon_2 + 1} \tag{30}$$

On the basis of the formula (30), there is:

$$\dot{V}(t) \le -2^{(\varepsilon_2 + 1)/2} k_2 V^{(\varepsilon_2 + 1)/2}(t). \tag{31}$$

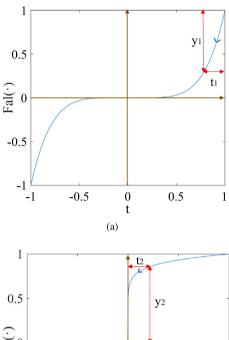
According to Lemma 2, when the convergence region of the FOSMS is as follows:

$$|\sigma(t)| \le \left(\frac{\Phi}{k_1}\right)^{\epsilon_1^{-1}} \tag{32}$$

the control system state converges to a neighborhood of the origin in finite time.

If  $|\sigma(t)| \le \delta_A$ , the formula (33) according to (22) is

$$\dot{V}\left(t\right) \leq \left[-k_{1} \delta_{\Delta_{1}}^{\left(\epsilon_{1}-1\right)} \sigma\left(t\right) - k_{2} \delta_{\Delta_{2}}^{\left(\epsilon_{2}-1\right)} \sigma\left(t\right)\right] \left|\sigma\left(t\right)\right|$$



-0.5 -1 -1 -0.5 0 0.5 t

Fig. 4. Reaching law function  $Fal(\cdot)$  analysis. (a) the distant reaching phase, (b) the close approximation phase.

$$+\Phi\left|\sigma\left(t\right)\right|,\tag{33}$$

and rewritten as follows:

$$\dot{V}\left(t\right) \leq -k_{2}\delta_{\Delta_{2}}^{\left(\varepsilon_{2}-1\right)}\sigma^{2}\left(t\right) - \left(k_{1}\delta_{\Delta_{1}}^{\left(\varepsilon_{1}-1\right)}\sigma\left(t\right) - \Phi\right)\left|\sigma\left(t\right)\right|.\tag{34}$$

When  $k_1 \delta_{A_1}^{(\epsilon_1 - 1)} \sigma(t) - \Phi \ge 0$ , based on the formula (34), we have

$$\dot{V}(t) \le -2\delta_{A_1}^{(\epsilon_2 - 1)} k_2 V(t). \tag{35}$$

According to Lemma 2, To ensure that the FOSMS is convergent in finite time, the convergence region is calculated as:

$$|\sigma(t)| \le \frac{\Phi}{k_1 \delta_{\Delta_1}^{(\epsilon_1 - 1)}}.$$
(36)

The convergence region of the FOSMS in the distant reaching phase is as follows:

$$\Omega_1: \left\{ |\sigma(t)| \mid |\sigma(t)| \le \min\{ (\Phi/k_1)^{\epsilon_1^{-1}}, \Phi/(k_1 \delta_{\Delta_1}^{(\epsilon_1 - 1)}) \} \right\}. \tag{37}$$

At this moment, the formula (28) satisfies  $\dot{V}(t) < 0$ , and the proposed whole control system is stable.

Case 2: when the system states are in the close approximation phase, as shown in Fig. 4(b), this phase can be represented from  $t_2 \to 0$ . At this moment,  $k_2 \operatorname{Fal}\left(\sigma\left(t\right), \varepsilon_2, \delta_{\Delta_2}\right)$  plays a key role due to  $0 < \varepsilon_2 < 1$ . The convergence region follows (38) to ensure the stability of the controller system using a similar calculation process.

$$\Omega_2: \left\{ |\sigma(t)| \mid |\sigma(t)| \le \min\{(\Phi/k_2)^{\varepsilon_2^{-1}}, \Phi/(k_2\delta_{A_2}^{(\varepsilon_2 - 1)})\} \right\}$$
 (38)

From the calculation above, the whole controller makes the system states converge to the following convergence region in finite time:

$$|\sigma(t)| \le \Psi = \min\{\Omega_1, \Omega_2\}. \quad \Box$$
 (39)

**Remark 2.** According to the formula (39), the steady-state error bound of the proposed FOSMC method is related to the  $\Phi$ , and the parameters  $k_1$ ,  $k_2$  reduce the steady-state error bound. Different from the [42,43], the lump disturbance  $\phi(t)$  is eliminated using the formula (21). Because the parameters of the two-power reaching law are obtained by trial-and-error method rather than the adaptive algorithm, the upper bound of the lump disturbance cannot be estimated. But it can reduce the "chattering phenomenon" and keep the system states on the sliding mode surface in a superior way.

**Remark 3.** It notes that the traditional reaching law of the SMC has the characteristics of "fast approach at long range, slow approach at close range". In this study, different from the existing reaching law, the proposed two-power reaching law (21) can make the states of the system reach the sliding mode surface at the faster speed and improve the robustness and convergence rate of the SMC. In generally, the system states entering the sliding surface at a faster speed will cause serious chattering phenomenon. However, the FOSMS can slow down the chattering amplitude when the system states reach  $\sigma(t)=0$ , it is shown as Fig. 3. Therefore, the proposed FOSMC method based on two-power reaching law not only speeds up the reaching rate but also reduces the chattering of the controller.

#### 4. Experimental facility

## 4.1. Description of experimental setup

The experimental setup of the PNS system and its corresponding structure diagram are illustrated in Fig. 5. The modeling and control schemes are verified on a commercial PNS (MPT-2MRL050, Boshi Robotics), which is composed of a piezoelectric actuator, a displacement sensor, and a mechanical structure. The system is built on a vibration reduction platform (J02-1809, Sunnylink), which avoids the negative influences of external disturbances. A host computer, equipped with 16-bit A/D and D/A converters (PCI-1710, Advantech), implements the proposed controller. An analog control input signal, which is produced by the DACs and subsequently amplified 15 times via the driver amplifier of the integral positioning controller (PPC-2CR0150, Boshi Robotics), is delivered to drive the PNS. The output displacement is measured in real-time by a built-in strain gauge sensor of the integral positioning controller, the actual output displacement value is converted into voltage signal by the ADCs, and it is used as the input of the proposed controller. Then, the control action of the proposed controller is obtained through calculation with the desired trajectory and the actual output displacement. Moreover, the proposed control strategies are carried out by Matlab/Simulink software (which is installed on the host computer and runs on Windows 10 and an Intel Core i7-1195G7 CPU) with the toolbox of the real-time windows target. The sampling time of the following experiments is set as 0.0001 s.

## 4.2. Identification of Duhem model

As mentioned above in the Duhem model, several parameters (i.e.,  $a_0$ ,  $a_1$ ,  $a_2$ ,  $f_i$ ,  $g_i$ , and  $\alpha$ ) are unknown, and they must be estimated using the optimization algorithm. For ease of identification of the parameters of the Duhem model, the formula (7) without the lump disturbance  $\phi$  is discretized and rewritten as:

$$\begin{cases} x(k) - x(k-1) = a_0 x(k) + a_1 u(k) + a_2 \hat{h}(u(k)) \\ \varphi(k) = \alpha |\tau(k)| \left[ \sum_{i=0}^{p} f_i u^i (k-1) - \hat{h}(u(k-1)) \right] \\ + \tau(k) \sum_{j=0}^{q} g_j u^j (k-1), \end{cases}$$
(40)

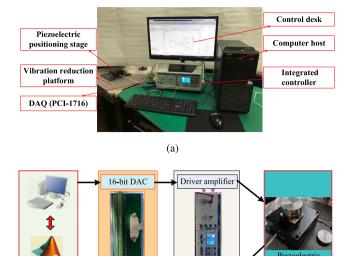


Fig. 5. Experimental setup. (a) picture of experimental platform, (b) structure diagram of experimental stage.

(b)

Integrated controller

16-bit ADC

DAO (PCI-1716)

nanopositioning

where  $\varphi(k) = \hat{h}(u(k)) - \hat{h}(u(k-1))$ ,  $\tau(k) = u(k) - u(k-1)$ , p = 3, q = 2, k represents the time state. In this study, to quantify the performance of the modeling and controller, we define the maximum (MAX), mean absolute (MA), and root-mean-square (RMS) error as follows:

$$RMS = \sqrt{\sum_{k=1}^{\eth} \frac{e^2(k)}{\eth}},$$
(41)

$$MAX = \max |e(k)|, \qquad (42)$$

$$MA = \frac{\sum_{k=1}^{\delta} |e(k)|}{\delta},$$
(43)

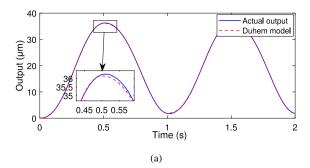
where ð is the number of datasets.

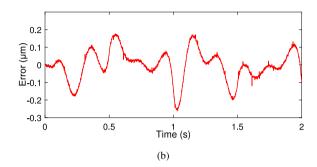
Then, we employ the interior point method in the MATLAB toolbox to obtain exact model parameters. The identified results of the Duhem model are shown in Fig. 6, and it is obvious that the Duhem model accurately captures the output of the actual PNS. The RMS error and MAX error of the identified Duhem model are 0.0761  $\mu m$  and 0.69%, respectively.

## 5. Experimental study and discussion

The performance of the proposed FOSMC method is evaluated by a series of experiments. For comparison purposes, the PID and SMC methods are likewise implemented as following formulae (44) and (45). The parameters of these three controllers are obtained using the trial-and-error method. In this process, with the increase of the parameter  $\lambda$ , the tracking error will decrease, but too large  $\lambda$  will also produce system shocks; in addition, the fractional order r should be balanced with the parameters  $\lambda$ . The parameters  $k_1$ ,  $k_2$ ,  $\eta$  are selected based on the lump disturbance. For the parameters  $\varepsilon_1$  and  $\varepsilon_1$ , we design that  $\varepsilon_1 = 1.00001$  and  $\varepsilon_2 = 0.99999$ . Then, while keeping the  $\varepsilon_2$  unchanged, increase the value of  $\varepsilon_1$  until the FOSMC system produces a slight chattering. At this point, reduce the value of  $\varepsilon_2$  to mitigate system chattering. According to the above rules, the parameters of these control methods are given in Table 1.

$$u_{\text{pid}}(t) = k_p e(t) + k_i \int e(t)dt + k_d \dot{e}(t), \tag{44}$$





**Fig. 6.** Real output of the PNS and output of the Duhem model under sinusoidal inputs. (a) model curves. (b) model error.

Table 1
Parameters of three different controllers.

Controller	Parameters	Value	
PID	<i>k</i> <sub>p</sub>	0.5030	
	$k_i$	1.1358	
	$k_d$	0.0015	
SMC	λ	2500	
	η	0.225	
FOSMC	λ	2000	
	r	0.6	
	$\epsilon_1/\epsilon_2$	2.1/0.9	
	$\delta_{A_1}/\delta_{A_2}$	0.1/0.01	
	$rac{\epsilon_1/\epsilon_2}{\delta_{f A_1}/\delta_{f A_2}} \ rac{k_1/k_2}{k_1/k_2}$	400/300	

where  $k_p$ ,  $k_i$ , and  $k_d$  represent the proportional, integral, and differential coefficients, respectively.

$$u_{\rm smc}(t) = \frac{1}{a_1} [\dot{x}_d(t) - a_0 x(t) - a_2 \hbar(t) + \lambda e(t)] + \eta \operatorname{sgn}(\sigma(t)). \tag{45}$$

where  $\eta$  is the switching gain, it is a constant.

## 5.1. Tracking experimental results of sinusoidal wave signal

To demonstrate the effectiveness of the proposed control scheme, the sinusoidal wave signal is used as the desired trajectory, which is denoted as  $x_{\rm dsin}(t)=18{\rm sin}(2\pi ft-0.5\pi)+18$  (f=1,50,100 Hz). For the purpose of comparison, we conducted tracking experiments with the other two types of controllers. Tracking results under different frequencies are presented in Figs. 7–9. It notes that the output displacement of the PNS is consistent with the desired trajectory, and the hysteresis effect and the lump disturbance are eliminated using the proposed controllers. The proposed FOSMC method was found to have the lowest positioning error, while the PID controller generates the maximum tracking error under different frequencies. When the frequency of the reference signal is 100 Hz, the RMS tracking errors of the PID, SMC, and FOSMC methods are 1.1520, 0.7766, and 0.4704  $\mu$ m, respectively. Evidently, the tracking performance of the proposed FOSMC method is improved by 59.17% and 39.42% compared with the PID and SMC

0.08

0.08

0.08

Ref.

PID

SMC

PID

SMC

PID SMC

**FOSMC** 

0.1

FOSMC

0.1

FOSMO

0.1

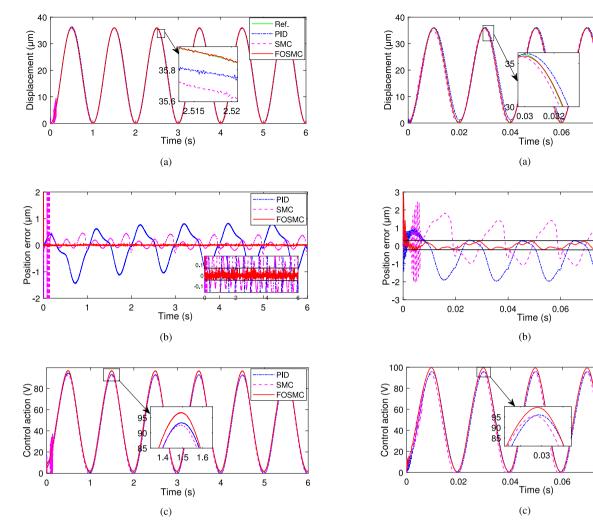


Fig. 7. Sinusoidal trajectory tracking results of (a) tracking curves, (b) error curve, and (c) control action with 1 Hz signal rate with proposed controller.

and (e) control action with 1 122 signal rate with proposed controller.

 $\begin{tabular}{ll} \textbf{Table 2} \\ \textbf{Tracking results of three different controllers under sinusoidal wave signal.} \\ \end{tabular}$ 

Frequency (Hz)	RMS/MAX of PID (µm/%)	RMS/MAX of SMC (µm/%)	RMS/MAX of FOSMC (µm/%)
f = 1	0.4772/2.64	0.1503/1.07	0.0064/0.19
f = 50	0.9100/4.46	0.7133/3.34	0.1409/0.76
f = 100	1.1520/5.81	0.7766/5.04	0.4704/2.48

method, respectively. The experimental data obtained with the elaborated controller are given in Table 2 and Fig. 10, where the trajectory tracking of the PNS is obtained with a great level of precision.

The control actions of different controllers are shown in Figs. 7(c)–9(c). When the frequency is 100 Hz, the control action of the SMC method produces some chattering, and the proposed FOSMC method has the smooth control actions from the magnified portion of Fig. 9(c). Hence, the chattering problem of the traditional SMC method is effectively resolved. The comparison results between the proposed FOSMC method and the ones in the literature are given in Table 3. The MAX tracking errors of the proposed FOSMC method are improved by more than 60% in comparison with existing control strategies.

## 5.2. Tracking experimental results of complex triangular wave

To further verify the control performance of FOSMC method, a complex triangular wave is employed as the desired reference trajectory.

Fig. 8. Sinusoidal trajectory tracking results of (a) tracking curves, (b) error curve, and (c) control action with 50 Hz signal rate with proposed controller.

 Table 3

 Comparison between FOSMC method and existing controller.

Method	Frequency (Hz)	MAX (%)	MAX of FOSMC (%)	Improved (%)
Controller in [5]	f = 1	1.20	0.19	84.17
Controller in [7]	f = 15	0.98	0.35	64.29
Controller in [19]	f = 50	2.11	0.76	63.98
Controller in [23]	f = 1 and 5	0.94	0.18	80.85

Fig. 11 depicts the experiment results by using the different controller. As shown in Fig. 11, the FOSMC shows the prominent control effect for tracking complex triangular wave trajectory. The RMS errors of SMC and FOSMC methods are 0.0331 and 0.0109  $\mu m$ , respectively. It is obvious that the tracking precision is greatly improved by FOSMC method. In addition, Fig. 11(c) shows that the sliding mode surface is constrained and the FOSMC drives the sliding mode surface to the range of  $\pm 0.4775~\mu m$ . However, the SMC drives the sliding mode surface to the range of  $\pm 0.8446~\mu m$ . In comparison with SMC, the proposed FOSMC method mitigates the sliding mode surface boundary by 43.46%.

## 5.3. Tracking experimental results with some different loads

To test the robustness of the proposed FOSMC approach, we carried out tracking control experiments by the PNS with different loads. In this experiment, the external loads are applied by five different weights.

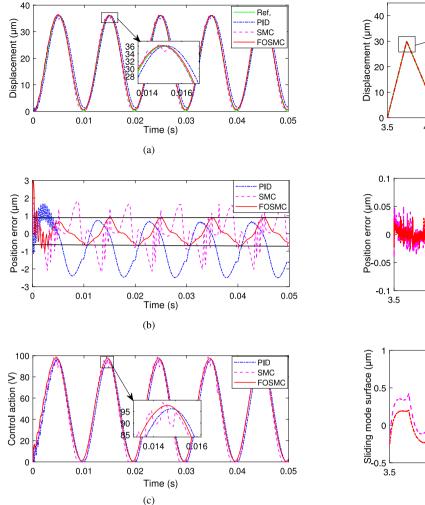


Fig. 9. Sinusoidal trajectory tracking results of (a) tracking curves, (b) error curve, and (c) control action with 100 Hz signal rate with proposed controller.

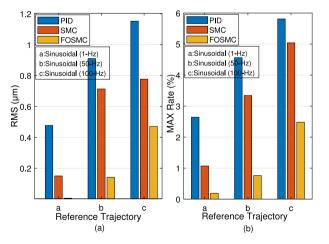


Fig. 10. Comparative tracking results of PID, SMC, and proposed FOSMC method. (a) RMS errors. (b) MAX rate.

With the reference signal as a 1 Hz sinusoidal wave signal, we mounted loads of 0, 100, 200, 300, 400, and 500 g onto the PNS to conduct the tracking experiments by the proposed FOSMC method. The MA errors and RMS tracking errors with different loads of the proposed control

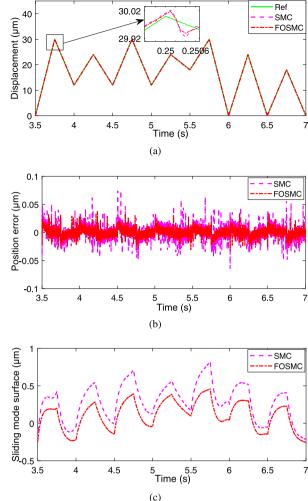


Fig. 11. Complex triangular wave trajectory tracking results of (a) tracking curves, (b) error curves, and (c) sliding mode surface.

approach barely change. The comparative experimental results are shown in Fig. 12, where the horizontal axis and vertical axis are the supporting weights and evaluation index. The supporting weights include 0, 100, 200, 300, 400, and 500 g weights. The evaluation index includes MA and RMS. From Fig. 12, when the frequency of the reference trajectory signal is 100 Hz, the fluctuation range of the MA and RMS errors, based on the FOSMC method, is from  $\pm 0.0041~\mu m$  and  $\pm 0.0059~\mu m$ . Thus, experimental results show that the developed FOSMC method is robust against the external load disturbance.

## 6. Conclusion and future prospects

A FOSMC scheme, based on a two-power reaching law, was developed in this study for application in PNS. The convergence and stability of the proposed controller are analyzed and verified by theoretical calculations. In contrast to the comment sliding mode surface, the FOSMS decays slower than that with the integer order, which improves the performance of the FOSMC method. The new two-power reaching law improves the robustness and convergence rate of the proposed controllers. The performance of the proposed FOSMC scheme has been validated by comparative experiments. Results show that the FOSMC method outperforms the PID and SMC methods. Moreover, load experiments demonstrate high robustness of the FOSMC method. From comparative experimental results, the percentage improvement in RMS

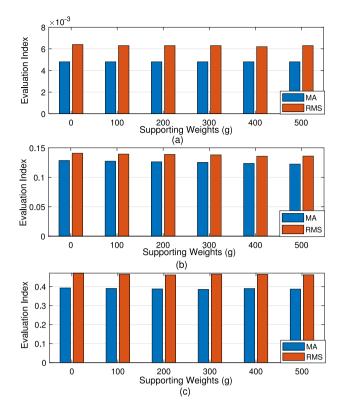


Fig. 12. Tracking experimental results of proposed controller with different loads. (a) 1 Hz, (b) 50 Hz, (c) 100 Hz.

error from SMC to FOSMC is 80.25% when the reference signal is sinusoidal trajectory with 50 Hz.

However, the parameters of the proposed controller are obtained by trial and error, which makes the design of controller parameters difficult. In the future, we will focus on the parameter tuning method of the FOSMC strategy and reduce this difficulty in obtaining controller parameters. In addition, the proposed FOSMC method is ultimately bounded stability. The appropriate adaptive law will be designed to estimate the upper bound of the lump disturbance and reduce the system stabilization error bounds.

## CRediT authorship contribution statement

Rui Xu: Conceptualization, Methodology, Validation, Software, Writing – original draft. Zhongshi Wang: Formal analysis. Miaolei Zhou: Data curation, Editing. Dapeng Tian: Supervision, Project administration.

## Declaration of competing interest

No potential conflict of interest was reported by the authors.

## Data availability

Data will be made available on request

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