Error analysis and optimization algorithm of focal shift on mode decomposition for few-mode fiber beam*

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Modal decomposition technology is an effective method to study the mode characteristics of laser beam in few-mode fibers. However, certain types of eigenmodes in the fiber can cause focal shift and affect the accuracy of modal decomposition. This article focuses on the influence of the focal shift of Laguerre-Gaussian mode and Linear Polarization mode on modal decomposition, and the research is based on correlation filter and the optimization algorithm of focal shift. The two-step ABCD algorithm is used to simulate and analyze the focal shift phenomenon of the two kinds of eigenmodes and the error influence of focal shift on the mode decomposition; Meanwhile, an iterative algorithm based on Fresnel diffraction is proposed to numerically calculate the light field distribution in focal plane to avoid the influence of focal shift errors. The focal shift analysis and its optimization algorithm make the modal decomposition technology be applicable to engineering applications.

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With the development of optical fiber technology, it is widely used in the fields of laser weapons[1], high-energy laser transmission[2], satellite communications[3] and other fields. Also, the energy transmitted in the optical fiber is gradually increasing. While high-energy density in optical fiber will cause nonlinear effects, mode competition and other unexpected phenomena result in poor beam quality and uneven power density[4]. Generally, the power density of the fiber output can be reduced by enlarging the core radius, but it will excite higher-order modes[5], causing signal crosstalk or noise increasing. Moreover, the reasons of the above beam quality deterioration cannot be analyzed by the beam quality evaluation factors. The modal decomposition technique can be exploited to calculate the weight and relative phase of eigenmodes, and then obtain the mode characteristics of the laser beam in optical fiber[6]. It is helpful for real-time monitoring of beam changes in the optical system, improving the design of the optical system, and providing a theoretical basis for optimizing beam performance parameters.

Mathematically, the eigenmodes in fiber is the solutions of the wave equation combined with different boundary conditions. In the ‘weakly guide’ fiber, the eigenmodes are considered to be the linear polarization modes (LP modes)[7]. When solving the wave equation in the polar coordinate system, the eigenmodes are considered to be the Laguerre-Gaussian modes (LG modes). However, the focal plane of LP mode and LG mode does not match the actual geometric focus during the focusing process[8,9], which affects the accuracy of the light intensity of rear focal plane of the lens. Because light intensity is the basis of modal decomposition, result of this calculation is also affected. In this article, the focal shift appearing in LP modes and LG modes is analyzed first; then, an iterative algorithm based on Fresnel diffraction is proposed to calculate the focused light intensity numerically and avoid the influence caused by inaccurate position of the rear focal plane, such as defocus, focus shift, etc.

Modal decomposition technology is actually a correlation filter based on eigenmode superposition. The correlation filter modulates the amplitude and phase of the beam excited from the end of the fiber. The liquid crystal phase modulator is used to modulate phase and corresponding algorithm dealing with the light intensity recorded by CCD camera to modulate amplitude[6]. Based on this, a correlation filtering laser beam can be achieved.

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The transmittance function of the correlation filter is shown as

$$T_n = \sum_n^{n_{\max}} \psi_n^*(u)\cdot \exp(\mathrm{i}\cdot f_n \cdot u), \tag{1}$$

where ψ_n and f_n refers to the eigenmode and the carrier frequency component, respectively. The transmittance function represents the superposition of eigenmode conjugates carrying different carrier frequencies. The imaginary part of $T_n(\mathrm{Im}[T_n])$ is coded as grayscale phase hologram uploaded in the Lc-SLM. The real part of $T_n(\mathrm{Re}[T_n])$ and the light intensity are handled by intensity algorithm[6]. The laser beam with a wavelength of 1 064 nm is coupled into a few-mode fiber, collimated and polarized after excited from fiber. Then the beam is vertically incident into the liquid crystal phase modulator (Lc-SLM), which is focused on the surface of the CCD camera through a lens to obtain the far-field diffracted light intensity. The CCD camera moves on the electronically controlled displacement platform to adjust the detection distance. The schematic diagram of the modal decomposition system is shown in Fig.1.

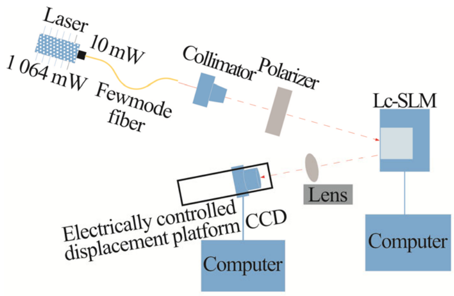


Fig.1 Simulation diagram of mode decomposition system

The incident light field can be referred as the superposition of eigenmode, which is shown as

$$U(u) = \sum_{n=1}^{n_{\max}} c_n \psi_n(u), \tag{2}$$

$$c_n = \rho_n \exp(\mathrm{i}\phi_n), \sum |c_n|^2 = \sum \rho_n{}^2 = 1, \tag{3}$$

where ρ_n, ρ_n^2 and φ_n represents the mode coefficient, the mode weight, and the inter-mode phase between the fundamental mode and the higher-order mode, respectively. The principle of modal decomposition could be simply summarized as follows. Since the transmittance function introduces the spatial carrier frequency f_n, the self-correlation distribution of eigenmodes are separated from each other in rear focal plane after the Fourier transform of the lens, according to the Fourier frequency shift theorem.

In the rear focal plane of the lens, the weight of the eigenmodes corresponds to the light intensity at its translation position:

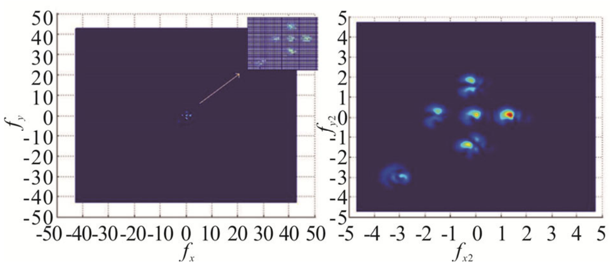
$$\left|\tilde{W}_f(u_n = \frac{\phi_n}{k_0} f)\right|^2 \propto |C_n|^2 = \rho_n^2 \cdot \tag{4}$$

The inter-mode relative phase can be obtained by combine Eqs.(5) and (6). Detailed derivation of these two formulas is introduced detailly in Ref.[10].

$$\left|W_f\left(u_n^{\cos} = \Delta\phi_n \frac{f}{k_0}\right)\right|^2 = \frac{1}{2}\left|A_0\left(C_0 + C_n\right)\right|^2 = \frac{1}{2}|A_0|^2\left[\rho_0^2 + \rho_n^2 + 2\rho_n\rho_0\cos(\Delta\phi_n)\right], \tag{5}$$

$$\left|W_f\left(u_n^{\sin} = \Delta\phi_n \frac{f}{k_0}\right)\right|^2 = \frac{1}{2}\left|A_0\left(C_0 + \mathrm{i}C_n\right)\right|^2 = \frac{1}{2}|A_0|^2\left[\rho_0^2 + \rho_n^2 - 2\rho_n\rho_0\sin(\Delta\phi_n)\right]. \tag{6}$$

From the theory, it can be seen that FFT is used to focus light beam, in which the size of image plane is restricted by the diffraction limit in simulation. Hence, the spot distribution acquired by Matlab software was too small and concentrated to observe, and the two-step ABCD algorithm was applied to adjust the size of sampling unit in image plane to improve the light pattern resolution[11,12]. The principle of two-step ABCD algorithm is based on double Fresnel diffraction and coordinate system substitution. Details of this algorithm is explained in Ref.[10]. In addition, the beam width is calculated by the second-order moment of light intensity. If the spot distribution is only a few bright spots, it is impossible to distinguish the change of light pattern. As a result, clear and definite light distribution is required for subsequent analysis. In simulation, it is assumed that there are 6 modes in the few-mode fiber and the initial weight coefficients and relative phases are set. The simulation results of direct FFT and two-step ABCD algorithm are illustrated in Fig.2. According to the result, it can be confirmed that two-step algorithm can obtain a higher resolution of light pattern than direct FFT in simulation.



(a) By FFT (b) By two-step ABCD algorithm

Fig.2 Separated light patterns by modal decomposition

It has been known from the researches that there is the focal shift in the lens and diaphragm system for Bessel beam[8,9]. In this part, we will analyze the focal shift in modal decomposition system for both LP and LG modes. Because of azimuth symmetry in both modes, the light intensity is zero at some points on the axis. Therefore, the second-order moment of light intensity of the rear focal plane is exploited to calculate the beam width. The position with the smallest beam width corresponds to the actual rear focal plane. The beam width in the x direction is defined as follows:

$$w_x^2 = \frac{4}{p}\int_{-\infty}^{+\infty} x^2 \left|E(x,z)\right|^2 \mathrm{d}x , \tag{7}$$

where p stands for total power.

$$p = \int_{-\infty}^{+\infty} \left|E(x,z)\right|^2 \mathrm{d}x . \tag{8}$$

The method for calculation of beam width in y direction is the same as above. It aims to calculate each beam width while changing the detecting position z. The relative distance is defined as

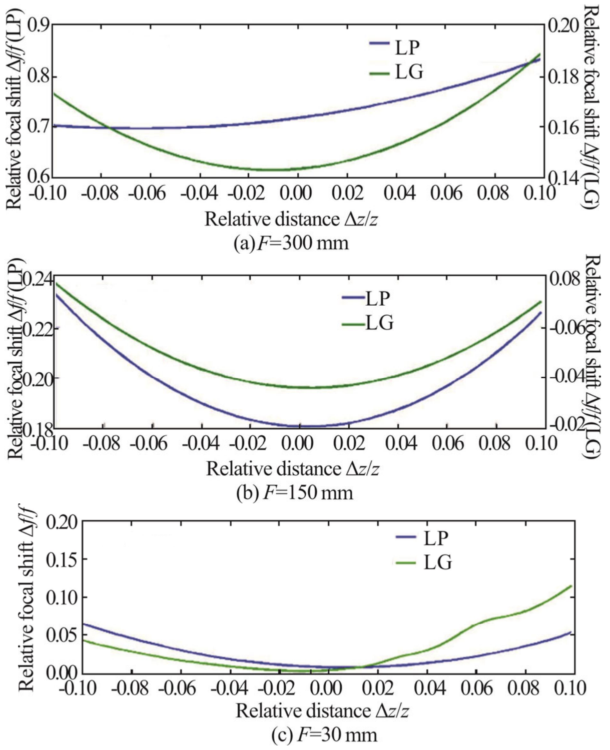
$$\frac{z-f}{f} = \frac{\Delta z}{f} , \tag{9}$$

where z represents detecting position of CCD camera, and f is the focal length. As result of focal shift, the beam is focused on other plane, named actual focal plan, not geometric focus. The relative focal shift between actual focal plane and geometric focus is defined as follows,

$$\frac{z_f - f}{f} = \frac{\Delta f}{f} , \tag{10}$$

where z_f represents actual focal plane.

The focal shift is related to the Fresnel number. However, the aperture size of the liquid crystal phase modulator is fixed, and only the focal length can be adjusted to change the Fresnel number. Therefore, multiple focal lengths are set in simulation. The focal shift phenomenon in the modal decomposition system is simulated and verified with results shown in Fig.3.



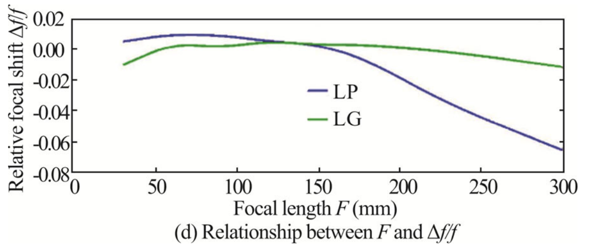


Fig.3 (a)-(c) Relationship between relative distance and relative focal shift; (d) Relationship between relative focal shift and focal length

It can be seen from Fig.3 that as for a smaller focal length (F=30 mm), the focal shift is not obvious, and the position with the smallest beam diameter is almost the same as the geometric focal position. As for larger focal lengths (F=150 mm and F=300 mm), there is a significant difference between the actual focus and geometric focus. Taking F=300 mm as an example, the relative focal shift is greater than 0.06, which indicates that the rear focal plane position shifts by 18 mm. This is a relatively large margin of error for practical engineering applications with high precision. When the focal length of the lens increases, the relative focal shift of the LP mode oscillates more, contributing to a smaller relative focal shift of LG mode. Also, the degree of focal shift is affected by different types of eigenmode. The modal decomposition system provides reference value for selecting lenses of different focal length and eigenmode mathematical models. It can be also applied in actual engineering applications to provide error reference.

The accuracy of the mode decomposition technology depends on the accuracy of rear focal plane light intensity, as only the rear focal plane can be regarded as a Fourier transform. This iterative algorithm is a simple one-way extreme value search algorithm, and can obtain the light field distribution of rear focal plane through numerical calculations. It is significant to avoid the focal shift error caused by the light intensity measurement at the geometric focal point. The basic principle of this algorithm is introduced in Fig.4.

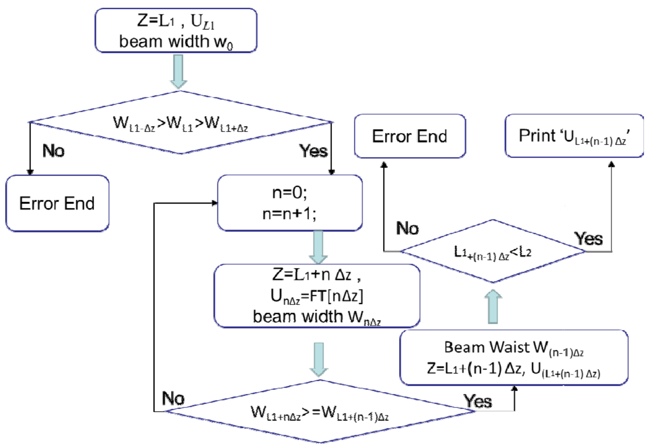


Fig.4 Principle of iterative algorithm

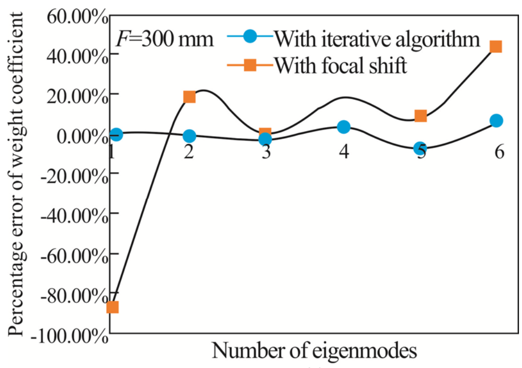
In Fig.4, FT stands for Fresnel operation, and Print stands for output. The amplitude distribution of U_{L1} can be obtained by taking the square root of light intensity at the position $z=L_1$ in front of the geometric focus. Setting a small displacement Δz along the propagation direction, the complex amplitude distribution $U_{L1+\Delta z}$ can be obtained according to Fresnel diffraction, which is illustrated as follows.

$$U_{L_1+\Delta z}=\iint U_{L_1}\cdot e^{i\frac{k}{2\Delta z}\left[(x-x_0)^2+(y-y_0)^2\right]}\mathrm{d}x_0\mathrm{d}y_0 . \quad (11)$$

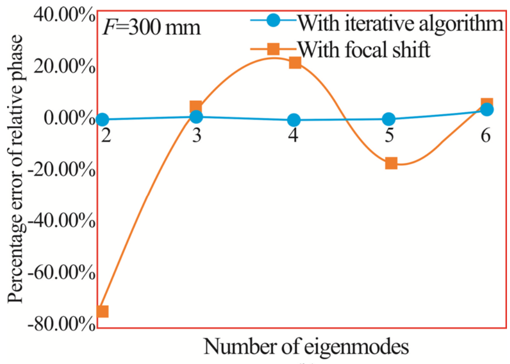
The beam width W is calculated by second-order moment of light intensity at each position. It is worth to know that it needs to determine the direction of the extreme value search, before running the algorithm. According to the characteristics of the focused beam, the beam width should decrease gradually, reaching a minimum at the focal plane. Only star running the iterative algorithm when the beam width is $W_{L1-\Delta z}>W_{L1}>W_{L1+\Delta z}$. The displacement of Δz is increased forward and calculating W (beam width) is repeated until the beam width stops reducing and becomes large. The minimum beam width is output and the actual focal plane position is found. The distance $z=L_2$ is selected behind the geometric focus. If the minimum beam width has not been found until position L_2, the algorithm has an error and should be terminated.

Next, the iterative algorithm is verified through simulation with result shown in Fig.5. The abscissae 1—6 in Fig.5 correspond to the 6 eigenmodes arranged in increasing order. Number 1 represents the fundamental mode of LP_{01} mode, and the initial phase is 0. So, there is no data at position of abscissa 1 in Fig.5(b).

It can be seen from the Fig.5 that the focal shift has a great influence on both the weight coefficient and relative phase, which is not beneficial for phase measurement. Optimized with the iterative algorithm, shown as the Bule Line in Fig.5, the error rate of both the weight coefficient and relative phase are significantly improved. Meanwhile, in an optical system where the incident beam has no focal shift, the errors caused by the misalignment of the detector position and the geometric focus can also be eliminated by this algorithm. Furthermore, the error analysis of focus shift provides reference value for the mode decomposition system when selecting lenses of different focal length and eigenmode types.



(a)



(b)

Fig.5 Comparison of error analysis between with focal shift and iterative algorithm

This article simulated and analyzed the effect of focal shift on the result of mode decomposition. It is found that phase of the mode decomposition result is influenced more by the focal shift than other factors. Also, change of the focal shift is affected by type of eigenmode and focal length of the lens. Besides, the focus shift of LP mode rather than LG mode is more sensitive to the focal length of lens. The simulation and analysis in this article provide reference and error correction for widely adoption of modal decomposition technology in engineering applications.

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