International Journal of Theoretical Physics (2021) 60:2066–2073
https://doi.org/10.1007/s10773-021-04823-y

Many-Body Localization of Haldane-Shastry Model with Periodic Driving

Haoyue Li[1] · Taotao Hu[1] · Kang Xue[1] · Hui Zhao[1] · Xiaodan Li[2] · Shuangyuan Ni[1] · Jiali Zhang[1] · Hang Ren[3]

Received: 28 January 2021 / Accepted: 28 April 2021 / Published online: 18 May 2021

Abstract

In this work, we study the property of many-body localization(MBL) in the Haldane-Shastry(HS) chain which is driven by an additional time-dependent perturbation periodically. The Haldane-Shastry (HS) model is the integrable one-dimensional quantum spin chain with long-range interactions, it is the generalized Heisenberg XXX model which only contain nearest two body interaction. By using HS model, we consider the global two-body interaction and expand the field of MBL. In this paper, we establish a Floquet operator by adding a time-periodic field formed as trigonometric function to a closed and disordered HS model in this periodic driven system. We use the exact matrix diagonalization to probe the property of MBL with different disorders and system sizes. When we drive the HS model in MBL phase, it shows that there is a significant change in the diagrams with when driving strength T reach to T_c which is the critical driving strength. We get that at large T ($T > T_c$), MBL phase will be broken and a transition from localized phase to delocalized phase will happen, conversely, at small T ($T < T_c$), MBL phase will be retained. The stronger disorder taken in system, the more stable the localized phase is and the higher T_c is needed to drive the transition. However, there is no MBL phase transition when we drive the HS model in ergodic phase with periodic driving. In contrast to the Heisenberg XXX model with the same situation which we have studied recently, the phase transition from delocalized phase to localized phase occurs. We also explore the non-disorder system of HS model with the same driving to explore the properties of MBL, it shows that under periodic driving, the non-disordered HS system has the quantum phase transition rather than MBL phase transition. This illustrates the important role of disorder on MBL.

Keywords Many-body localization · Haldane-Shastry model · Periodic driving

✉ Haoyue Li
hutt262@nenu.edu.cn

[1] School of Physics, Northeast Normal University, Changchun 130024, People's Republic of China

[2] College of Science, University of Shanghai for Science and Technology, Shanghai 200093, People's Republic of China

[3] Key Laboratory of Airborne Optical Imaging and Measurement, Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun 130033, People's Republic of China

1 Introduction

Mang-body localization(MBL) is the extensive form of Anderson localization [1] considering the interaction between particles, which has been the subject of intense investigation in quantum information field. Many features of MBL phase have been widely explored, e.g. entanglement entropy, which has exponential growth in MBL system [2, 3], and averaged energy gap ratio $\langle r \rangle$, by whose value one can confirm the phase of system, when $\langle r \rangle$ approximately equal to 0.386 system follows poisson statistics of quasienergy levels and be localized [2]. Disorder h is an important index to distinguish the MBL phase and ergodic phase [2, 4], when h is less than the critical disorder h_c, the system with weak disorder is in the ergodic phase and could be heated up to thermalization. On the contrary, the system is in MBL phase, in which quantum and energy transport is suppressed and eigenstate-thermalization hypothesis loses efficiency. Similar to the random field which bring disorder to MBL system can effect quantum phase, we argue that time-periodic field which drive MBL system also can lead to a transition to ergodic behavior. Periodically driven systems have been extensively researched over last decades both in classical and quantum levels [5–12], and whose application in inducing phase transitions [13–17] has been widely concerned. From the aspect of experiment,there are nuclear magnetic resonance (NMR)experiments [18], AC-driven electron glass [19] and relativistic-electrons accelerated by lasers [20], etc, which provides strong objective basis for effectiveness of periodically driven. On the other hand, progress has also been made in theory. The proposal of kicked rotor is a representative case, which can induce dynamical Anderson localization and the transition between chaotic and regular behaviors [21, 22]. Most of the early studies are focus on nointeracting systems. With the development of experiments in ultracold atomic, trapped ions and molecular, the interactions between particles of isolated systems have been gradually applied to the driven researchs [23]. Periodically driven MBL system have been the subject of growing interest, more relevant research need to be explored to enrich the theory. In this paper, we explore MBL by considering the long range interactions in one dimensional periodically driven system. The study of periodically driven system provides new ideas to probe thermalization and can be a flexible experimental knob to investigate MBL transition. In fact, the essence of periodically driven is that the system Hamilton changes periodically as the form of $\hat{H}(t) = \hat{H}(t+T)$, caused by the coupling with the external field. If we calculate the integration of the evolution operator over one period, we will get the important unitary operator, the Floquet operator [24]

$$\hat{\mathrm{F}} = \mathcal{T}\exp\int_0^{\mathrm{T}}(-\mathrm{i}\hat{\mathrm{H}}(\mathrm{t}))\mathrm{dt} \tag{1}$$

Here $\mathcal{T}\exp$ is a time-ordered exponential. In the eigenstate of $\hat{\mathrm{H}}(\mathrm{t})$, $|\varphi_{\mathrm{n}}\rangle$, can be written as

$$\hat{\mathrm{F}} = \sum_{\mathrm{n}=1}^{\mathcal{D}} \mathrm{e}^{-\mathrm{i}\theta_{\mathrm{n}}} |\varphi_{\mathrm{n}}\rangle \langle\varphi_{\mathrm{n}}| \tag{2}$$

where $\mathcal{D}$ is the dimension of the system, and θ_{n} is the quasienergy of $\hat{\mathrm{H}}(\mathrm{t})$ in $|\varphi_{\mathrm{n}}\rangle$. One can introduce Floquet Hamilton $\mathrm{H_F}$ which effectively determines the properties of driven system, as $\hat{\mathrm{F}} = e^{-i\mathrm{H_F}T}$, consequently, the eigenstate of $\hat{\mathrm{F}}$ is also as $|\varphi_{\mathrm{n}}\rangle$. It has been confirmed in ref. [12] that period can be used as an important factor determining whether the drive of kicked rotor can cause a transition from MBL to delocalized phase. We have reason to speculate that in new driven there will still be a critical period T_c corresponding the phase transition point, just like h_c. One of the most commonly used forms of driven in recent years is periodic replacement of two different hamiltonians, here we adopt the new type of driven by adding a time-dependent trigonometric functions as external control field to many-body system to structure a periodic time-dependent Hamiltonian [25, 26]. It can be expressed as:

$$\hat{\mathrm{H}}(\mathrm{t}) = \mathrm{H}_0 + \hat{\mathrm{V}}\cos(\omega\mathrm{t} + \varphi) \tag{3}$$

H_0 is the hamiltonian of HS model in MBL system which consider the global two-body interaction, we will introduce the model in detail in Section 2, here H satisfies the relation, $\hat{\mathrm{H}}(\mathrm{t}) = \hat{\mathrm{H}}(\mathrm{t}+\mathrm{T})$, $\mathrm{T} = 2\pi/\omega$. In ref. [14], the utilization of this type of driving in delocalized many-body system of hard-core bosons model has been confirmed that high frequency can induce the transition to MBL phase. We can predict that MBL phase will transit to ergodic phase if the frequency of driving is sufficiently low or the period of driving is sufficiently strong. The purpose of this paper is to explore the properties of MBL in the HS chain which is driven periodically. Through the numerical simulation of several efficiency physics, we can argue when driving strength is less than T_c, the state will retain the local memory of the initial state. In contrast, strong driving will break the localized phase and heat the system to thermalization. This result has been confirmed in different scale sizes and we can see that they have same tendency although slightly difference caused by scale are reflected in the images. In Section 3, we will detailed presentative relevant calculation results. Section 4 is devoted to discuss the conclusions we get and what to further.

2 Model

We study the HS model in ergodic phase and MBL phase driven periodically in this paper. Currently, the research of the nearest neighbor interacting of spin-1/2 chain has been very common [12, 16], and a wider range of interactions such as long-range interaction needs to be taken into account in order to complete the theory of driving many-body system. As the generalization of one-dimensional XXX Heisenberg model, HS model can give full play to its characteristic of considering the global interactions among many-body system to meet above needs [24, 27]. Due to the close relationship between mathematics and physics, more attention is focused on HS model. There are lots of hot theories are related to it, e.g. Yangian quantum groups, quantum halls effect and Yang-Mills theory. Therefore it is important to characterize the dynamics in this model particularly under standing the specifics of many-body localization there. Here we consider the global two-body interactions. The form of Hamilton in that system is

$$\mathrm{H}_2 = \sum_{\mathrm{ij}}{}' \left(\frac{\mathrm{Z_i Z_j}}{\mathrm{Z_{ij}} - \mathrm{Z_{ji}}}\right)\left(\mathrm{P_{ij}} - \mathrm{I}\right), \mathrm{Z_{ij}} = \mathrm{Z_i} - \mathrm{Z_j}, \mathrm{Z_j} = \exp\left(\frac{\mathrm{i}2\pi}{\mathrm{N}}\mathrm{j}\right) \tag{4}$$

$\sum{}'$ represents the summation of all cases of $i \neq j$, and P_{ij} is an operator exchanging the state on site i and j, namely the spin operators. For spin 1/2 chain, that is, the SU(2)

$$\mathrm{P_{ij}} = \mathrm{S_i^+ S_j^-} + \mathrm{S_i^- S_j^+} + 2\mathrm{S_i^z S_j^z} + \frac{1}{2} = \frac{1}{2} + 2\overrightarrow{\mathrm{S_i}} \cdot \overrightarrow{\mathrm{S_j}} \tag{5}$$

In terms of spin operators, it can be rewritten as

$$\mathrm{H}_2 = \sum_{\mathrm{ij}}{}' \frac{1}{4\sin^2\theta_{ij}}\left(-\frac{1}{2}\mathrm{I} + 2\overrightarrow{\mathrm{S_i}} \cdot \overrightarrow{\mathrm{S_j}}\right) \tag{6}$$

N is the total particle number, $\theta_{ij} = \frac{(i-j)\pi}{N}$ Then we can investigate the MBL transition in H_2 with disordered external fields, namely the disordered HS spin-1/2 chain with global two-body interaction, form as

$$H' = H_2 + \sum_i h_i S_i^z \tag{7}$$

The last step is to drive the external field periodically with added item we mentioned at Section 1, with the form

$$H(t) = H' + V_0 \cos \omega t \sum_i S_i^z \tag{8}$$

The floquet operator is written as

$$\hat{F} = \mathcal{T} \exp \int_0^T (-i\hat{H}(t))dt \tag{9}$$

3 Numerical Simulations

In order to study the critical point of the phase transition in this periodically driven system, we first choose representative physical quantity, fidelity [28], which is widely used in the characterization of phase transition. Following the definition of the ground-state fidelity in ref. [29], we can generalize the fidelity of the n-th excited state, defined as the overlap between $|\psi_n(\lambda)\rangle$ and $|\psi_n(\lambda + \delta)\rangle$

$$F_n(\lambda, \delta) = |\langle \psi_n(\lambda)| \psi_n(\lambda + \delta)\rangle| \tag{10}$$

where δ is only a 10^{-3} order of magnitude small value for λ. Usually the value of the fidelity is approaching to 1, but near the critical point of the phase transition, a significant variation in trends will appear for the fidelity because the difference between the two state of same energy level on both sides of the critical point is the maximum. Comparing with ref. [29] which research based on ground-state fidelity, current studies show that the characterization of phase transition by the fidelity of excited state is more obvious [30]. We choose the fidelity for exited state in the middle one third of the spectrum which represent a higher temperature to observe such a more persuasive phase transition in high energy level.

In Fig. 1a, b and c, we plot the averaged excited-state fidelity E[F] as a function of the driven strength T, respectively, we drive the disordered HS systems with different disorder

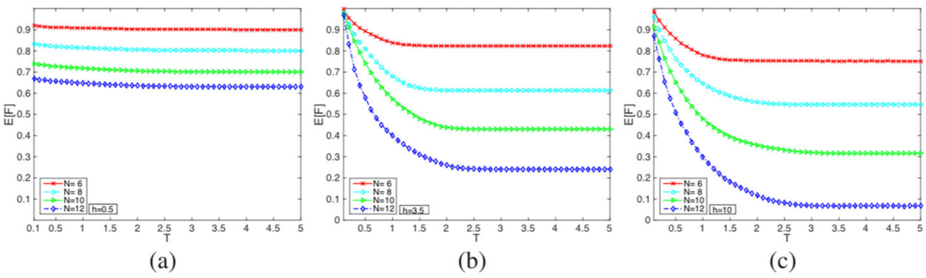

Fig. 1 **a** Average fidelity E[F] as a function of the driven strength T when we take h=0.5. Data are for system sizes N = 6; 8; 10; 12, and averaging is performed over 1000 disorder realizations. **b** Average fidelity E[F] as a function of the driven strength T when we take h=3.5. **c** Average fidelity E[F] as a function of the driven strength T when we take h=10. The values of N are indicated in the legend

strengths to observe the effect of periodic driving on phase transition. In Fig. 1a, we drive the system with a small disorder strength as h=0.5, which is in ergodic phase. It can be seen that as the driven strength increases, the trend of plot does not change obviously even in different system sizes. The pronounced data change shown in Fig. 1a indicates that phase transition does not happen in this periodic driven system and the system is still in a ergodic phase. However, in Fig. 1b, in the localized phase (small T) E[F] decays substantially until T approaches to the critical point T_c, then in the ergodic phase (large T) E[F] stay at the minimum value and the trend of plot does not change. When the disorder strength h= 3.5 which is close to the critical value of the localized phase, the trend of averaged excited-state fidelity changes obviously on the both side of critical driven strength T_c which takes 1.2 for N=6, 1.9 for N=8, 2.2 for N=10, 2.5 for N=12. So one can get the critical driving strength $T_c \in [1.2, 2.5]$, which indicates that the localized HS system has a transition from localized phase to ergodic phase. It can be seen from the comparison that the size of the system will affect the critical point of phase transition. The larger the system is, the larger the critical driven strength needed for phase transition, because the more particles in the system, the more complex interaction between the global two particles and the more difficult it is to change from localized phase to ergodic phase. In Fig. 1c, we once again increase the disorder strength to let h=10, it can be seen that the trend of the plot presented in Fig. 1b are further strengthened, here the phase transition is more obvious and the critical driven strength obtain $T_c \in [1.5, 3]$. And this is consistent with the conclusion mentioned In ref. [14] which argue that the system in large driven frequency is in the localization phase and the system in small frequency is in the ergodic phase. In picture 1(c), we once again increase the disorder strength to let the h take as 10. It can be seen that the trend of the plot presented in Fig. 1b are further strengthened, in which the phase transition is more obvious and we get the critical driven strength $T_c \in [1.5, 3]$. It is worth noting that is consistent with the conclusion mentioned In ref. [14] which give that the system retains the localization phase when the driving frequency is large and the system breaks to the ergodic phase when the driving frequency is small.

Compared the critical disorder strength T_c of 1(b) with 1(c), it can be seen that for a MBL system with larger disorder strength, correspondingly, a larger driven strength is needed to promote the phase transition. In order to confirm the relationship between disorder strength h and critical driving strength, we plot the averaged excited-state fidelity E[F] under the same system size as a function of the driven strength in different disorder strength as shown in Fig. 2. we can obtain the approximate critical driven strength T_c for different system size h. For h = 3.5, $T_c \to 2.2$, h = 5, $T_c \to 2.4$, h = 10, $T_c \to 2.7$. It can be seen that with the increase of disorder strength, the localized property of the system becomes more stable and a larger required critical driving strength is needed.

In Fig. 1a, the periodic driving does not bring phase transition from ergodic phase to the localized phase, which is different from the periodically driven Heisenberg XXX model with the same situation. It just because that here the HS model has global two-body interaction. To further study the property of MBL phase, under the same periodic driving, we also investigate the corresponding non-disordered system when h =0.5. Here h is constant, not disordered. In Fig. 3, one can see the E[F] versus T shows sudden drop at the critical point and then immediately rise to 1. It indicates that the non-disordered system has the quantum phase transition rather than a MBL transition. This illustrates that disorder plays an important role on MBL transition.

For the purpose of further studying the properties of ergodic phase under periodic driving, we study the phase transition properties of Heisenberg XXX model for the same

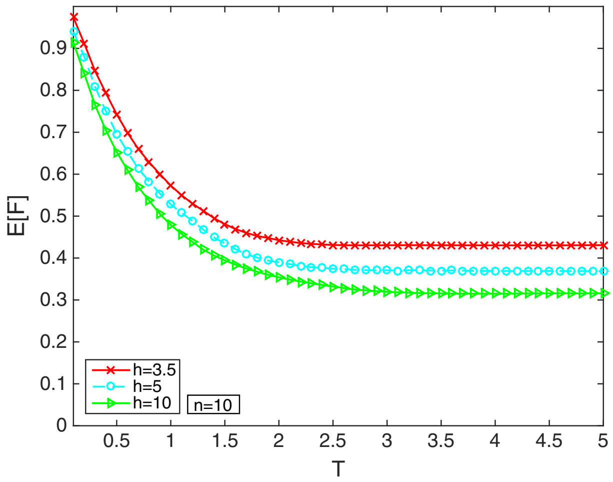

Fig. 2 Average fidelity E[F] as a function of the driven strength T when we take N=10. Data are for disorder strength h = 3.5; 5; 10, and averaging is performed over 100 disorder realizations

driving which only contain nearest neighbor two-body interaction. In Fig. 4, the trend of plot changes obviously on the both side of critical driven strength. It shows that the MBL transition does happen in Heisenberg XXX model, which is different from the HS model. This finding suggests that the interaction does affect the properties of the MBL under periodically driven. The more complex the interaction between particles, the more stable the delocalized properties of the system, the more difficult it is to drive MBL transition.

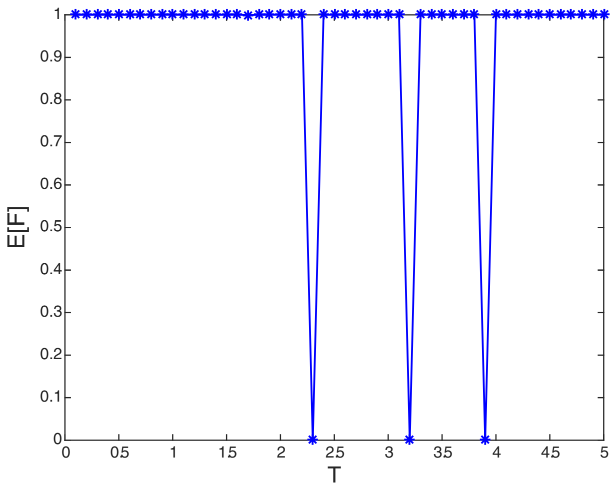

Fig. 3 Average fidelity E[F] as a function of the driven strength T when we take the same strength as Fig. 1a of non-disordered system. Data are for system sizes N = 6, and averaging is performed over 1000 disorder realizations. E[F] versus h show three sudden drops at the critical points and then immediately rise to 1

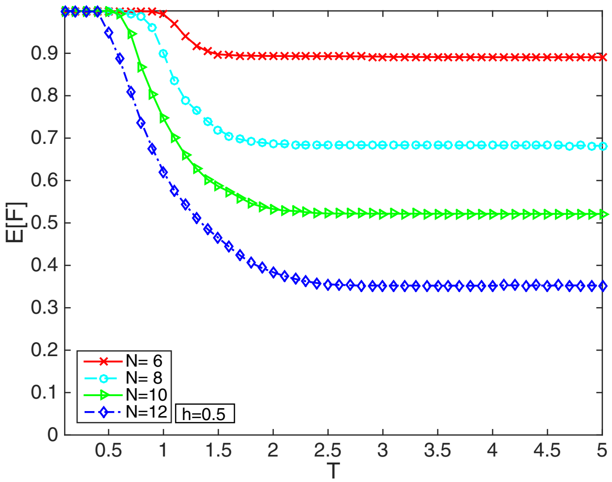

Fig. 4 Average fidelity E[F] as a function of the driven strength T when we take h=0.5 in Heisenberg model. Data are for system sizes N = 6; 8; 10; 12, and number of averages is 10^4 for N=6, 1000 for N=8, 100 for N=10 and N=12

4 Summary

In this paper, we use exact matrix diagonalization to explore the property of MBL in the Haldane-Shastry(HS) with periodic driving.We test the fidelity between two excited states by a small parameter perturbation δT, and use it to explore the phase transition. Numerical simulations performed show that periodic disturbance can induce the transition of the disordered HS systems with long-range interactions from MBL phase to ergodic phases. We get that if the driving strength, which is the period of periodic disturbance, is larger than critical driving strength T_c, the phase transition from MBL phase to ergodic phase will occur in this HS system, and the amplitude and critical point of the phase transition are related to the size of the system. In addition, the disorder strength also affects the critical point and amplitude of the phase transition by affecting the localization of the system. It is worth noting that when we drive the disordered HS model in the ergodic phase periodically, there is no phase transition under periodic disturbance which is opposite to the Heisenberg XXX model. While for the situation of the non-disordered system which takes the same strength of h as the ergodic phase, the system has the quantum phase transition rather than a MBL transition under periodic disturbance. It illustrates that disorder plays an important role on MBL transition. By comparing the results of Heisenberg XXX model with nearest neighbor interaction in the same situation, it shows that the interaction has an important effect on property of MBL. We hope that this work would future the exploration of the property of MBL with periodical driving, and expand the related research in our future work.

Acknowledgments This work is supported by “the Fundamental Research Funds for the Central Universities (No. 2412019FZ037).

References

1. Anderson, P.W.: Absence of diffusion in certain random lattices. Phys. Rev. Lett. **109**, 1492 (1958)
2. Pal, A., Huse, D.A.: Many-body localization phase transition. Phys. Rev. B **82**, 174411 (2010)

3. Kjall, J.A., Bardarson, J.H., Pollmann, F.: Many-body localization in a disordered quantum Ising chain. Phys. Rev. Lett. **113**, 107204 (2014)
4. Huo, T.T., Xue, K., Li, X., Zhang, Y., Ren, H.: Fidelity of the diagonal ensemble signals the many-body localization transition. Phys. Rev. E **94**, 052119 (2016)
5. Petsch, S., Schuhladen, S., Dreesen, L., Zappe, H.: The engineered eyeball, a tunable imaging system using soft-matter micro-optics. Light Sci. Appl. **5**, e16068 (2016)
6. Ponte, P., Chandran, A., Papic, Z., Abanin, D.A.: Periodically driven ergodic and many-body localized quantum systems. Ann. Phys. **353**, 196–204 (2015)
7. Biasco, S., Beere, H.E., Ritchie, D.A., Li, L., Giles Davies, A., Linfield, E.H., Vitiello, M.S.: Frequency-tunable continuous-wave random lasers at terahertz frequencies. Light Sci. Appl. **8**, 43 (2019)
8. Mishra, U., Prabuh, R., Rakshit, D.: Quantum correlations in periodically driven spin chains: revivals and steady-state properties. J. Magn. Magn. Mater **491**, 165546 (2019)
9. Rubin, S., Hong, B., Fainman, Y.: Subnanometer imaging and controlled dynamical patterning of thermocapillary driven deformation of thin liquid films. Light Sci. Appl **8**, 77 (2019)
10. Alessio, L.D., Polkovnikov, A.: Dynamically induce mamy-body localization. Ann. Phys. **333**, 19–33 (2013)
11. Etezadi, D., Warner, J.B. IV., Ruggeri, F.S., Dietler, G., Lashuel, H.A., Altug, H.: Nanoplasmonic mid-infrared biosensor for in vitro protein secondary structure detection. Light Sci. Appl. **6**, e17029 (2017)
12. Ponte, P., Papic, Z., Huveneers, F., Abanin, D.A.: Many-body localization in periodically driven system. Phys. Rev. Lett. **144**, 140401 (2015)
13. Qu, Y., Li, Q., Lu, C., Pan, M., Ghosh, P., Du, K., Qiu, M.: Thermal camouflage based on the phase-changing material GST. Light Sci. Appl. **7**, 26 (2018)
14. Choi, S., Abanin, D.A., Lukin, M.D.: Dynamically induce mamy-body localization. Phys. Rev. B **97**, 100301(R) (2010)
15. Dutt, A., Minkov, M., Williamson, I.A.D., Fan, S.: Higher-order topological insulators in synthetic dimensions. Light Sci. Appl. **9**, 131 (2020)
16. Bairey, E., Refael, G., Lindner, N.H.: Driving induce mamy-body localization. Phys. Rev. B **96**, 020201(R) (2017)
17. Strek, W., Cichy, B., Radosinski, L., Gluchowski, P., Marciniak, L., Lukaszewicz, M., Hreniak, D.: Laser-induced white-light emission from graphene ceramics-opening a band gap in graphene. Light Sci. Appl **4**, e237 (2015)
18. Hahn, E.L.: Spin echoes. Phys. Rev. **80**, 580 (1950)
19. Ovadyahu, Z.: Suppression of inelastic Electron-Electron scattering in anderson insulators. Phys. Rev. Lett. **108**, 156602 (2012)
20. Tochitsky, S.Y., Narang, R., Filip, C.V., Musumeci, P., Clayton, C.E., Yoder, R.B., Marsh, K.A., Rosenzweig, J.B., Pellegrini, C., Joshi, C.: Enhanced acceleration of injected electrons in a Laser-Beat-Wave-Induced plasma channel. Phys. Rev. Lett. **92**, 095004 (2004)
21. Fishman, S., Grempel, D.R., Prange, R.E.: Chaos, quantum recurrences,and anderson localization. Phys. Rev. Lett. **49**, 509 (1982)
22. Matrasulov, D.U., Milibaeva, G.M., Salomov, U.R., Sundaram, B.: Ralativistic kicked rotor. Phys. Rev. E **72**, 016213 (2005)
23. Alessio, L.D., Rigol, M.: Long-time behavior of isolated periodically driven interacting lattice systems. Phys. Rev. X **4**, 040048 (2014)
24. Grifoni, M., Hanggi, P.: Driven quantum tunneling. Phys. Reports **304**, 229–354 (1998)
25. Eckardt, A., Weiss, C., Holthaus, M.: Superfluid-insulator transition in a periodically driven optical lattice. Phys. Rev. Letts. **95**, 260404 (2005)
26. Katan, Y.T., Podolsky, D.: Modulated floquet topological insulators. Phys. Rev. Letts. **110**, 016802 (2013)
27. Polychronakos, A.P.: Lattice integrable systems of Haldane-Shastry type. Phys. Rev. Letts. **70**, 15 (1993)
28. You, W.L., Li, Y.W., Gu, S.J.: Fidelity, dynamic struture factor, and susceptibility in critical phenomena. Phys. Rev. E **76**, 022101 (2007)
29. Zanardi, P., Paunkovic, N.: Ground state overlap and quantum phase transitions. Phys. Rev. E **74**, 031123 (2006)
30. Chen, S., Wang, L., Gu, S.J., Wang, Y.P.: Fidelity and quantum phase transition for the Heisenberg chain with next-nearest-neighbor interaction. Phys. Rev. E **76**, 061108 (2007)