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REGULAR PAPER

Optical system design of fully symmetrical Fourier transform lens

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Abstract

The design requirements of the Fourier transform optical system required for synthetic aperture radar(SAR) are analyzed and the specific optical parameters are determined. The design of the Fourier lens adopts a fully symmetrical structure to avoid simultaneous optimization of the forward and reverse directions of the optical path and can complete the correction of the conjugate positional aberrations of the two pairs of object images. A Fourier lens composed of four spherical mirrors was designed and reached the diffraction limit. To verify that the optical lens can realize the Fourier transform, a 4f optical system was built in the laboratory using the lens. First, the 4f system built by the optical element was demonstrated to be able to process the signal by demodulating a one-dimensional linear frequency modulation(LFM) signal. Then, the echo signal of the SAR point target is demodulated, Fourier transform is realized by the lens, and the imaging of SAR single point target signal is completed, which plays a role in the practical application of the fully symmetrical Fourier optical lens.

Keywords Fourier transform · Synthetic aperture radar · Symmetrical structure · Fourier lens · 4f optical system

1 Introduction

Optical information processing mainly uses optical methods to realize various transformations or processing of input information [13]. The input information can be light information, such as an image recorded on a photosensitive film, which uses a coherent light source or an incoherent light source to illuminate the picture, and the input information is expressed in the form of spatial modulation of light intensity. It can also be an electrical signal or an acoustic signal (such as radar or sonar), but it needs to use related electro-optical or acoustic-optical devices to convert them into optical signals and enter the optical system. Various transformation operations can be realized by optical methods [7, 10]. For example, Fresnel diffraction can realize Fresnel transformation, and Fraunhofer diffraction produced by the lens can realize Fourier transformation [3].

Because SAR works in the microwave band, it has the ability to work around the clock and acquire target information in a way that traditional optical imaging does not [2, 15]. However, the SAR data volume is large [14], and the data model involved in the calculation during transmission, compression, storage, and processing is huge, and the digital signal processing relies on high-performance processing chips, which is both time-consuming and energy-consuming, and has greater difficulties for real-time imaging of satellite-based SAR [4, 9].

The core calculation of SAR is the Fourier transform, and one of the most fundamental mathematical operations in optical information processing is the Fourier transform [8]. Optical processing possesses inherently parallel computation and implements the Fourier transform at the speed of light [1, 6]. Optical lenses can be used both as imaging and information transfer tools and as computational elements, which have the ability to perform Fourier transforms [12], and the use of lenses instead of computers to process this redundant information in a way that the processing speed is theoretically related only to the optical path, allowing for real-time data processing [18].

To make the system compact and has high resolution imaging capabilities, Fourier transforms lens with a short focal length; in order to improve resolution, the use of smaller image elements of the spatial light modulator (SLM)

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as the objective [19, 20], which requires a Fourier transform lens with a large field of view. Fourier-transformed lenses with a short focal length and a large field of view generally use asymmetric 5 or more spherical lens structure form.

The characteristics of Fourier transform lenses for diffracted light imaging dictate that the lenses meet the following design requirements [11, 16, 17]: (1) Control the aberration of the conjugate position of two pairs of object images. The first pair of object-image co-choke positions:an object at infinity, the diaphragm on the front focal plane, and the image on the back focal plane. The second pair of object-image co-choke position: an object in the front focal plane, the diaphragm in the back focal plane, the image at infinity. (2) The primary light rays that exit parallel to the optical axis satisfy the sine condition. (3) Eliminate all kinds of monochromatic aberrations, achieve diffraction-limited, and wave aberration less than 0.25λ in the whole field of view.

This paper analyzes the design requirements of the Fourier transform the optical system and the determination of optical parameters in the SAR echo signal real-time imaging system, and adopts a fully symmetric approach to the design, thus realizing the control of aberration in the co-choke position of two pairs of objects, thus avoiding the trouble of optimizing the forward and reverse optical paths of the Fourier transform lens at the same time using the method of multiple structures. Finally, the design results of a set of Fourier lenses with four spherical structures of focal length 125 mm meeting the diffraction limit requirement are given, and successfully applied to the SAR processing optical system, and good imaging results are obtained.

2 Determination of the optical parameters of the Fourier transform lens

Optical information processing is usually done with the help of a 4f system, as shown in Fig. 1 is a simple 4f system schematic, there are two focal lengths are f_1 and f_2 lens composition, plane 1, plane 2, plane 3 are object plane, spectrum plane, and image plane respectively. There is a one-to-one correspondence between the image and the spatial spectrum transmitted by the system. The degree of similarity between image and object depends on how many frequency components of the object can be transmitted to the image plane by the system. The structure of the output image will change when different devices such as slits or small holes are placed in the spectral plane 2 to change the spectrum. In this paper, SLMs are placed on the object plane 1 and the spectrum plane 2, and the SLM on the object plane are loaded with the information to be processed and the SLM on the spectrum plane are loaded with the filtering function. The information on the object plane is propagated to the spectrum plane through the first Fourier lens and interacts with the filter function on the spectrum plane. The filtered information is then propagated to the image plane through the second Fourier lens and received by the complementary metal oxide semiconductor(CMOS). The Fourier lens is the main component of the 4f optical system, and its function is to realize the Fourier transform. The ability of the lens to accurately realize the Fourier transform is the basis for the high-quality processing of the 4f system. For this reason, the Fourier lens needs to be specially designed to meet the standard of use.

The parameters of the Fourier lens are related to the specifications of the SLM placed on the object plane 1 and the spectrum plane 2.The pixel size of the input surface SLM 1 is 8 μm,and the number of pixels is 1980*1080, so the diagonal length is about Φ=18 mm. Since the structure is fully symmetric, the same size of SLM is adopted in the spectral plane, and the detector is selected as long as the pixel size is smaller than the pixel size of SLM and the size is larger than the size of SLM. According to the device parameters and experimental measurement parameters provided by the spatial light modulator manufacturer, the wavelength of 532 nm has the best linear modulation performance, so the green

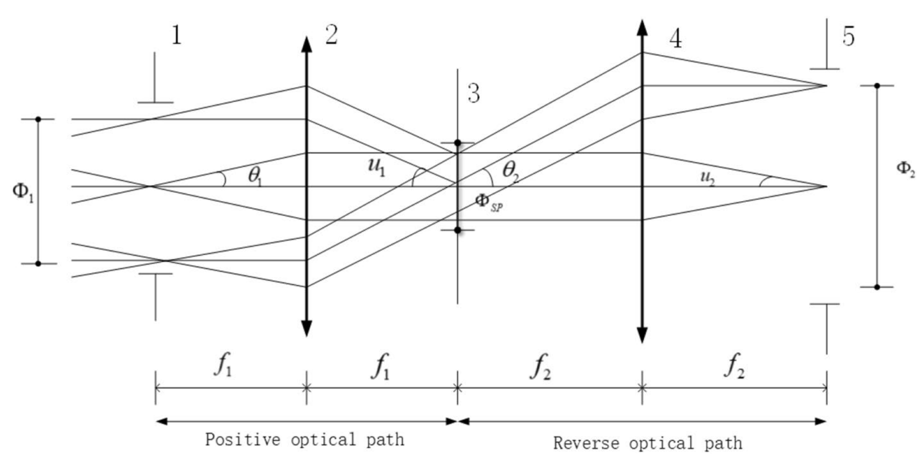


Fig. 1 Fourier transform optical system. 1—Input surface, 2—front group Fourier transform lens, 3—spectrum surface, 4—rear group Fourier transform lens, 5—output surface

laser of 532 nm is used as the light source in the selection of the operating wavelength of the system.

We have selected the input surface and the spectrum surface to have the same specifications on the device, that is, the aperture of the spectrum surface and the aperture of the input surface SLM 1 are equal. The Fourier transform lens images the diffracted light and the SLM 1 on the front focal plane can be regarded as a periodic grating. Suppose the image element size of SLM 1 is δ_{SLM} , then the grating constant is δ_{SLM} . The diffraction occurs after the illumination of SLM 1 by parallel coherent light, and the Fourier lens images the diffracted light. If the diffraction angle of the diffracted light of the first level is θ , and the wavelength of the coherent light is λ , then we have $\sin\theta = \frac{\lambda}{\delta_{\text{SLM}}}$. In order to let the first level of diffracted light pass smoothly, the object-squared numerical aperture of the lens $\text{NA} \geq \sin\theta = \frac{\lambda}{\delta_{\text{SLM}}}$,the size of the spectral plane is $\Phi_{\text{SP}} = 2 \times \text{NA} \cdot f$.

As shown in Fig. 1, the optical parameters of the forward optical path include the half field of view angle θ of the system , the image square numerical aperture NA_{I}, F number, according to the parameters of the SLM and the focal length of the system, it is known that $\theta = \arcsin\left(\frac{\Phi_{\text{SP}}}{2f}\right)$, $\text{NA}_{\text{I}} = \sin u_{\text{I}} = \Phi/2f$, $F = f/\Phi$.

As we mentioned in the first part, the Fourier lens needs to meet three requirements, which need to be satisfied in both forward and reverse directions, but for the convenience of optical design, we adopt a fully symmetric design idea, which only needs to follow the conventional optical design method to meet the design needs, avoiding the trouble of optimizing forward and reverse directions at the same time. After adopting the fully symmetric design method, the optical parameters of the forward optical path are also the optical parameters of the reverse direction. Table 1 lists the parameters required by the design ideas of a normal Fourier lens, and we find that all four sets of data are the same, which can be simplified to Table 2 after taking a fully symmetrical structure, which does not differ from the normal optical design approach and greatly simplifies the optical design work.

Table 2 Simplified optical parameters of Fourier lens designed with symmetrical structure

f_1=125 mm	f_2=125 mm	Φ=18 mm
F	θ	NA
6.94	3.82°	0.072

3 Example of the design of a fully symmetric Fourier lens

As shown in Fig. 1, the first pair of object conjugate position corresponds to the object at infinity, the aperture diaphragm at the front focal plane, the exit pupil at infinity, and the image on the spectral plane. The first pair of conjugate positions corresponds to the forward optical path in Fig. 1. The second pair of conjugate positions corresponds to the object in the front focal plane, the aperture diaphragm in the back focal plane, and the pupil at infinity. In Fig. 1, the same set of lenses can be used for both the Fourier transform (position 2), can also be used for the inverse Fourier transform (position 4, $f1 = f2$), the lens for the inverse Fourier transform when the structure of the structure of the Fourier transform flip. In the inverse Fourier transform lens object at infinity, the aperture diaphragm in the spectral plane, imaging in the output plane. The second pair of object conjugate position control aberration is equivalent to the inverse optical path control aberration of the lens in Fig. 1.

From the above analysis, it is clear that when setting up the Fourier transform lens, the aberration should be corrected for both the forward and reverse optical paths, and the common way to do this is to adopt a dual structure lens modeling and optimization method. The method we adopted is a fully symmetrical method, which is no longer divided into a forward optical path and a reverse optical path. It is no longer necessary to set different optical parameters. It is only necessary to design according to the usual optical design method.

The results of the design of the fully symmetric Fourier transform lens are shown in Figs. 2 and 3.

The system data are shown in Table 2. The focal length of the system is 125 mm, a single group of Fourier lens

Table 1 Optical parameters that need to be considered in the optical design of asymmetric Fourier lenses

f_1=125 mm			f_2=125mm		Φ=18 mm
The first set of positive			The first set of reverse		
F	θ	NA	F	θ	NA
6.94	3.82°	0.072	6.94	3.82°	0.072
The second set of positive			The second set of reverse		
F	θ	NA	F	θ	NA
6.94	3.82°	0.072	6.94	3.82°	0.072

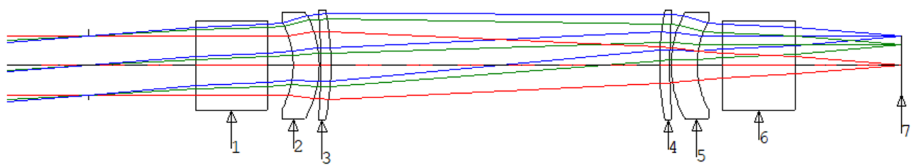


Fig. 2 Two-dimensional optical design of Fourier lens (The number under the component represents the order of the components appearing in the light path)

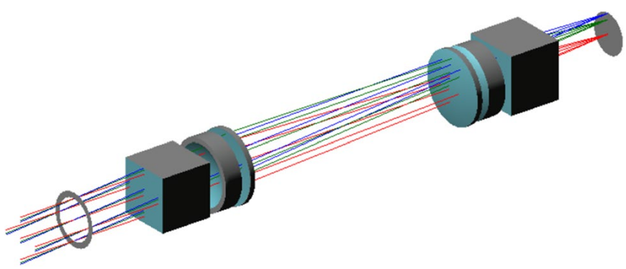


Fig. 3 Three-dimensional optical design of Fourier lens

consists of four pieces of glass, and all are spherical lenses, not added aspheric, where elements 1 and 6 are split beam cubes.The analysis of the results of the optical design is shown in Figs. 4 and 5.

The wave aberration in the maximum field of view is less than 0.02λ,the maximum root mean square radius of the image point is 0.002 mm, within the Airy spot, while the design reaches the diffraction limit, due to the symmetric structure, the distortion is balanced, as shown in Fig. 4b, the maximum distortion is less than 0.4%. The 4f optical path built with this Fourier lens is shown in Fig. 6.

4 Experiment

The schematic diagram of the experimental structure is shown in Fig. 7 for a classical 4f system, where SLM 1 is the input plane, SLM 2 is the spectral plane for filtering, and the plane where the CMOS is located in the image plane. The first SLM 1 is used to load the signal, we load the echo signal of SAR, the light wave carrying the signal propagates to the lens for Fourier transform, the light wave after Fourier transform acts with the filter function loaded by SLM 2 for modulation, and then passes through the second lens to realize the Fourier inverse transform, and finally receives the experimental results at the image plane.

Since the SLM we used is reflective, the optical path built in the laboratory is shown in Fig. 8. For the sake of experimental simplicity, the 4f system was built using a beam splitting cube with only one set of Fourier transform lenses, and the structure is schematically shown in Fig. 9.

Before imaging the SAR echo data, we first process the linear frequency modulation signal. The linear frequency modulation signal is used to evaluate the feasibility of the 4f system composed of optical lenses for information processing. At the same time, the optical path can be adjusted according to the imaging results of the linear frequency modulation signal, so that the experimental optical path can achieve good alignment, which makes preparation for SAR information processing. The purpose of designing the Fourier lens is to be able to process certain information, and our main purpose is to apply it to a synthetic aperture radar imaging system, instead of doing the Fourier transform in the traditional electrical way. Compared with SAR echo information, the LFM signal is relatively simple, and after the Fourier transform, filtering at the spectrum will give a known experimental phenomenon, which is used to test whether the 4f system composed of Fourier lens can be competent to match the filtering work.

Therefore, before processing the SAR echo signal, we first carry out experiments on a one-dimensional LFM signal, which can be expressed as

$$s(t) = \mathrm{rect}\left(\frac{t}{T}\right)\exp\left(j\pi Kt^2\right) = \mathrm{rect}\left(\frac{t}{T}\right)\exp[j\varphi(t)] \tag{1}$$

where $\varphi(t) = \pi Kt^2$,K means frequency modulation, T means duration of LFM signal.

We feed this signal to the SLM 1 by rewriting the signal $s(t)$ as

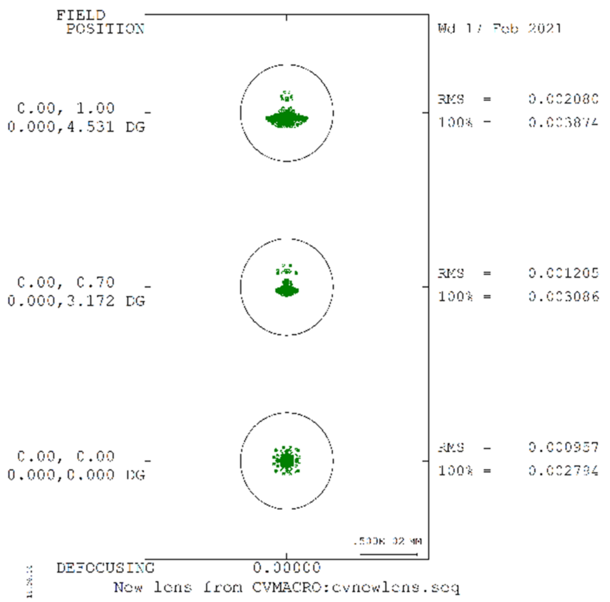
$$f(x,y) = \mathrm{rect}\left(\frac{x}{W_x/2}\right)\exp\left(j\pi K_x x^2\right)\mathrm{rect}\left(\frac{y}{W_y/2}\right) \tag{2}$$

where $W_x = 1920 \times 8\,\mu$m is the length of the spatial light modulator, $W_y = 1080 \times 8\,\mu$m is the width of the spatial light modulator; the modulation frequency is taken as $K_x = \frac{1}{\lambda_l f_l} = \frac{1}{532\,\mathrm{nm} \times 250\,\mathrm{mm}}$, and the plane origin of this signal is at the center of the lens.

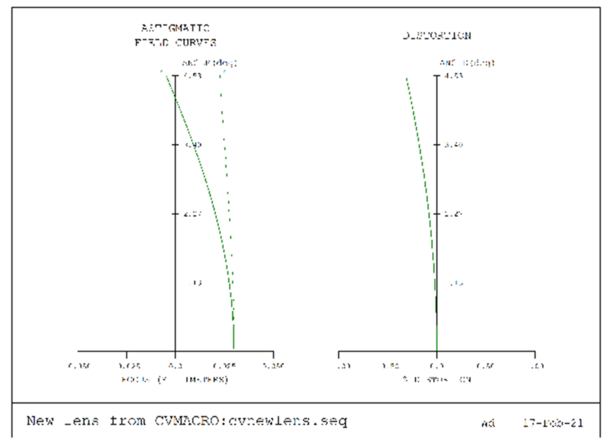
We Fourier transform Equation 4-1, then we know that the optical field of the LFM signal in the spectral plane is

$$F(u,v) = \mathrm{rect}\left(\frac{u}{W_x/2}\right)\exp\left(-j\pi\frac{u^2}{\lambda f}\right)\cdot\frac{2\sin\left(2\pi\frac{v}{\lambda f}\frac{W_y}{2}\right)}{2\pi\frac{v}{\lambda f}} \tag{3}$$

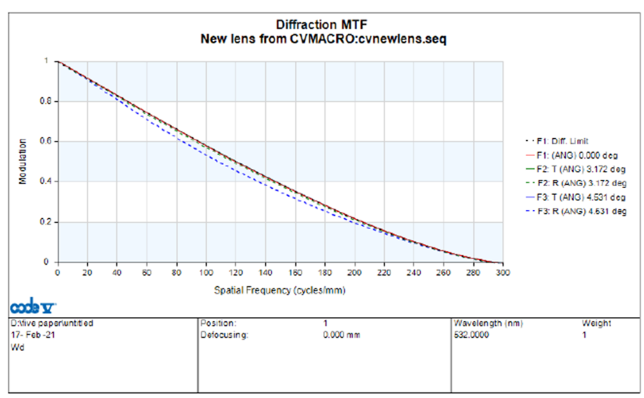
We will use the second spatial light modulator to filter the LFM signal, and we will only do matched filtering on the transverse side, so the applied phase is



(a) spot diagram



(b) ray aberration curves



(c) MTF

Fig. 4 Analysis of the results of Fourier lens optical design

$$G(u,v) = \mathrm{rect}\left(\frac{u}{W_x/2}\right)\exp\left(j\pi\frac{u^2}{\lambda f}\right) \tag{4}$$

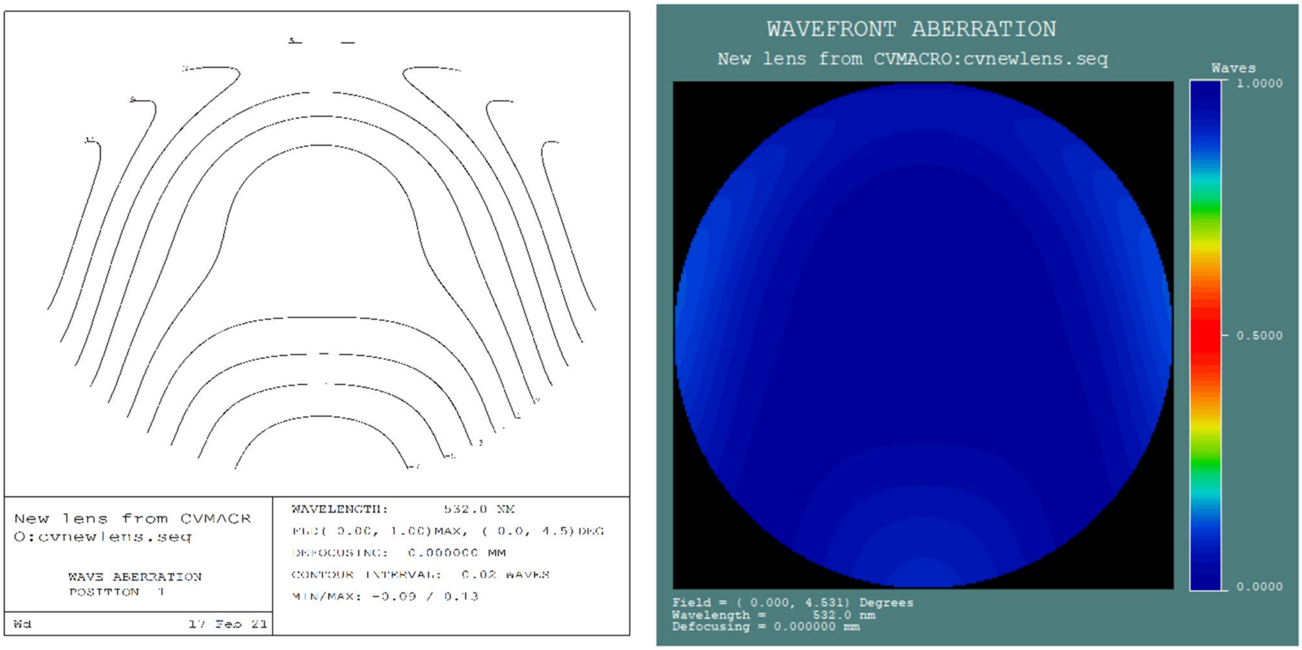
so we would end up with a bright line in the image plane of the 4f system.

SLM is an 8-bit device, which needs to quantize the input information to 0-255. Simulate Eqs. (2) and (4) with MATLAB, take their phase, and quantify the gray value to 0–255, then get Figs. 10 and 11, where Eq. (2) is one-dimensional LFM signal, Eq. (4) is one-dimensional LFM signal filtering function, then the figure corresponding to Eq. (2) is loaded into the first space optical modulator, which means that 4f system input is LFM signal, when the figure corresponding to Eq. (4) is loaded into the second SLM, it means that the spectrum plane of the 4f system is the filtering function for the one-dimensional LFM signal.

The phase diagram of the signal loaded into the LFM of the first SLM is shown in Fig. 10. The phase diagram of the filter function loaded into the second spatial light modulator is shown in Fig. 11.

When we adjust the optical path of the 4f system, load Figs. 10 and 11 into the corresponding SLM, respectively, and adjust the exposure of the CMOS, the collected experimental results are shown in Fig. 12, we can observe that there is a bright line at the middle position, and the experimental phenomenon is consistent with the conclusion.

After successfully processing the one-dimensional LFM signal, the accuracy of the experimental optical path is verified, and then the SAR echo data is processed. Using the SAR point target formula [5] and the SAR two-dimensional filter function formula [18] for simulation, Figs. 13 and 14 are obtained respectively.The phase of the SAR echo signal



(a) Contour map of wavefront aberration (b) Color display of wavefront aberration

Fig. 5 Wavefront aberration of Fourier lens optical design

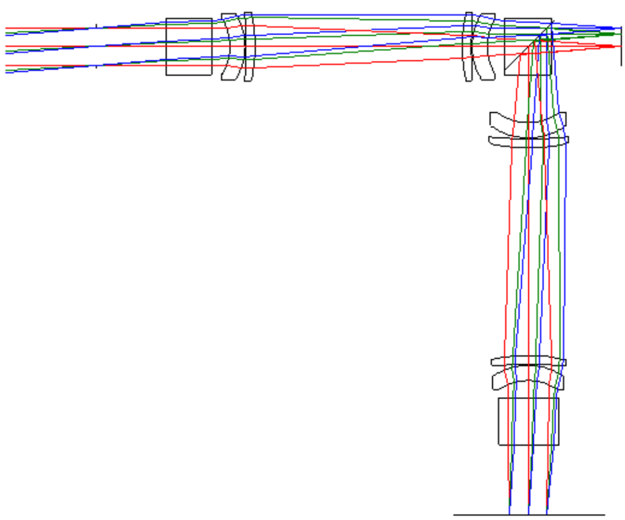


Fig. 6 Simulation of a 4f system composed of two sets of Fourier lenses

of the simulated single-point target is input to the first SLM, and the SAR filter function is loaded into the second SLM. The result obtained through the optical 4f system is shown in Fig. 15a. Figure15b shows the image of the single point target of SAR echo signal obtained by Matlab simulation.By comparing Fig. 15a with Fig. 15b, we find that the experiment of using the lens to implement the Fourier transform for SAR imaging yields good results.

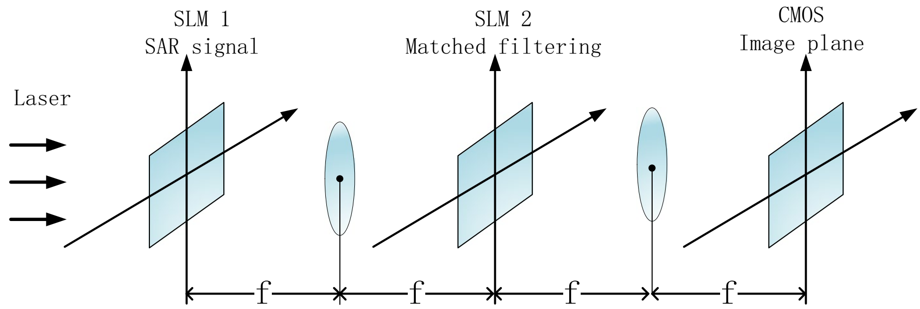


Fig. 7 Schematic diagram of the signal processing flow of the optical 4f system

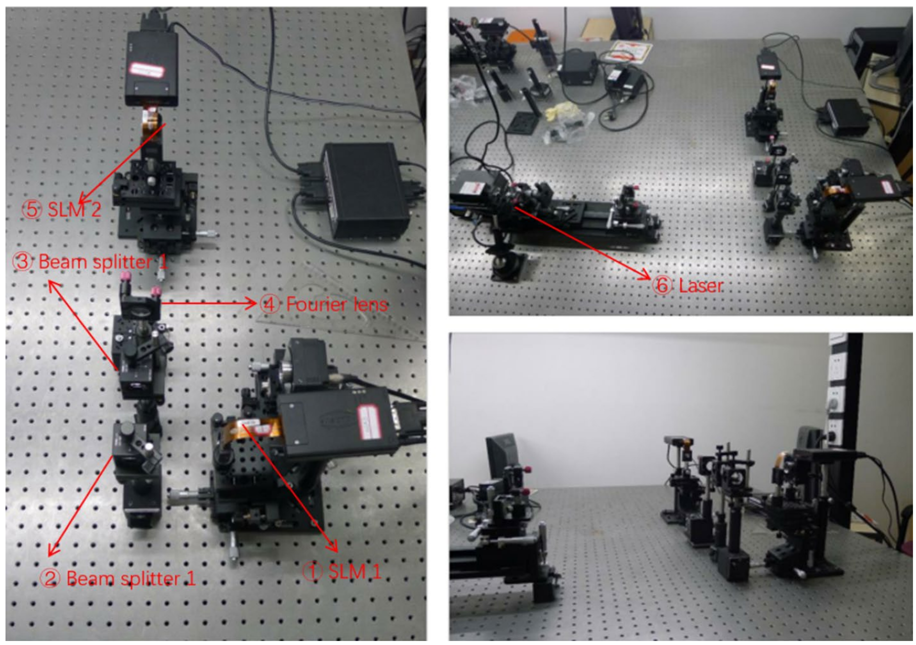


Fig. 8 Actual 4f optical system built in the laboratory. The system consists of two spatial light modulators and a Fourier transform lens as well as optical components (e.g. CMOS and beam splitter). The 4f system uses a beam splitter cube to simplify the optical path and uses only one Fourier lens for the two Fourier transforms. The signal data to be processed is loaded by the computer onto the input SLM 1, and the Fourier transform lens is used to complete the Fourier transform. The matrix multiplication is achieved by loading a filter function using the SLM 2, and the Fourier transform is inverted by the Fourier transform lens after reflection. The obtained image is acquired by CMOS

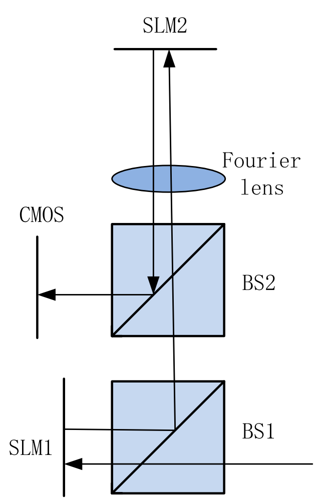


Fig. 9 Schematic diagram of a 4f system built with a set of Fourier lenses using a split beam cube to simplify the optical path

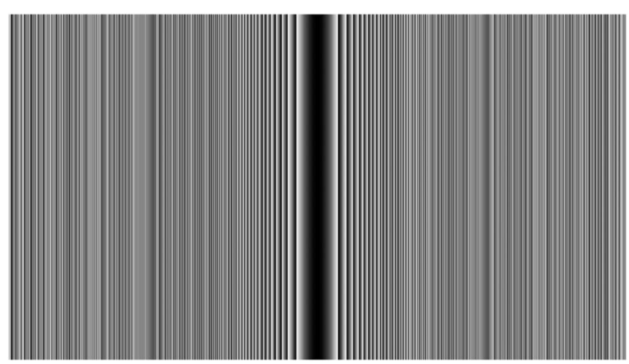


Fig. 10 Phase diagram of the LFM signal loaded into SLM 1

Fig. 11 Phase diagram of the filter function loaded into SLM 2

Fig. 12 Experimental results obtained with CMOS by loading Figs. 10 and 11 into slm1 and SLM2 in the 4f system, respectively, a bright line in the vertical direction is obtained because only the horizontal filtering is done

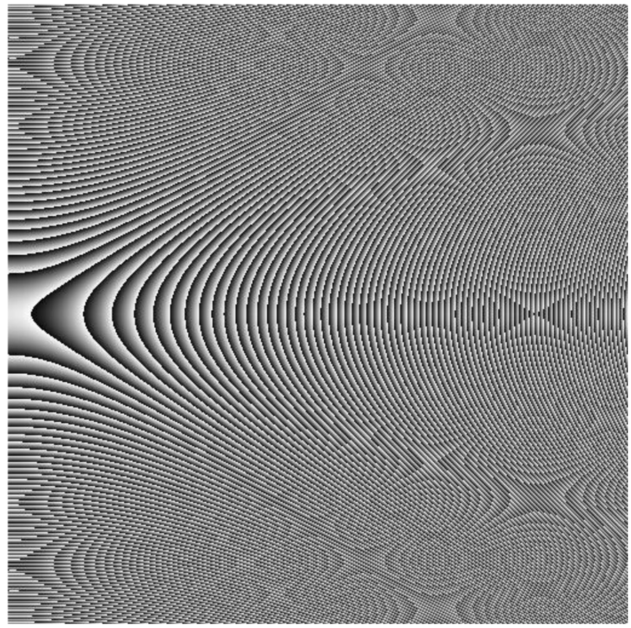


Fig. 13 Phase diagram of single-point SAR echo signal

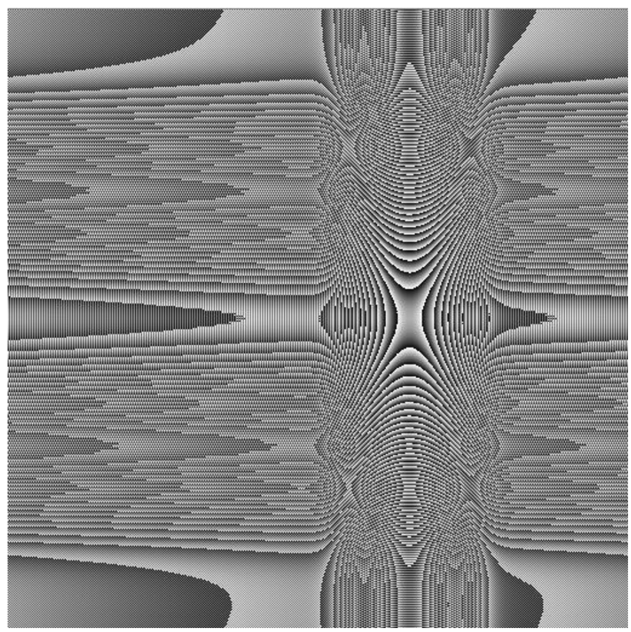


Fig. 14 Phase diagram of the filter function for modulation of a single-point target SAR signal

Fig. 15 Results of demodulating single-point target SAR echo signals using different methods

(a) The demodulation result of single-point target SAR echo signal using optical 4f system (b) The result of demodulating single-point target SAR echo signal with matlab simulation

5 Conclusion

In this paper, a fully symmetric 4-piece spherical Fourier transform optical lens is designed to avoid the trouble of simultaneous optimization in both forward and reverse directions. A typical 4f optical system was built in the laboratory using an optical lens, and SLMs of the same size were placed on the input side and the spectrum side, with the SLM on the input side used for loading the signal and the SLM on the spectrum side used for filtering. We used this 4f system to demodulate LFM signals and SAR single-point target signals and obtained good experimental results. In the future, we intend to use an optical lens to complete the operation of the Fourier transform of the SAR signal, because the calculation speed of optics is only related to the optical path, which can be said to be a real-time operation to solve the problem of slow imaging speed of SAR due to the huge amount of operations, for which we designed a fully symmetric Fourier lens and carried out the verification work of SAR imaging in the early stage.

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