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Optimal Control of Passive Cascaded Liquid Crystal Polarization Gratings

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Abstract: The incidence angle of a beam entering a liquid crystal variable retarder (LCVR) and the liquid crystal polarization grating (LCPG) is greater when the LCPG is utilized in the quadrature cascade, which reduces the diffraction efficiency. Therefore, this study investigates the best method for controlling a cascaded LCPG. By optimizing the control coefficient and the method for controlling the voltage of the LCVR, this study reduces the angle between the incident LCVR and the polarization grating, and increases the diffraction efficiency of the grating. It was confirmed that this strategy can increase the diffraction effectiveness of the quadrature cascaded LCPG through the design of the experimental setup.

Keywords: liquid crystal polarization grating; liquid crystal variable retarder; diffraction efficiency; quadrature cascading; optimized control

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1. Introduction

In contrast to other mechanical deflection devices, a Liquid Crystal Polarization Grating (LCPG) is a device that introduces a continuous gradient tilt phase distribution based on the geometric phase to produce beam deflection [1,2]. It has the advantages of being tiny, light, and having low power consumption. In contrast to non-mechanical beam deflection devices such as liquid crystal space light modulators, fiber phased arrays, and optical waveguide phased arrays, which have the advantage of large deflection angles, orthogonal cascaded liquid crystal polarization gratings (OC-LCPGSs) can achieve two-dimensional deflection of beams with a large angle range and small angle intervals. There are numerous potential applications in industries including liquid crystal displays, laser communication, and others [3].

The orthogonal cascade polarization grating's diffraction efficiency is a crucial parameter, and it decreases as the deflection angle increases. Diffraction efficiency refers to the ratio of light intensity in a certain diffraction direction to the incident light intensity. To address this issue, numerous researchers both domestically and internationally have conducted studies on how to use light control orientation technology and new light control orientation materials to improve LCPG's diffraction efficiency. The field of view of LCPG was expanded by the Hong Kong University of Science and Technology, who investigated the diffraction efficiency of LCPG at oblique incidence, examined the situation when converting the conventional oblique incidence LCPG to positive incidence distortion LCPG based on the equivalent distortion model at oblique incidence, and designed a double distortion structure similar to the former based on the theory of double layer compensation. However, there have been no reports of enhancing the liquid crystal phase delayer's control voltage and control coefficient to enhance the diffraction effectiveness of cascaded polarization gratings [4].

In accordance with the theory of the liquid crystal polarization grating (LCPG) and liquid crystal variable retarder (LCVR) [5–7], the effect of the angle of incidence on the liquid crystal variable retarder is investigated, and the phase delay of the liquid crystal variable retarder (LCVR) controlled by different incidence angles is obtained.

2. Theory

2.1. Diffraction Characteristics of Monolithic Liquid Crystal Polarization Grating

To achieve beam deflection, a liquid crystal polarization grating introduces a continuous gradient tilt phase distribution based on the geometric phase. A planar uniaxial birefringent liquid crystal molecule, which is the same as a grating, makes up the structure of a monolithic LCPG. The liquid crystal molecule vector varies periodically with its position [8], and its structure is depicted in Figure 1.

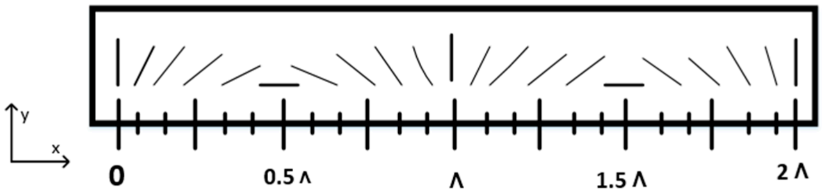


Figure 1. Structure of a monolithic LCPG.

The LC director of its two polarization states is as follows:

$$n(y) = [\sin(y/\Lambda), \cos(y/\Lambda), 0] \quad (1)$$

wherein Λ is the grating period, and the azimuth of the liquid crystal molecular vector $\varphi = \pi y/\Lambda$ linearly changes periodically along the y axis [9].

The ideal diffraction efficiency at positive incidence can be derived from the Jones matrix [10]:

$$\begin{aligned} \eta_0 &= \cos^2\left(\frac{\pi \Delta n d}{\lambda}\right) \\ \eta_{\pm 1} &= \frac{1 \mp S_3'}{2} \sin\left(\frac{\pi \Delta n d}{\lambda}\right) \end{aligned} \quad (2)$$

where η_m is the m-class diffraction efficiency and $S_3{}' = S_3/S_0$ is the normalized Stokes parameter. From Equation (2) it can be seen that the liquid crystal polarization grating has only three diffraction stages: 0 and ±1, with the intensity distribution between the diffraction stages depending on the phase delay δ and incident polarization. When the birefringence phase retarder of the liquid crystal retarder is an odd multiple of π, the zero-level diffraction will be zero and all light will be biased to ±1 level. When the incident light is a right-handed circular polarized light, $S_3{}' = -1$, then the diffraction efficiency will be $\eta_{+1} = 1$ and $\eta_{-1} = 1$, all light passing through the LPG is diffracted to +1 level, as shown in Figure 2a. When the incident light is an L-circle-polarized light, $S_3{}' = +1$, all light is diffracted to −1 level, as shown in Figure 2b. When the LCPG is at a high voltage, the incident light is transmitted along the axis (zero-level diffraction direction), as shown in Figure 2c [11]. It is theoretically possible to achieve a single stage with a diffraction efficiency of 100%, in fact; however, due to the influence of error, experimentally it is not.

The diffraction effectiveness of the cascaded LCPGs is affected by two factors. First, the adjustable 1/2 wave plate's real polarization state change, as per the original 0° incidence design, is no longer 1/2 wavelength. Second, the diffraction efficiency of the polarization grating is decreased because of the increase in incidence angle.

Li Tan created a liquid crystal polarization grating with a similar structure to the double twist layer, which makes the liquid crystal polarization grating in the 20° incidence field of view of the first level diffraction efficiency of more than 90%, thereby improving the diffraction efficiency. In fact, after design optimization and process improvement, the diffraction efficiency of the 0° beam into the monolithic polarization grating can reach 97% [4]. The relation curve between diffraction efficiency and incidence angle is shown in

Figure 3. According to the curve between the diffraction efficiency and the incidence angle, the diffraction efficiency will decrease with the increase in the incidence angle. According to its research results, the diffraction efficiency will decrease.

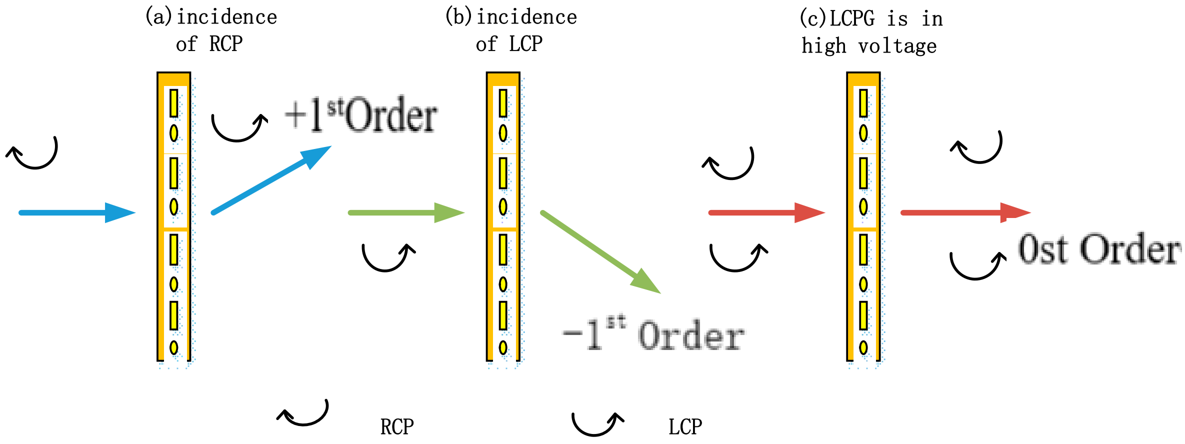


Figure 2. Diffraction characteristics of LCPG.

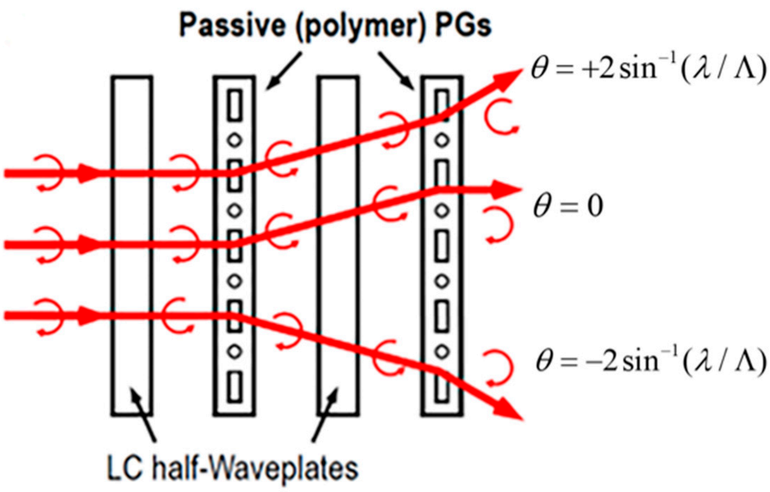


Figure 3. Two liquid crystal half-wave wafers and two LPGGs consist of an LCPG layer.

The polarization state of the incident light has an impact on the diffraction efficiency of the LCPG, in addition to the liquid crystal material and structure. The liquid crystal variable retarder is usually used for polarization modulation of incident light, and the ideal liquid crystal variable retarder ideal phase modulation amount is

$$\begin{aligned} \varphi(V_{idea}) &= \mathrm{d}(n_e - n_0)\tfrac{2\pi}{\lambda} \\ &= \pi \end{aligned} \tag{3}$$

When the angle of incidence is angled in the actual cascade situation, the incident adjustable half-beam wave angle is variable, and the following actual phase is produced:

$$\begin{aligned} \varphi_{\text{act}} &= \varphi(V_{idea})f(\theta_{in}) \\ &= \pi f(\theta_{in}) \end{aligned} \tag{4}$$

When $f(\theta_{in}) < 1$, the emitting light is elliptically polarized, and the Stokes parameter becomes

$$S_3' = \sin\left(\pm\frac{\pi}{2} + \pi f(\theta_{in})\right) \tag{5}$$

According to formula (2), the actual efficiency is

$$\eta_{\pm 1} = \frac{1 + \sin\left(\frac{\pi}{2} \pm \pi f(\theta_{in})\right)}{2} \sin\left(\frac{\pi \Delta n d}{\lambda}\right) \quad (6)$$

This implies that the diffraction efficiency is decreased by a change in the angle of incidence.

2.2. Diffraction Characteristics of Quadrature Cascaded Liquid Crystal Polarization Grating

A multi-layer LCPG combination can accomplish a wide range of deflection by cascading several LCPG layers together in a sequence. Each LCPG layer can achieve a fixed angle of deflection. By varying the phase of the adjustable liquid crystal half-wave, it is possible to deflect a "LCPG layer" made up of two identical passive LCPGs and two adjustable liquid crystal half-wave elements in series, as shown in Figure 3.

Cascaded LCPGs combine multiple LCPG layers in series. Each LCPG layer can achieve a fixed angle of deflection. Multilayer LCPGs can achieve deflection in a large angle range. The combination forms include binary, quasi-binary, and ternary. The most common combination form is binary.

The binary combination method for large angle deflection determines the resolution for the first layer, and the diffraction angle of each layer is twice that of the previous layer (for example: 1.25°, 2.5°, 5.0°, 10.0°, 20.0°), The total angle is an approximate integer multiple of the resolution. For an N-layer cascaded LCPG, 2^{N+1} angles can be controlled theoretically. In order to achieve 1.25° resolution and ±40° angle deflection control, a total of five layers of LCPG (2 × 40°/1.25° = 64, 2^{N+1} = 64, N = 5) are required. In order to achieve two-dimensional beam deflection, the same five-layer LCPGs are required in the orthogonal direction. The best series connection mode is: pitch 1.25°, azimuth 1.25°, pitch 2.5°, azimuth 2.5°

The control parameters include the layer control coefficient and slice control coefficient. The layer control coefficient mainly indicates that the deflection angle of a layer is 0°, positive or negative, and the values are 0, −1, or +1; The plate control coefficient mainly indicates that the deflection angle of a liquid crystal adjustable half-wave plate in the layer is positive and negative, and the values are −0.5 and +0.5. The middle layer control coefficient of an LCPG layer is the sum of two slice control coefficients.

For example, to achieve beam deflection with an azimuth angle of 22.5° and an elevation angle of −16.25°, the layer control coefficient and plate control coefficient of the adjustable liquid crystal half-wave plate in each layer of LCPG in the azimuth direction and elevation are shown in Table 1, Table 2 is the Orthogonal cascaded liquid crystal polarization gratings structural cloth.

Table 1. Layer control coefficient and plate control coefficient of adjustable liquid crystal half-wave plate.

Direction	**Controlparameters**	**Floor 1 (1.25°)**		**Floor 2 (2.5°)**		**Floor 3 (5.0°)**		**Floor 4 (10.0°)**		**Floor 5 (20.0°)**	
Totalpitch 22.5°	Floorcontrolcoefficient	0		1		0		0		1	
	Slicecontrol coefficient	0.5	0.5	0.5	0.5	0.5	0.5	−0.5	0.5	−0.5	0.5
Azimuthsummation −16.25°	Floorcontrol coefficient	−1		0		−1		−1		0	
	Slicecontrol coefficient	−0.5	−0.5	0.5	0.5	−0.5	−0.5	−0.5	−0.5	−0.5	0.5

Table 2. Orthogonal cascaded liquid crystal polarization gratings.

Floor 1 (1.25°)		**Floor 2 (2.5°)**		**Floor 3 (5.0°)**		**Floor 4 (10.0°)**		**Floor 5 (20.0°)**	
slice $_{11}$ (0.625°)	slice $_{12}$ (0.625°)	slice $_{21}$ (1.25°)	slice $_{22}$ (1.25°)	slice $_{31}$ (2.5°)	slice $_{32}$ (2.5°)	slice $_{41}$ (5.0°)	slice $_{42}$ (5.0°)	slice $_{51}$ (10.0°)	slice $_{52}$ (10.0°)

The total diffraction efficiency of orthogonal cascaded polarization gratings is the product of each liquid crystal polarization grating in the cascade structure.

2.3. Vertical Incident Liquid Crystal Adjustable Phase Delayer Phase Delay Theory

Figure 4 shows a schematic of the LCVR structure. The transparent electrode ITO is typically plated on the inside of two parallel quartz glass (S_iO_2) plates, followed by coating with a transparent material, PI. Finally, liquid crystal molecules were filled between the two parallel plates and enclosed in a bracket. When the drive voltage exceeds the critical voltage V_C of the LCVR, the LC molecules start to spin away from the plate, and the rotation angle depends on the drive voltage at both ends of the plate. In addition, the liquid crystal molecules do not rotate when the driving voltage is below the critical value V_C, or when the rotation angle is 0. Consequently, the driving voltage that the liquid crystal controller applies to the two ends of the LCVR can be altered to alter the phase delay.

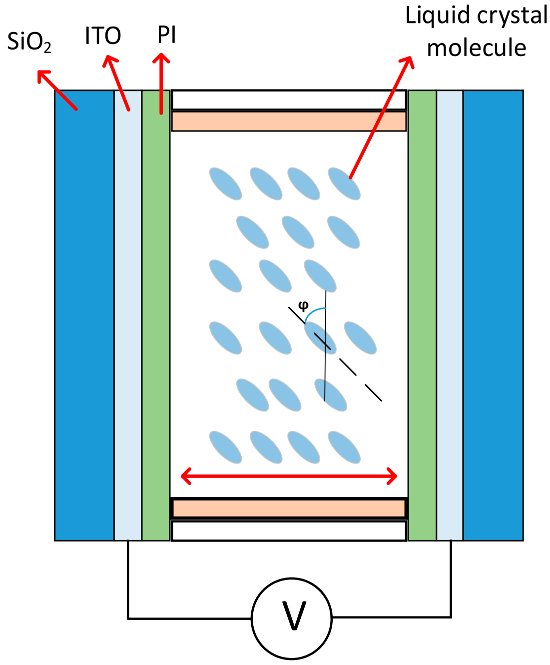


Figure 4. Schematic diagram of a typical liquid crystal adjustable phase delayer.

The relationship between the rotation angle φ of the liquid crystal molecule and the drive voltage V can be expressed as follows [12]:

$$\beta(V) = \begin{cases} \varphi_C; V \leq V_C \\ \frac{\pi}{2} - 2\tan^{-1}\exp\left[-\left(\frac{V-V_C}{V_0}\right)\right] + \varphi_C; V > V_C \end{cases} \tag{7}$$

where V_o is the voltage constant, φ_c is the initial rotation angle, V and V_c are the drive and critical voltages of the liquid crystal adjustable phase delayer, respectively.

When incident light is vertical, LCVR is frequently utilized. In this scenario, the phase delay of the light changes with the driving voltage and wavelength of the incident light, and the relationship between the two can be written as follows:

$$\delta = \frac{2\pi \cdot d \cdot \left[n_{ef}(\lambda, \varphi) - n_0\right]}{\lambda} \tag{8}$$

Here, $n_{ef}(\lambda, \varphi)$ represents the effective refractive index, which is related to the dispersion characteristics of the liquid crystal adjustable phase delayer and the liquid crystal driving voltage, n_0 is the refractive index of o light, d is the thickness of the liquid crystal layer, when the light is incident, the thickness d is equal to the optical path, and λ is the wavelength of the incident light.

$n_{ef}(\lambda, \varphi)$ is also related to the e-light refractive index n_e, o-light refractive index n_0, and tilt angle φ of liquid crystal molecules. The relationship between them can be expressed as follows [13,14]:

$$\frac{1}{n_{ef}^2(\lambda,\varphi)} = \frac{\cos^2\varphi}{n_e^2} + \frac{\sin^2\varphi}{n_0^2} \tag{9}$$

From vertical Equations (8) and (9), the phase delay LCVR quantity can be expressed as follows:

$$\delta = \frac{2\pi \cdot d}{\lambda} \cdot \frac{n_0 n_e}{\sqrt{n_e^2 + (n_0^2 - n_e^2)\cos^2\varphi}} \tag{10}$$

The amount of delay δ can be approximated as follows:

$$\delta = \frac{2\pi \cdot d}{\lambda} \cdot (n_e - n_0)\cos^2\varphi \tag{11}$$

$\Delta n(\lambda)$ is the birefringence of the liquid crystal layer, $\Delta n(\lambda) = n_e(\lambda) - n_0(\lambda)$, which is wavelength dependent and can be represented by the dispersion equation; its simplified form can be expressed as follows [15]:

$$\Delta n(\lambda) = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} \tag{12}$$

where a, b, and c represent the regression coefficients of the dispersion equation, and n_e and n_0 are related to wavelength. Therefore, δ' can be expressed as follows:

$$\begin{aligned} \delta' &= \tfrac{2\pi\cdot d}{\lambda} \cdot \Delta n(\lambda) \cdot \cos^2\varphi \\ &= \tfrac{2\pi\cdot d}{\lambda} \cdot \left(a + \tfrac{b}{\lambda^2} + \tfrac{c}{\lambda^4}\right) \cdot \cos^2\varphi \end{aligned} \tag{13}$$

2.4. Tilt Incident Liquid Crystal Adjustable Phase Delayer Phase Delay Theory

For an oblique incident, the degree of delay is influenced by the angle of incidence and azimuth of the incident light, in addition to the voltage and wavelength, in a coordinate system with oblique incident light. Figure 5a shows the geometric relationship between the angle of incidence of the beam and the guided axis of the liquid crystal molecule. The liquid crystal molecular guide axis is established according to the LCVR plane and its normal, as shown in the x-y-z coordinate system. The xoy plane is parallel to the phase delayer method plane; the xoy plane contains the liquid crystal molecular guide axis, and the angle between the x-axis is η.

For the incident light σ the angle of incidence is oblique to the LCVR plane, and refraction occurs at the interface between the air and the liquid crystal layer, as shown in Figure 5b. According to Snell's law, for the angle of refraction ξ with two components of ξ_o and ξ_{ef} is expressed as [16]

$$\begin{cases} \xi_o = \arcsin\left(\frac{\sin\sigma}{n_0}\right) \\ \xi_{ef} = \arcsin\left(\frac{\sin\sigma}{n_{ef}(\lambda,\phi)}\right) \end{cases} \tag{14}$$

where n_{ef} is the refractive index of e-light,

$$n_{ef} = \frac{n_0 n_e}{\sqrt{n_e^2\cos^2\tau + n_0^2\sin^2\tau}} \tag{15}$$

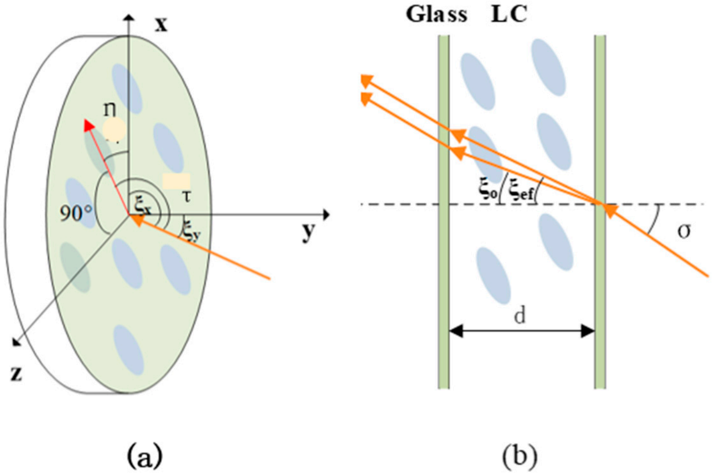


Figure 5. (**a**) Schematic diagram of the internal structure of LCVR under oblique incident light. (**b**) Schematic diagram of the refraction phenomenon that occurs when incident light is obliquely incident LCVR.

The angle between the wave normal line of e-light and the optical axis of liquid crystal molecule is τ. According to the literature [11], the angle is obtained by using the coordinate analysis method, with the general expression for τ as follows:

$$\tau = \arccos[\cos\varphi \sin\xi_x - \sin\varphi \cos\xi_x \cos\xi_y] \quad (16)$$

The optical path difference between o light and e light can be expressed as follows [17]:

$$\begin{aligned} d\delta &= \frac{2\pi d}{\lambda}\left(\frac{n_0}{\cos\xi_o} - \frac{n_e}{\cos\xi_{ef}}\right) \\ &\approx \frac{2\pi d}{\lambda} \cdot \Delta n(\lambda) \cdot \frac{\cos^2\tau}{\cos\xi_o} \\ &= \frac{2\pi d}{\lambda}\left(a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}\right) \cdot \frac{\cos^2\tau}{\cos\xi_o} \end{aligned} \quad (17)$$

Make formula (17) $d\delta = \pi$, then it can be calculated that the included angle between the molecular axis and the incident vector is τ, so the included angle η can be calculated by Formula (16). Finally, the driving voltage V can be calculated by Formula (7).

3. Optimal Control Method of the Orthogonal Cascaded LCPG

Assume that the total angles ($M\Delta\theta$, $N\Delta\theta$), M and N are decimal deflection multiples of pitch and azimuth directions of orthogonal cascaded polarization gratings, where $\Delta\theta$ is the angular resolution. The main control steps are as follows:

Step 1: binarize the angle to be achieved, and calculate the layer control coefficients required for each layer of the cascaded liquid crystal polarization gratings with different deflection angles.

Let M and N be converted into binary deflection multiples to obtain the weights of different bits in the meridian direction: n_1, n_3, n_5, n_7, n_9 and the weights of different bits in the sagittal direction: n_2, n_4, n_6, n_8, n_{10}.

Relationship between n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10} and M, N:

$$\begin{cases} M = n_1 + 2n_3 + 4n_5 + 8n_7 + 16n_9 \\ N = n_2 + 2n_4 + 4n_6 + 8n_8 + 16n_{10} \end{cases} \quad (18)$$

Step 2: calculate the plate control coefficient of the liquid crystal adjustable half-wave plate.

The plate control coefficient of the liquid crystal adjustable half-wave plate cannot be given randomly, as shown in the following figure. With the liquid crystal polarization grating layer, the incidence angle of the first liquid crystal polarization grating is ω (ω is positive), and the incident angle of the second liquid crystal polarization grating can be $(\omega+\theta,\beta)$. It can also be $(\omega-\theta,\beta)$. The diffraction efficiency of the liquid crystal polarization grating is different at different incidence angles, and the diffraction efficiency is higher at small incidence angles. Therefore, when the control coefficient of the liquid crystal adjustable half-wave plate in front of the first liquid crystal polarization grating deflects the first liquid crystal polarization grating, $-\theta$. In this way, the diffraction efficiency is better. Figure 6 is the Optimization control coefficient rendering.

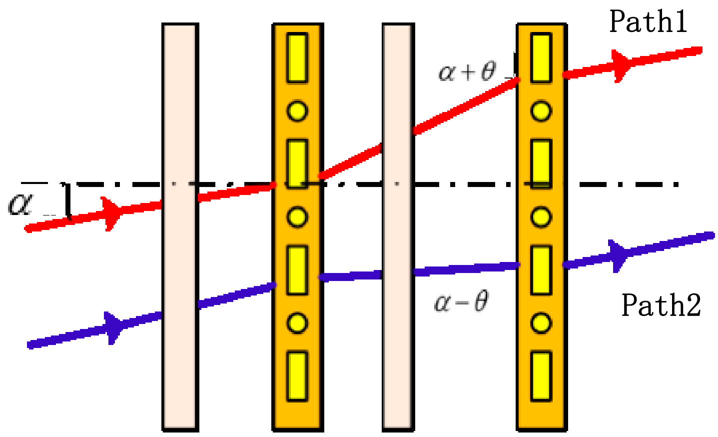


Figure 6. Optimization control coefficient rendering.

Two control coefficients n_{i1} and n_{i2} need to be determined for each layer of passively cascaded liquid crystal polarization gratings. n_{i1}, n_{i2} as defined as follows:

$$n_{i1}+n_{i2}=n_i \tag{19}$$

For the first and second layers, since the incidence angle is 0° and the layer control is 0°, the control coefficient of the first liquid crystal adjustable half-wave plate can be set to be positive, that is, n_{11} = 0.5, n_{12} = −0.5, n_{21} = 0.5, and n_{22} = −0.5.

For layers with $i \geq 3$ and $n_i = 0$, when the total angle is positive, n_{i1} = −0.5, n_{i2} = 0.5. When the total angle is negative, n_{i1} = 0.5, n_{i2} = 0.5. Calculate the incidence angle of each liquid crystal adjustable half-wave plate (α_{ij},β_{ij}), where i represents the serial number of the layer, 1~10 in the example, and j represents the serial number of two liquid crystal adjustable half-wave plates in the LCPG layer, 1 or 2:

$$\begin{cases} \alpha_{11}=0 \\ \alpha_{12}=\frac{\Delta\theta}{2} \\ \beta_{11}=0 \\ \beta_{12}=\frac{\Delta\theta}{2} \\ \alpha_{i1}=\sum\limits_{j=1}^{i-1}\left(2^{2j-2}\cdot n_{2j-1}\right)\Delta\theta \\ \alpha_{i2}=\sum\limits_{j=1}^{i-1}\left(2^{2j-2}\cdot n_{2j-1}\right)\Delta\theta+n_{i1}\Delta\theta \\ \beta_{i1}=\sum\limits_{j=1}^{i-1}\left(2^{2j-1}\cdot n_{2j}\right)\Delta\theta \\ \beta_{i2}=\sum\limits_{j=1}^{i-1}\left(2^{2j-1}\cdot n_{2j}\right)\Delta\theta+n_{i1}\Delta\theta \end{cases} \tag{20}$$

Step 3: calculate the power application coefficient m_{ij} of each liquid crystal adjustable half-wave plate piece by piece from the transmitting end (m_{ij} is taken as 0 or 1, and 1 is the power application) and the optimal power application voltage V_{ij}.

When the incident light is right-handed circularly polarized light, the application coefficient m_{11} = 0; when the incident light is left-handed circularly polarized light, the application coefficient m_{11} = 1, and other application coefficients are as follows:

$$\begin{cases} m_{11} = 0 \\ m_{12} = \mathrm{mod}(2\mathrm{n}_{11} + 2n_{12}) \\ m_{\mathrm{i}1} = \mathrm{mod}\left(2\mathrm{n}_{(\mathrm{i}-1)2} + 2n_{21}\right) \\ m_{\mathrm{i}2} = \mathrm{mod}(2\mathrm{n}_{\mathrm{i}1} + 2n_{22}) \end{cases} \quad (21)$$

Step 4: according to the electric application coefficients m_{i1} and m_{i2} of each liquid crystal adjustable half-wave plate and the incident light angle (α_{ij},β_{ij}), calculate the control voltage of each liquid crystal adjustable half-wave plate, so that the outgoing light of each layer is standard circularly polarized light.

The incident angle is obtained by simultaneous calculation of Equations (15) and (17) (α_{ij},β_{ij}) angle with molecular axis:

$$\frac{\cos(\sqrt{\alpha_{ij}^2 + \beta_{ij}^2})}{\left(\cos\psi_{ij}\sin\alpha_{ij}^2 - \sin\psi\cos\alpha_{ij}\cos\beta_{ij}\right)^2} = \frac{2d}{\lambda}\cdot(a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}) \quad (22)$$

The corresponding molecular axis angle of each adjustable half-wave plate can be calculated as ψ_{ij}.

Through the angle of molecular axis ψ_{ij} the optimal control voltage V_{ij} can be obtained by combining it with the liquid crystal driving voltage Formula (7).

4. Diffraction Efficiency Test of the Quadrature Cascaded LCPG

4.1. Experiment on Optical Efficiency Optimization Control Technique

The test was conducted based on the idea of maximizing the diffraction effectiveness of the cascaded LCPGs. The test system is shown in Figure 7. The polarization state of the light emitted after the LCVR is further modulated, acting as angle modulation, by controlling the deflection angle of liquid crystal molecules inside the LCVR by controlling the voltage modulation of the LCVR. To increase the diffraction effectiveness of the cascaded LCPG at large modulation angles, the control voltage of each liquid crystal tunable phase retarder was optimized for different angles.

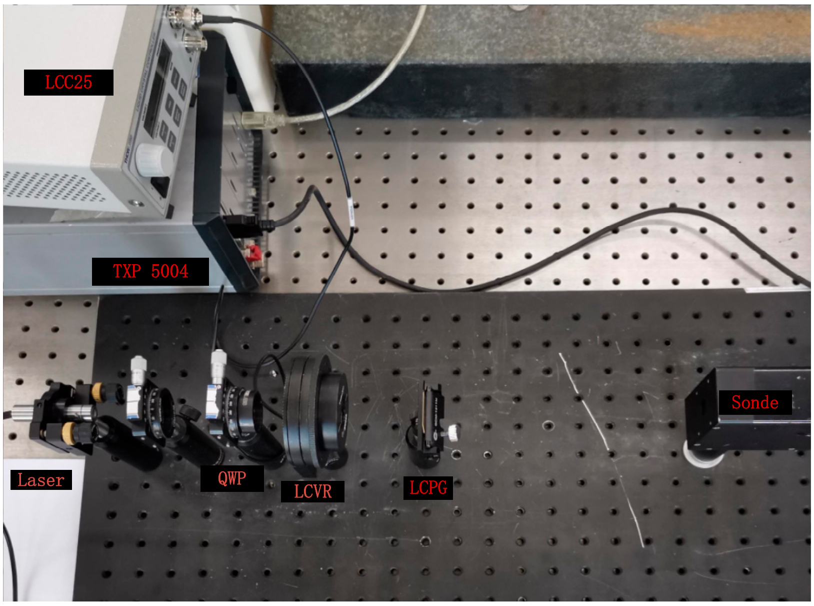
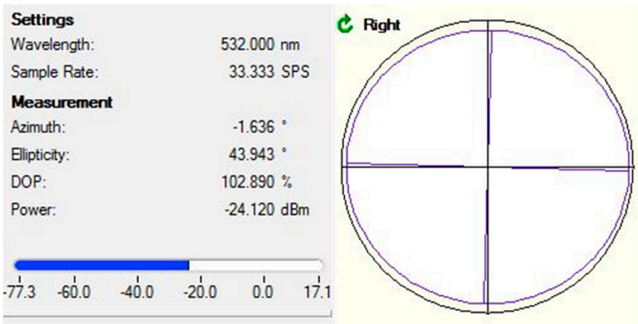
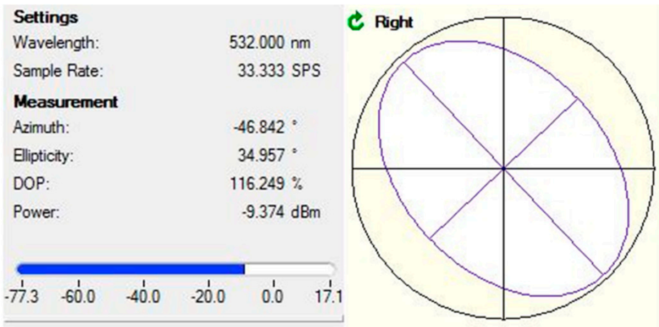


Figure 7. Test System Diagram.

When the beam is vertically incident on the LCVR, as illustrated in Figure 8a, the circularly polarized light incident on the LCPG is obtained, and its diffraction efficiency is the best. The cascaded LCPGs are shown in Figure 8b, where it is shown that the beam will slant into the LCVR to obtain elliptically polarized light before injecting it into the LCPG, at which point the diffraction efficiency decreases. The circularly polarized outgoing light is created by modifying the voltage put into the liquid crystal adjustable half-wave plate, and is subsequently incident on the liquid crystal polarized grating, at which point the diffraction efficiency recovers, as shown in Figure 8c. That is, the diffraction effectiveness of the cascaded LCPGs at large modulation angles is enhanced by optimizing the control voltage of each liquid crystal tunable phase retarder.



(**a**)



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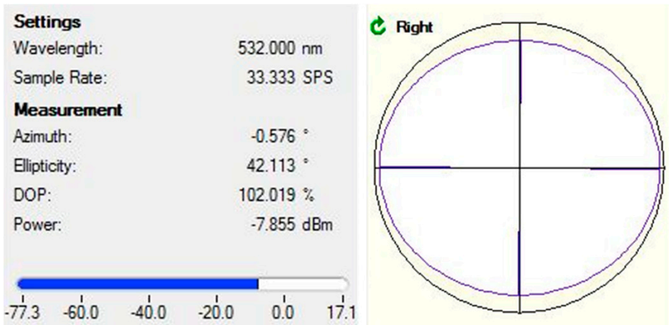

(**c**)

Figure 8. (**a**) Circularly polarized light obtained by vertically incident LCVR. (**b**) Elliptically polarized light obtained by oblique incidence. (**c**) Optimizing the control voltage to obtain circularly polarized light.

Figure 8 illustrates how to maximize the management of the LCVR voltage while reducing the diffraction efficiency under ideal diffraction efficiency and oblique incidence to increase the diffraction efficiency. The theoretical and experimental findings were consistent. This demonstrates the efficiency and viability of the LCPG-based cascaded optical efficiency optimization control approach.

4.2. Diffraction Efficiency Test of the Orthogonal Cascaded LCPG

An experimental test system was developed using a laser, polarization state generator, 1/4 wave plate, LCVR, LCPG, and an optical power meter to confirm the efficacy of the proposed approach, as shown in Figure 9. Table 3 presents the fundamental characteristics of the experimental apparatus. The angle of incidence of the incident light can be adjusted by turning the scale chassis while the LCVR is fixed to a base marked with a scale.

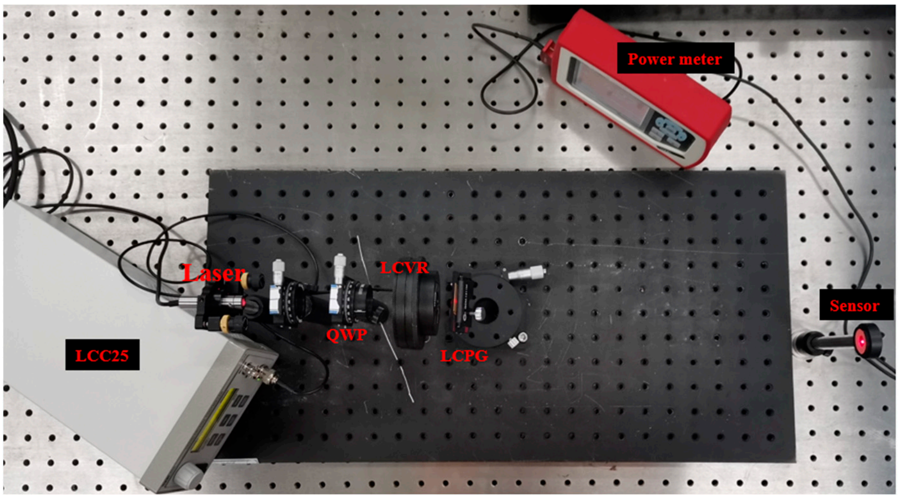

Figure 9. Diagram of the experimental apparatus.

Table 3. Basic parameters of the experimental device.

Serial Number	Devices	Parameter
1	Laser	The wavelength 532.8 nm Spot diameter < 2 mm
2	Polarizer	Parallel single polarized light transmittance > 70%@533 nm
3	1/4 wave plate	The wavelength 532.8 nm size ϕ25.4
4	LCVR	LCC25
5	LCPG	5 degrees deflection The efficiency of 98%
6	Optical power meter	Power test, no more than 0–5000 mw/cm^2

A rotary table was used to control the injection of the laser beam into the liquid crystal adjustable phase delayer at various angles. The voltage of the liquid crystal adjustable phase delayer was then adjusted to achieve the best voltage corresponding to the various angles of incidence, and the exiting beam was then passed through the liquid crystal adjustable phase delayer. The light intensity before and after the optimization control of the LCPG was investigated to confirm that the optimal control approach may enhance the diffraction efficiency of the cascaded LCPG.

The transmissivity test results before and after optimal control were obtained by simulating multiple measurements of liquid crystal polarization gratings with various angles of incidence, as shown in Figure 10. The above method was used to improve the diffraction efficiency of LCPGs at different incident angles.

Figure 10. Diffraction efficiency diagram before and after optimization control.

By adjusting the liquid crystal adjustable phase retarder voltage under various deflection angles, the diffraction efficiency of the cascaded LCPGs can be increased (as illustrated by the experimental findings in Figure 8). It has been demonstrated that the technique described in this study can increase the diffraction efficiency of high-order LCPGs, and the increase can be as high as 10% depending on the deflection angle.

5. Summary and Discussion

The diffraction efficiency of the orthogonal cascaded LCPGs decreased with an increase in the deflection angle. To address this issue, many studies have been conducted on how to use light-controlled orientation technology and new light-controlled orientation materials to improve the diffraction efficiency of LCPGs. In this study, a liquid crystal tunable phase retarder was introduced in front of a tilted LCPG. By tuning the control voltage of the liquid crystal tunable phase retarder, the diffraction efficiency was increased in the case of large-angle deflection. The experimental results demonstrate that by adjusting the voltage of the liquid crystal phase retarder at various deflection angles, the diffraction efficiency can be increased. It has laid the foundation for realizing non mechanical beam deflection with large angle, high accuracy, and high diffraction efficiency. It is believed that the excellent characteristics of liquid crystal polarization grating will certainly shine in the future communication or display fields.

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