



## Research Article

# Enhancing Whale Optimization Algorithm with Chaotic Theory for Permutation Flow Shop Scheduling Problem

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## ABSTRACT

The permutation flow shop scheduling problem (PFSSP) is a typical production scheduling problem and it has been proved to be a nondeterministic polynomial (NP-hard) problem when its scale is larger than 3. The whale optimization algorithm (WOA) is a new swarm intelligence algorithm which performs well for PFSSP. But the stability is still low, and the optimization results are not too good. On this basis, we optimize the parameters of WOA through chaos theory, and put forward a chaotic whale algorithm (CWA). Firstly, in this paper, the proposed CWA is combined with Nawaz–Enesco–Ham (NEH) and largest-rank-value (LRV) rule to initialize the population. Next, chaos theory is applied to WOA algorithm to improve its convergence speed and stability. On this basis, we also use cross operator and reversal-insertion operator to enhance the search ability of the algorithm. Finally, the improved local search algorithm is used to optimize the job sequence to find the minimum makespan. In several experiments, different benchmarks are used to investigate the performance of CWA. The experimental results show that CWA has better performance than other scheduling algorithms.

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## 1. INTRODUCTION

The flow shop scheduling problem (FSSP) [1] is a job set allocation for a set of available machines in time to satisfy a set of performance indicators. Traditional FSSP includes a set of jobs, and each job is processed on all machines in a certain order. The goal of scheduling is to arrange jobs to each machine reasonably, and to arrange the processing order and start time of jobs reasonably, so that the constrained conditions can be used and some performance indicators can be optimized. Permutation flow shop scheduling problem (PFSSP) is an important problem in the actual production process, and it is one of the most difficult NP-hard problem [2]. In essence, it is a classical combinatorial optimization problem, involving  $n$  jobs and  $m$  machines. It is generally described that  $n$  jobs which needs to go through  $m$  processes need to be processed on  $m$  machines. Each process requires different machines, and  $n$  jobs have the same processing sequence on  $m$  machines which is different from FSSP. The traditional method is difficult to solve these problems, so the intelligent algorithms are required to solve these problems. Inspired by nature, these strong metaheuristic algorithms are applied to solve NP-hard problems such as scheduling [3–7], image [8–10], feature selection [11–13] and detection [14–16], path planning [17,18], cyber-physical social system [19,20], texture discrimination [21], factor evaluation [22], saliency detection [23], classification [24,25], object extraction [26], gesture segmentation [27], economic load

dispatch [28,29], shape design [30], big data and large-scale optimization [31–33], signal processing [34], silencing efficacy prediction [35], multi-objective optimization [36,37], unit commitment [38,39], vehicle routing [40], knapsack problem [41–43] and fault diagnosis [44–46], and test-sheet composition [47].

Many methods are used to solve FSSP, such as ant colony optimization (ACO) [48,49], artificial bee colony (ABC) [50], genetic algorithms (GAs) [51,52], and some hybrid algorithms [53–57]. Among hybrid methods, how to schedule resources reasonably and efficiently is an important factor affecting the efficiency of the model, which has been widely concerned for a long time. Up to now, the study of swarm has been very widely. Furthermore, the method of biotechnology was explored and proved to be effective. The above algorithms are optimized iteratively to simulate the evolution of nature, so that the result approaches to the optimal solution. These algorithms are mainly realized by simulating the biological characteristics of nature, e.g., whale optimization algorithm (WOA) [58] simulates predation behavior of humpback whales, and it is used to solve PFSSP [59]. There are many methods to solve PFSSP before, but they have some problems such as slow convergence and easy to fall into local optimum. WOA is a new type of algorithm to solve this problem, and it is widely used in various fields, such as wind speed forecasting system [60], image segmentation [61], economic dispatch problem [62], and neural network [63], which imitates the behavior characteristics of animal neural network and carries out distributed parallel information processing. According to the

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different industrial environment, the heuristic algorithm mainly optimize the following two parameters in the quality of service (QoS):

1. Makespan—optimize the maximum makespan of the job scheduling so as to ensure the highest efficiency of the job processing.
2. Running time—the algorithm requires lower running time.

However, the stability of WOA is poor, and the experimental results also have a lot of room for optimization, so we proposed an algorithm—chaotic whale optimization algorithm (CWA) which achieves a better stability and a smaller makespan than WOA. This algorithm uses the WOA model and improves it on this basis. Firstly, in initialization stage, WOA is combined with Nawaz-Enesco-Ham (NEH) algorithm, and largest-rank-value (LRV) rule is used to transform continuous whale population into discrete job sequence. In this way, a more suitable initial population can be generated. Secondly, CWA adopts cross selection strategy, including cross operator and reversal-insertion operator, which makes the generated search agent better. Thirdly, we improve the local search strategy so that the algorithm can escape from local optimal solutions. Finally, and most importantly, we adopt the chaotic mapping strategy, which makes the generation of the next search agent affected by the last search agent. We incorporated the above strategies into WOA algorithm, and a new variant of WOA namely CWA was proposed for solving PFSSP problem. The experimental results indicated that CWA has better performance than WOA.

The remainder of the paper is organized as follows: Some methods reported in the literatures for FSSP are provided in Section 2. The PFSSP is represented in Section 3. The standard WOA is described in Section 4 and CWA algorithm is discussed in Section 5. The experimental results of CWA are shown in Section 6. This work ends in Section 7 with the conclusions and future work.

## 2. RELATED WORK

In this paper, we introduced chaos theory into WOA and proposed CWA method to solve PFSSP. Therefore, the latest literatures on PFSSP, search algorithms, and chaos theory are summarized and reviewed as follows.

### 2.1. Permutation Flow Shop Scheduling Problem

In the field of PFSSP, bio-inspired algorithms have achieved a lot of excellent results and PFSSP was proposed by Johnson in 1954 [64]. Over the next few years, there are also many mature algorithms that have been applied. Rossit *et al.* [65] reviewed the related work of PFSSP past decades and provided the future work orientation of PFSSP. Han *et al.* [66] proposed a bi-objective model consisting of robustness and stability, which improved the efficiency of flow shop scheduling and optimized the problem. At the same time, in order to reduce the makespan, Ruiz *et al.* [67] improved the initialization and processing of the algorithm, and enhanced the ability of local search. Liu *et al.* [68] proposed a different priority rule using skewness used in NEH [69] algorithm to reduce the

makespan of PFSSP. Santucci *et al.* [70] made a special treatment of the search operator and changed the selection strategy, which made the algorithm reach an excellent level. ABC algorithm [71] is swarm intelligence algorithm that mimic natural creatures. Li *et al.* [72] studied bionic principles and applied ABC to solve PFSSP to reduce the makespan. Sioud and Gagné [73] chosen a suitable neighborhood and improved the migrating bird optimization algorithm by Tabu list and restart method. Finally, it is applied to PFSSP to complete the optimization of this problem. Pagnozzi and Stützle [74] redefined the framework of random local search algorithm (RLSA), and automatically matched the search algorithm to improve the performance of RLSA. In this way, the minimum makespan can be obtained efficiently. All the above algorithms have improved the flow shop scheduling, which provided a good reference for us to reduce makespan of PFSSP.

## 2.2. Search Algorithms

In terms of the implementation of search algorithm, there have been many fruitful research results. Wang and Tan [75] investigated a new idea of search algorithm, which is to reuse the previous individual search information. This method improved the quality of the subsequent population and had a good application in PFSSP. Benavides and Ritt [76] observed that the optimal scheduling usually includes a few local job inversion permutation structures, so they proposed an iterative local search algorithm. Gao *et al.* [77] proposed a new hybrid search algorithm, which introduced the balance strategy into the improved particle swarm optimization algorithm, and then combined with tabu search to achieve efficient search for FSSPs. Wang and Wang [78] improved the search algorithm according to the characteristics of other search algorithms. It included the improved initialization scheme based on NEH and other properties based on knowledge-based search algorithm. In addition, it also used critical path to improve the performance of the search algorithm. Kwong *et al.* [79] optimized the algorithm by changing the search direction of the search algorithm, and combined learning strategies to optimize the performance of the algorithm. In addition, we can combine different strategies to study new search algorithms. Fu *et al.* [80] combined different strategies such as explosive spark and selective solution, proposed a firework algorithm, and has a good optimization for flow shop scheduling.

Through the study on the search algorithms of flow shop scheduling, we proposed a strategy to change the search agent, and introduced a probability function to make the search results escape from the local optimal solution. These methods can achieve good results for PFSSP problems.

## 2.3. Chaos Theory

Chaos theory is the focus of this paper, and many achievements have been achieved in this field. Wang *et al.* [81] proposed a chaotic krill herd (KH) algorithm by studied the behavioral specialty of krill swarm and combined chaos theory. They used 12 chaotic mapping methods to improve the algorithm. Mokhtari and Noroozi [82] studied the chaotic map attraction characteristics in particle swarm optimization (PSO) and applied it to flow shop scheduling. Alatas [83] used chaotic mapping to improve the convergence of the algorithm and enabled ABC algorithm to escape from local optimal

solution. At the same time, chaos theory can also be used in hybrid algorithm. Wang *et al.* [84] combines chaos theory with cuckoo algorithm to improve the performance of the algorithm.

In addition, chaos theory has a very important application in cryptography and image field, through which, the useful knowledge has been investigated. Xie *et al.* [85] combined cryptography with chaos and proposed a new encryption scheme, which is not inferior to the previous one. Abbasinezhad-Mood and Nikooghadam [86] used chaotic mapping to solve the key exchange problem in the process of cryptographic authentication. Chaotic mapping not only improved the security of the problem, but also made the method anonymous. Xu *et al.* [87] mainly proposed a bit-level image encryption algorithm based on chaotic mapping, which converted one image into two, and then replaced each other by chaotic control, thus it can complete image encryption. This encryption method has good encryption. The application of chaos theory is very extensive. We have applied the above ideas to PFSSP and achieved outstanding results.

Although there are many researches on flow shop scheduling, PFSSP still does not achieve the minimum makespan. By further improving the search algorithm and introducing chaos theory, the stability of the algorithm is greatly increased and the makespan is reduced.

### 3. PERMUTATION FLOW SHOP SCHEDULING PROBLEM

The mathematical description of the PFSSP with minimizing makespan is as follows.

The machine  $M = \{M_1, M_2, M_3, \dots, M_m\}$  processing jobs  $N = \{J_1, J_2, J_3, \dots, J_n\}$ , in which each processing job contains an operator set  $O = \{O_{i1}, O_{i2}, O_{i3}, \dots, O_{im}\}$ , and each processing job passes through  $m$  machines in turn. The processing time of the processing job  $J_j$  on the machine  $M_i$  is given, and it is recorded as  $P_{ij}$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ).

PFSSP has the following requirements:

1. Each job  $j = 1, 2, \dots, n$  has the same machining sequence on the machine;
2. Each machine processes each job in the same processing sequence;
3. Each machine can only process one job at a time;
4. Each job can only be processed by one machine at a time;
5. The job must not be interrupted during the processing;
6. The buffer is infinite.

The processing sequence of jobs is  $S = \{S_1, S_2, S_3, \dots, S_n\}$ , and  $n$  jobs will follow  $m$  machines in turn. The completion time of job  $S_j$  on machine  $m$  is represented by  $C(s_j, m)$ . According to the processing sequence  $S = \{S_1, S_2, S_3, \dots, S_n\}$ , the completion time of the PFSSP can be given as follows:

$$C_{(s_1,1)} = P_{s1,1} \quad (1)$$

$$C_{(s_j,1)} = C_{(s_{j-1},1)} + P_{s_j,1}, j = 2, 3, \dots, n, \quad (2)$$

$$C_{(s_j,i)} = C_{(s_1,i-1)} + P_{s_j,i}, i = 2, 3, \dots, m, \quad (3)$$

$$C_{(s_j,i)} = \max \left\{ C_{(s_{j-1},i)}, C_{(s_j,i-1)} \right\} + P_{s_j,i}, \quad (4)$$

$$j = 2, 3, \dots, n; i = 2, 3, \dots, m.$$

Therefore, the maximum completion time (makespan) can be defined as:  $C_{\max} = C(s_n, m)$ . The goal of PFSSP is to find the most appropriate permutation  $S^*$  to minimize the maximum completion time of the sequence, i.e., to minimize makespan.

## 4. WOA AND CHAOTIC MAPS

### 4.1. Whale Optimization Algorithm

WOA [58] is a new swarm intelligence algorithm, which simulates the process of humpback whale predation, identifies prey, and circles around it. Algorithm 1 shows the implementation process of WOA. The WOA mainly consists of the three following processes: encircling prey, bubble-net attacking, and search for prey.

#### 4.1.1. Encircling prey

Humpback whales swirl around prey in the course of predation and weave foam nets, and prey is gathered by bubbles. The voice makers of humpback whales make specific sounds at the bottom to push their prey to the surface. The best search location is determined by the location of the prey or the location close to the prey, and then updated according to the best search location. The strategy can be shown below:

$$D = |C \cdot x^* - x^t| \quad (5)$$

$$x^{t+1} = x^* - A \cdot D \quad (6)$$

$$A = 2a \cdot r - a \quad (7)$$

$$C = 2 \cdot r \quad (8)$$

where  $t$  is number of iterations,  $x^*$  is the best location of search agent,  $x^t$  is the present location of search agent at iteration  $t$ . The parameter  $a$  decreases linearly from 2 to 0, and  $r$  is a random value between 0 and 1, so the range of  $A$  is  $-a$  to  $a$ .

#### 4.1.2. Bubble-net attacking

Bubble-net attacking includes the following two behaviors: shrink ring of encirclement and continue to encircling prey. These two behaviors are modeled as the following equation:

$$x^{t+1} = E \cdot e^{b \cdot l} \cdot \cos(2\pi l) + x^* \quad (9)$$

$$E = |x^* - x^t| \quad (10)$$

where  $b$  describes the shape of the logarithmic spiral,  $l$  is a random number between 0 and 1, and  $E$  is the distance between  $x^*$  and  $x^t$ .

Humpback whales surround their prey and shrink their range, so the values of both behaviors are 1/2.

$$x^{t+1} = \begin{cases} x^* - A \cdot D & k > 0.5 \\ E \cdot e^{bl} \cdot \cos(2\pi l) + x^* & k \leq 0.5 \end{cases} \quad (11)$$

#### 4.1.3. Search for prey

The exploration of WOA can be achieved by random search agent  $x_{rand}$ , which is different from exploitation using  $x^*$ . This process can be given as follows:

$$D = |C \cdot x_{rand} - x^t| \quad (12)$$

$$x^{t+1} = x_{rand} - A \cdot D \quad (13)$$

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#### Algorithm 1: The standard WOA

```

Initialize a population of  $n$  random search agents
Evaluate all  $n$  search agents
 $w^*$  = the best search agent
 $t = 0$ 
While ( $t <$  iterations)
  for each search agent
    Update WOA parameters ( $a$ ,  $A$ ,  $C$ ,  $l$ , and  $k$ )
    if ( $k < 0.5$ )
      if ( $|A| < l$ )
         $x^{t+1} = w^* - A \cdot D$ 
      else if ( $|A| \geq l$ )
         $x^{t+1} = x_{rand} - A \cdot D$ 
      end if
    else if ( $k \geq 0.5$ )
       $x^{t+1} = E \cdot e^{bl} \cdot \cos(2\pi l) + w^*$ 
    end if
  end for
  Evaluate the search agent  $x^{t+1}$ 
  Update  $w^*$  if  $x^{t+1}$  is better
   $t = t + 1$ 
end while
return  $w^*$ 
```

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## 4.2. Chaotic Maps

In this paper, the concept of chaotic optimization algorithm (COA) [88] is introduced, chaotic variables are used. Random variables make the results of functions randomly, while the use of chaotic variables avoids randomness, which makes the results more stable [81]. The nonrepetition and ergodicity of chaos makes the method using chaotic variables have faster convergence speed than the method using random variables [88]. In this paper, a one-dimensional irreversible chaotic mapping function is investigated. Table 1 lists twelve different chaotic maps [81].

## 5. THE PROPOSED ALGORITHM

As a new intelligent algorithm, chaos optimization algorithm (COA) [81] has been widely studied and has been applied in many fields. In this paper, COA is firstly applied to the WOA of PFSSP, and a new algorithm called CWA is then proposed. CWA maps chaotic variables linearly to the value range of optimization variables, and optimizes the search process by using the randomness, ergodicity, and regularity of chaotic variables. For random variables, chaotic mapping changes the stability of variables and optimizes the convergence speed and performance of WOA. There are four processes that will be discussed below. The first three processes of CWA and WOA are similar, but CWA has more optimization of chaotic mapping process than WOA, so CWA has better performance than WOA. The time complexity of the four processes are  $O(N)$ ,  $O(N^2)$ ,  $O(N)$ ,  $O(N)$ , therefore, the time complexity of CWA is  $O(N^2)$ .

### 5.1. Initialization

For the initialization of PFSSP, firstly, we need to solve the problem of transforming continuous search agents into discrete job sequences. To solve this problem, we use the LRV rule [89] in CWA, which can effectively transform continuous values into discrete sequences. Secondly, we use the NEH algorithm [90] to initialize population. It includes the following steps:

1. These  $n$  jobs are arranged according to the decreasing order of machining time on the machine;
2. Take the first two jobs in the operation sequence and select the job sequence that minimized the makespan;
3. From the beginning of the third job, each job is inserted into each location sequence that already exists, and the sequence having smallest makespan is selected.

Afterwards, we need to calculate the maximum makespan of the generated sequence, i.e., to have a direct research of search agent performance, so as to get the best search agent. For example, we design a work sequence as shown in Table 2 and show the optimal scheduling sequence of this sequence, as shown in Table 3. Among them, the calculation of optimal scheduling sequence 1-4-3-2 is obtained by Eqs. (1–4). Some other sequences of this job scheduling are also shown in Table 4. The optimal job scheduling sequence is 1-4-3-2, corresponding to the completion time of this scheduling sequence, i.e., the optimal completion time, 64. Figure 1 shows the Gantt chart of the sequence 1-4-3-2.

### 5.2. Cross Selection

In this strategy, two operators are adopted in the experiment, namely cross operator and reversal-insertion operator.

#### 5.2.1. Cross operator

This operator mainly exchanges the sequence of any two jobs and searches again. The main steps are as follows:

- (a) Two locations are randomly selected in the search agent, which are two jobs in the job sequence;

**Table 1** | Chaotic maps.

No.	Name	Definition
1	Chebyshev map	$x_{k+1} = \cos(k \cdot \cos^{-1}(x_k))$
2	Circle map*	$x_{k+1} = x_k + 0.2 - (1/4\pi) \sin(2\pi k) \text{mod}(1)$
3	Gaussian map	$x_{k+1} = \begin{cases} 0 & x_k = 0 \\ 1/x_k \text{mod}(1) & \text{otherwise}, 1/x_k \text{mod}(1) = \frac{1}{x_k} - \left[ \frac{1}{x_k} \right] \end{cases}$
4	Intermittency map	$x_{k+1} = \begin{cases} \varepsilon + x_k + cx_k^n & 0 < x_k \leq P \\ \frac{x_k - P}{1-P} & P < x_k < 1 \end{cases}$
5	Iterative map	$x_{k+1} = \sin\left(\frac{m\pi}{x_k}\right), m \in (0, 1)$
6	Liebovitch map	$x_{k+1} = \begin{cases} \alpha x_k & 0 < x_k \leq P \\ \frac{P-x_k}{P_2-P_1} & P_1 < x_k \leq P_2 \\ 1 - \beta(1 - x_k) & P_2 < x_k \leq 1 \end{cases}$
7	Logistic map	$x_{k+1} = 0.5x_k(1 - x_k)$ $\begin{cases} \frac{x_k}{P} & 0 \leq x_k < P \\ \frac{x_k - P}{0.5 - P} & P \leq x_k < 0.5 \\ \frac{1 - P - x_k}{0.5 - P} & 0.5 \leq x_k < 1 - P \\ \frac{1 - x_k}{P} & 1 - P \leq x_k < 1 \end{cases}$
8	Piecewise map	$x_{k+1} = \begin{cases} \frac{x_k}{0.7} & x_k < 0.7 \\ \frac{10}{3}(1 - x_k) & x_k \geq 0.7 \end{cases}$
9	Sine map	$x_{k+1} = \frac{a}{4} \sin(\pi x_k), 0 < a \leq 4$
10	Singer map	$x_{k+1} = \mu(7.86x_k - 23.31x_k^2 + 28.75x_k^3 - 13.302875x_k^4)$
11	Sinusoidal map	$x_{k+1} = 0.5x_k^2 \sin(\pi x_k)$
12	Tent map	$x_{k+1} = \begin{cases} \frac{x_k}{0.7} & x_k < 0.7 \\ \frac{10}{3}(1 - x_k) & x_k \geq 0.7 \end{cases}$

**Table 2** | Job scheduling example.

	M1	M2	M3
J1	5	6	11
J2	8	4	7
J3	11	9	3
J4	14	15	20

**Table 3** | Evaluation of optimal scheduling sequence.

	M1	M2	M3
J1	5	11	22
J4	19	34	54
J3	30	43	57
J2	38	47	64

**Table 4** | Evaluation of different job scheduling.

Number	Job Sequence	Makespan
1	2-3-4-1	79
2	4-1-3-2	70
3	3-1-2-4	73
4	4-3-2-1	70
5	1-4-3-2	64

(b) Exchange the order of these two jobs;

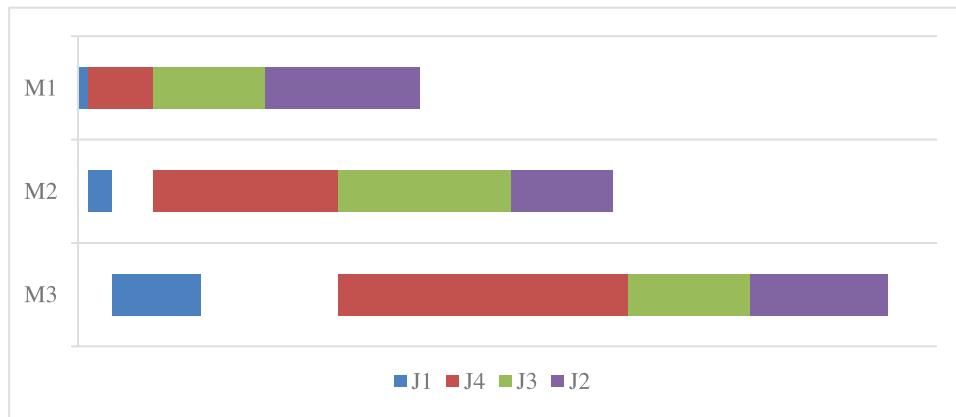
(c) Evaluate makespan for new sequences.

The details of cross operator can be shown in Figure 2. The number of cross operators is related to the number of jobs. In this experiment, we set the number of cross operators as one percent of the number of jobs.

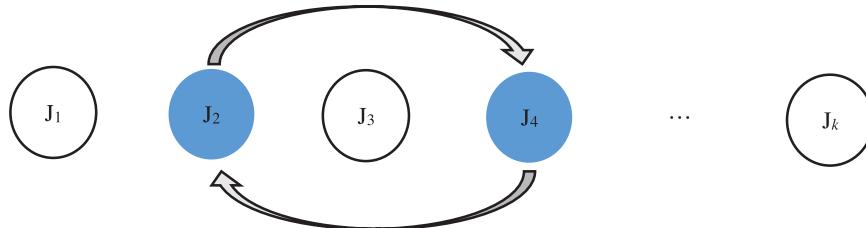
### 5.2.2. Reversal-insertion operator

This operator is mainly to exchange the sequence of two adjacent jobs and insert them into other locations. The main steps are as follows:

- (a) Two adjacent locations are selected in the search agent, which are two adjacent jobs in the job sequence;
- (b) Swap the sequences of these two jobs and remove them from the current sequence which includes  $k (1 \leq k \leq n)$  jobs;
- (c) Insert the exchanged jobs into the  $k - 2$  possible locations in the sequence;
- (d) Evaluate makespan for new sequences.



**Figure 1** | The Gantt chart for the job sequence 1-4-3-2.



**Figure 2** | Cross operator.

The details of cross operator can be shown in Figure 3.

### 5.3. Local Search

The improved local search strategy is used to find better results than those obtained by WOA. For each job at generation  $t$ , move the job to each position in the sequence and find the minimum makespan. After the above strategies, an optimal search agent has been generated. The operator process can be provided as follows:

1. All search agents are orderly selected, and each selected search agent is removed from the job sequence which includes  $n$  jobs;
2. Insert the exchanged jobs into the  $n - 1$  possible locations in the sequence and the makespan of the new sequence obtained by each insertion operator is calculated, while the minimum makespan is selected;
3. In order to avoid redundant computations, we make this strategy with small probability  $P$ .

### 5.4. Chaotic Mapping

According to Eqs. (11–13), the variable  $r$  affects the generation of search agents at the next iteration, which changes the performance and convergence of global search. Therefore, it is of great significance for the convergence of WOA and the generation of the next population.

Although the performance of standard WOA is satisfactory, there are still some deviations from the optimal results for PFSSP. In WOA, the variable  $r$  is randomly chosen between 0 and 1, which makes the next search agent more volatile. After introducing chaotic

theory, we used chaotic mapping to change these parameters, which reduced the randomness of the algorithm and improved the performance and stability of the algorithm. Therefore, WOA using chaotic mapping is called chaotic WOA.

Algorithm 2 gives the basic process of CWA, and Figure 4 gives the basic framework of CWA.

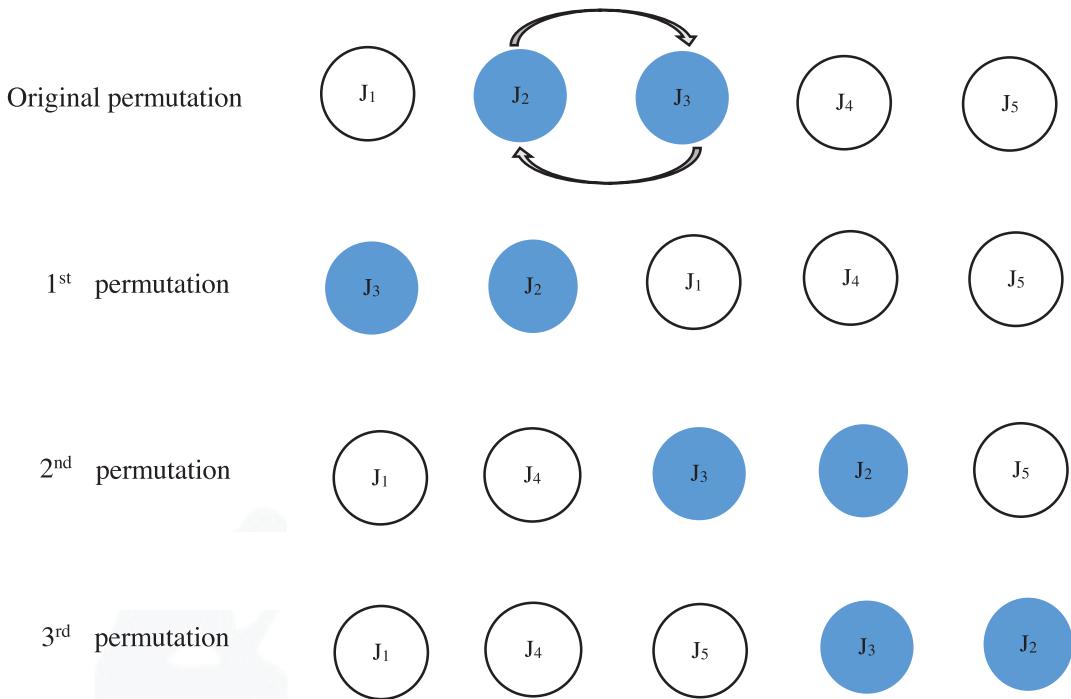
From Tables 5 and 6, we compare twelve different chaotic maps for WOA optimization through Reeves and Taillard benchmarks. M2 (Circle map) shows the best performance, so we get the Eq. (14), which is the best chaotic mapping of the variable  $r$ .

$$r^t = r^{t-1} + b - (a/2\pi) \sin(2\pi t) \bmod(1) \quad (14)$$

where  $r^t$  is the variable  $r$  at iteration  $t$ , and in CWA,  $a = 0.5$  and  $b = 0.2$ .

## 6. EXPERIMENTAL RESULTS

In the field of flow shop scheduling, there are some well-known test functions studied in this paper to investigate the performance of CWA. Carlier consists of eight instances [91] and there is Reeves datasets [92] with twenty-one instances. In addition, there is only two instances, Heller data [93]. All of them can be found at <http://people.brunel.ac.uk/~mastjeb/jeb/orlib/files/flowshop1.txt>. The last one can be found at <http://mistic.heig-vd.ch/taillard/problems.dir/ordonnancement.dir/ordonnancement.html> by Taillard [94] which has the largest number of instances (120). The proposed algorithm was written in Java and was run in the following conditions: Intel(R) Core(TM) i3-8100 CPU 3.60GHz which possesses 8G of RAM on Windows 10. The parameters of the proposed algorithm are presented in Table 7.



**Figure 3 |** Reversal-insertion operator.

## 6.1. The Comparison of WOA and CWA

Table 8 and Figure 5 show the standard deviation (STD) of WOA and CWA, which is defined as follows:

$$STD = \sqrt{\frac{\sum_{k=1}^{num} (x_k - x_{k+1})^2}{num-1}} \quad (15)$$

where *num* is the number of runs per problem,  $x_k - x_{k-1}$  represents the difference between two adjacent experimental results.

Table 8 shows the STD of the Reeves and Taillard benchmarks. The lower the numerical value, the higher the stability of the algorithm. In Figure 5, the numerical value of CWA is always lower than that of WOA, which proves that the addition of chaotic factor increases the stability of the algorithm.

## 6.2. The Comparison with Carlier, Reeves, and Heller Benchmarks

In this section, we compare the following algorithm with our CWA algorithm, as shown in Table 9. The comparative algorithms are as follows:

- Hybrid genetic algorithm (HGA) [95]. HGA is a hybrid GA, which is used to solve the no wait FSSP with completion period as the goal.
- Hybrid backtracking search algorithm (HBSA) [96]. Based on backtracking search algorithm (BSA), a HBSA is proposed for PFSP with the objective to minimize the makespan.

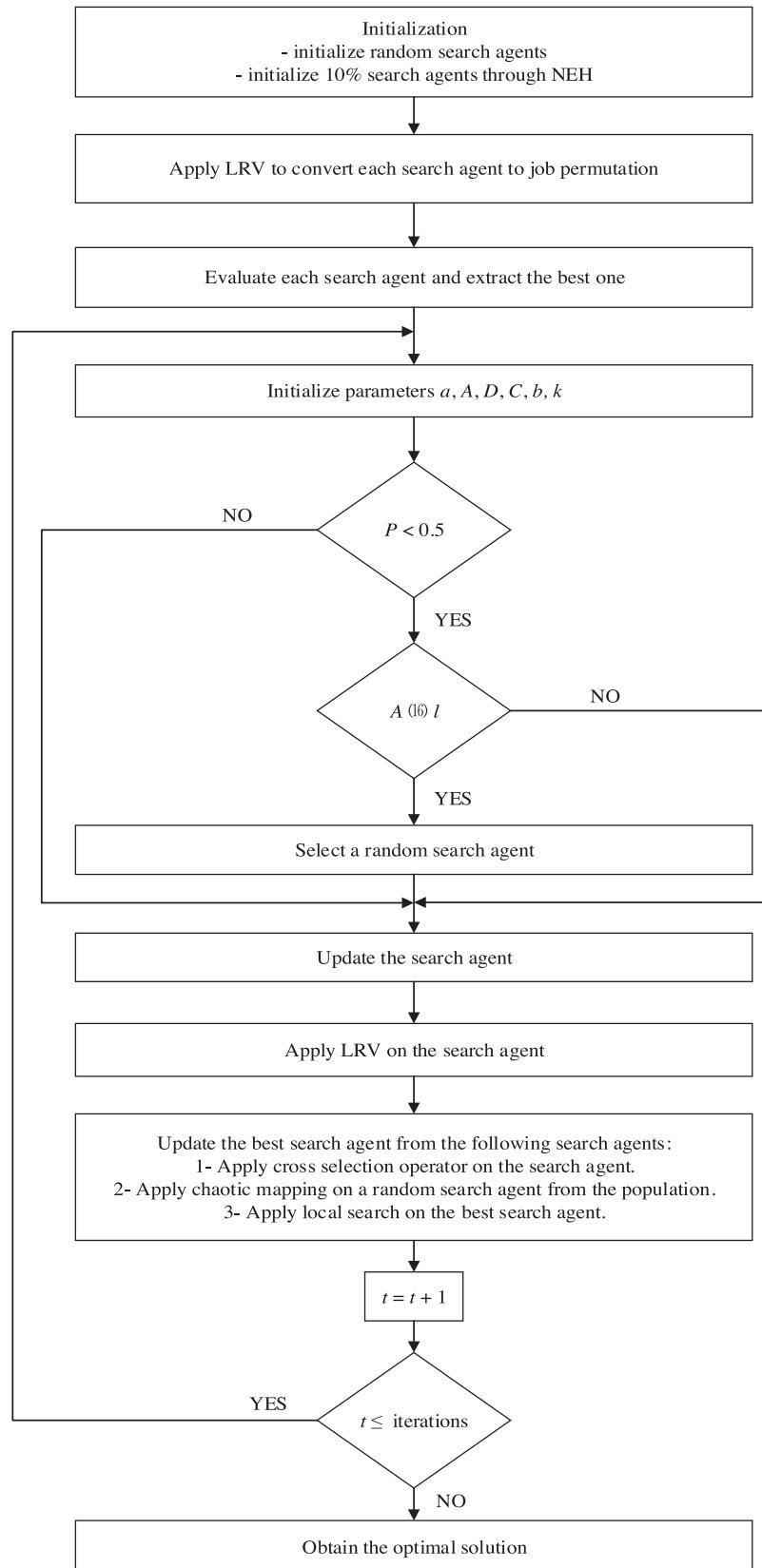
- Discrete bat algorithm (DBA) [97]. DBA is constructed based on the idea of basic bat algorithm (BA), which divide whole scheduling problem into many subscheduling problems and then NEH heuristic be introduced to solve subscheduling problem.
- Particle swarm optimization-based memetic algorithm (PSOMA) [98]. In the proposed PSOMA, both PSO-based searching operators and some special local searching operators are designed to balance the exploration and exploitation abilities. And the PSOMA utilizes several adaptive local searches to perform exploitation.
- Particle swarm optimization based on variable neighborhood search (PSOVNS) [99]. A very efficient local search, called variable neighborhood search (VNS), was embedded in the PSO algorithm to solve the well-known benchmark suites in the literature.

In Table 9, we use the best relative error (BRE), the average relative error (ARE), and the worst relative error (WRE) to measure the algorithm [59]. They are provided as follows:

$$BRE = \frac{v^* - v_{best}}{v^*} \quad (16)$$

$$ARE = \frac{v^* - v_{avg}}{v^*} \quad (17)$$

$$WRE = \frac{v^* - v_{worst}}{v^*} \quad (18)$$



**Figure 4** | Framework of chaotic whale algorithm (CWA).

**Algorithm 2:** The proposed CWA

---

Initialize a population of  $n$  random whales search agents  
Apply LRV on each search to be mapped into job permutation  
Solve the PFSSP using NEH  
Choose about 10% random search agents from the population and replace them with NEH  
Evaluate all  $n$  search agents  
 $w^*$  = the best search agent  
find  $w^*$   
 $t = 0$   
**While** ( $t < \text{iterations}$ )  
  **for** each search agent  
    Update WOA parameters ( $a, A, C, l$ , and  $k$ )  
    **if** ( $k < 0.5$ )  
      **if** ( $|A| < l$ )  
         $x^{t+1} = w^* - A \cdot D$   
      **else if** ( $|A| \geq l$ )  
         $x^{t+1} = x_{rand} - A \cdot D$   
      **end if**  
    **else if** ( $k \geq 0.5$ )  
       $x^{t+1} = E \cdot e^{b \cdot l} \cdot \cos(2\pi l) + w^*$   
    **end if**  
  **end for**  
  Apply LRV on each search agent  $x^{t+1}$   
  Perform cross selection strategy on the search agent  $x^{t+1}$   
  **if**  $f(x^{t+1}) < f(w^*)$   
     $w^* = x^{t+1}$   
  **end if**  
  Perform Chaotic mapping strategy on the search agent  $x^{t+1}$   
  **if**  $f(x_r) < f(w^*)$   
     $w^* = x_r$   
  **end if**  
   $w^*$  = Perform local search strategy on the best search  $w^*$   
  Evaluate the search agent  $x^{t+1}$   
  Update  $w^*$  if  $x^{t+1}$  is better  
   $t = t + 1$   
**end while**  
**return**  $w^*$

---

**Table 5** | The comparison of twelve chaotic maps on the Reeves benchmark.

No.	REC07	REC13	REC19	REC27	REC31	REC39
M1	1566.9	1964.2	2120.7	2418.4	3072.3	5170.3
M2	<b>1566.9</b>	<b>1938.8</b>	<b>2104.3</b>	<b>2396.8</b>	<b>3070.2</b>	<b>5167.0</b>
M3	1568.1	1966.1	2121.8	2416.2	3078.3	5177.2
M4	1568.7	1966.5	2118.9	2417.6	3079.8	5170.1
M5	1568.9	1950.6	2110.3	2402.0	3080.7	5167.8
M6	1568.2	1956.7	2120.3	2416.6	3077.1	5173.6
M7	1567.8	1966.9	2120.6	2417.0	3085.8	5169.9
M8	1568.7	1965.3	2120.7	2398.7	3075.2	5170.1
M9	1571.4	1961.0	2125.4	2414.7	3079.8	5168.2
M10	1566.9	1962.3	2118.4	2416.8	3071.8	5168.6
M11	1568.8	1969.7	2122.7	2411.4	3071.1	5169.0
M12	1569.2	1940.4	2106.2	2410.8	3070.9	5183.7

The best experimental results are shown in **bold**.

where  $v^*$  is the best scheduling result up to date,  $v_{best}$  is the best experimental result in this experiment,  $v_{avg}$  is the average value of the experimental results,  $v_{worst}$  is the worst experimental result in this experiment.

**Table 6** | The comparison of twelve chaotic maps on the Taillard benchmark.

No.	Ta001	Ta011	Ta021	Ta031	Ta041	Ta051
M1	1281.8	1610.6	2346.4	2730.3	3054.3	3967.8
M2	<b>1278.2</b>	<b>1604.0</b>	<b>2346.0</b>	<b>2720.6</b>	<b>3043.6</b>	<b>3966.7</b>
M3	1279.9	1612.1	2355.9	2729.7	3045.8	3966.8
M4	1278.2	1604.1	2348.2	2732.4	3051.0	3974.7
M5	1278.2	1610.7	2356.6	2734.0	3045.8	3966.9
M6	1281.8	1610.7	2359.9	2730.0	3046.0	3966.8
M7	1279.9	1607.3	2354.8	2732.1	3047.9	3972.2
M8	1279.9	1608.5	2355.3	2730.3	3046.8	3971.0
M9	1281.9	1608.5	2354.8	2731.4	3043.6	3975.7
M10	1280.9	1609.0	2348.5	2731.0	3047.1	3968.9
M11	1279.9	1609.6	2349.5	2730.4	3049.1	3967.5
M12	1279.9	1610.5	2353.5	2729.4	3049.8	3971.3

No.	Ta061	Ta071	Ta081	Ta091	Ta101	Ta111
M1	5493.1	5804.0	6382.4	10939.0	11417.6	26572.2
M2	<b>5493.0</b>	<b>5798.4</b>	<b>6370.0</b>	<b>10914.0</b>	<b>11401.0</b>	<b>26550.1</b>
M3	5493.0	5826.0	6374.7	10927.9	11404.3	26554.3
M4	5493.0	5809.1	6380.1	10931.2	11411.2	26557.0
M5	5493.1	5799.3	6374.6	10927.2	11401.4	26540.2
M6	5493.0	5803.0	6384.4	10933.0	11415.8	26566.3
M7	5493.0	5801.8	6370.6	10934.2	11407.6	26557.4
M8	5493.0	5800.6	6387.8	10921.1	11403.7	26551.7
M9	5493.0	5800.3	6382.0	10938.6	11406.8	26554.2
M10	5493.1	5801.2	6383.0	10922.6	11408.4	26572.7
M11	5493.0	5799.6	6385.2	10927.9	11408.6	26550.1
M12	5493.0	5803.0	6371.5	10926.2	11412.0	26576.5

The best experimental results are shown in **bold**.

**Table 7** | The parameters of the proposed algorithm.

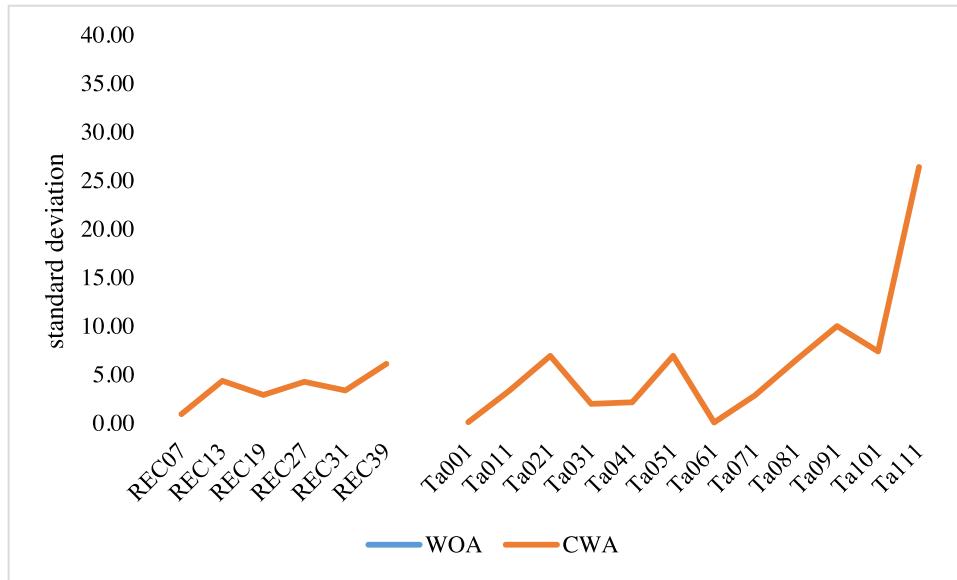
Parameter	Value
Population size	50
Number of iterations	3000
Small probability $P$	0.01
Number of runs	20

**Table 8** | The standard deviation of WOA and CWA.

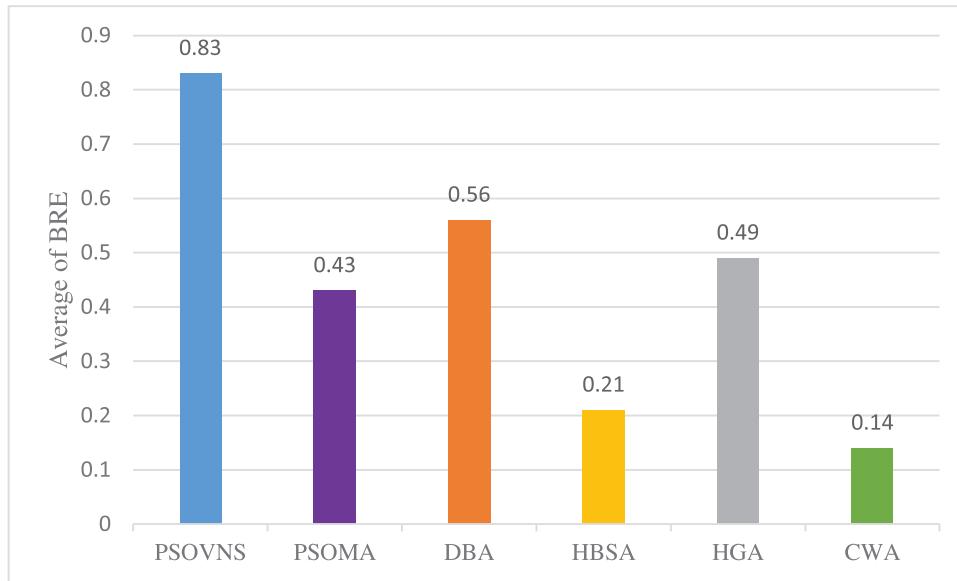
Problem	WOA	CWA
REC07	9.44	0.92
REC13	11.57	4.36
REC19	11.43	2.91
REC27	13.21	4.26
REC31	15.56	3.36
REC39	13.06	6.09
Ta001	6.39	0.10
Ta011	7.26	3.32
Ta021	12.69	6.93
Ta031	13.05	1.98
Ta041	12.75	2.14
Ta051	13.48	6.92
Ta061	18.99	0.05
Ta071	18.99	2.85
Ta081	10.79	6.48
Ta091	26.94	9.99
Ta101	18.37	7.38
Ta111	33.48	26.38

Figures 6–8 shows the average value of BRE, ARE, and WRE on Table 9 about PSOVNS, PSOMA, DBA, HBSA, and CWA. The average values of CWA of BRE, ARE, and WRE are 0.14, 0.44, 0.73, respectively.

Table 9 shows the results of comparisons with other functions on Car and Rec benchmarks. The value in **bold** represents the best result. Looking at the Table 8, the scheduling results for each



**Figure 5** | The standard deviation of whale optimization algorithm (WOA) and chaotic whale algorithm (CWA).



**Figure 6** | Average of best relative error (BRE).

instance are different, but CWA's scheduling results are the best in both Car and Rec benchmarks.

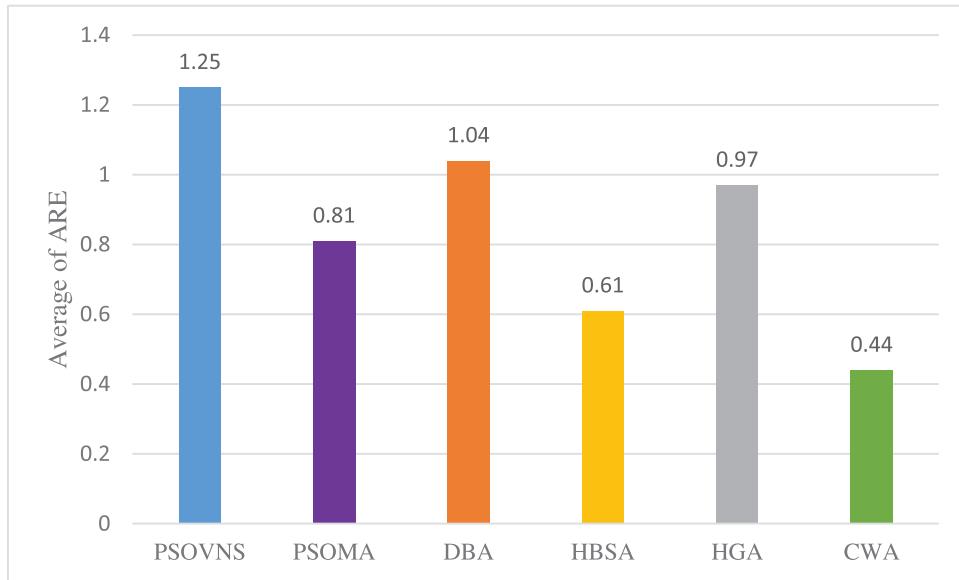
Figure 6 reveals the average value of BRE. From Figure 6, CWA shows a shorter maximum completion time than the other five algorithms, and the scheduling results of HBSA are second only to CWA in terms of scheduling results.

Figure 7 displays the average of ARE. Compared with the best and worst values, the average value is the most suitable value to represent the performance of the algorithm. From Figure 7, for this case, CWA scheduling results are still better than other algorithms.

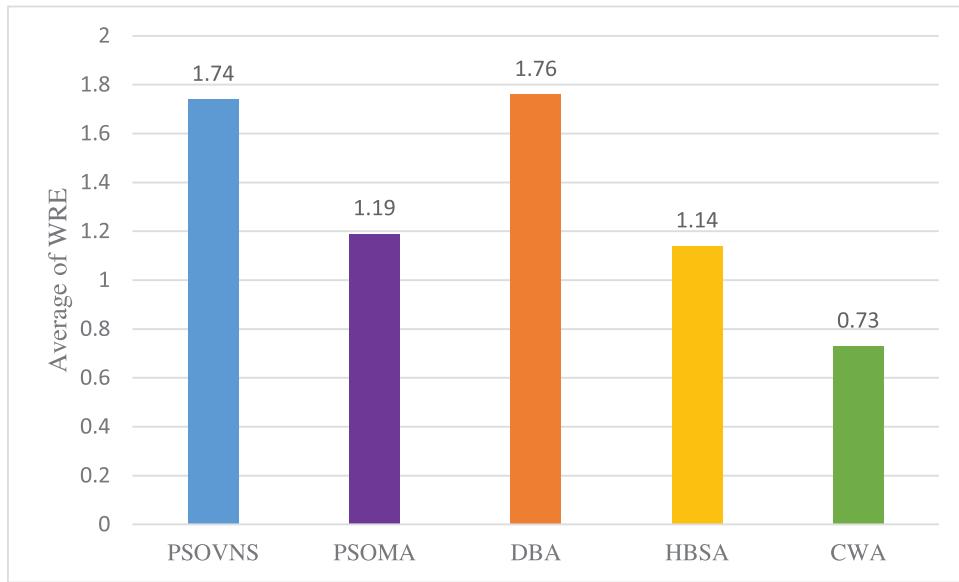
Figure 8 reveals the average value of WRE. It can be seen that even in the worst case, CWA is still the best method.

CWA is superior to the other five algorithms. It can be seen from the experimental results, CWA is very effective in the field of PFSSP.

Table 10 shows the experiments on Heller benchmark instances and compares them with NEH, SG [59], and DSOMA [100]. The CWA algorithm is not only superior to the three methods, but also better than the current known best values. Therefore, the 516 of Hel1 and 136 of Hel2 can be used as better-known results for later experimenters.



**Figure 7** | Average of average relative error (ARE).



**Figure 8** | Average of worst relative error (WRE).

### 6.3. Comparison on Benchmark Taillard

In this section, we measure the performance of CWA on the Taillard benchmark instances and compare it with other algorithms. The best known value comes from [101]. This benchmark contains 120 examples, from the smallest  $20 \times 5$  to the largest  $500 \times 20$ . We compare the Taillard function from the three aspects of the best scheduling, the average scheduling and the worst scheduling. In Table 11, we listed the experimental results of CWA and the following methods for best scheduling:

- Discrete self-organizing migrating algorithm (DSOMA) [100]. New sampling routines have been developed that propagate the space between solutions in order to drive the algorithm.

- Iterated greedy with referenced insertion scheme (IG\_RIS) [102]. IG\_RIS propose a constructive heuristic to generate an initial solution. In addition, an iteration jumping probability is proposed to change the neighborhood structure from insertion neighborhood to swap neighborhood.
- Improved iterated greedy algorithm (IIGA) [103]. In the proposed IIGA, a speed-up method for the insert neighborhood is developed to evaluate the whole insert neighborhood of a single solution with  $(n - 1)^2$  neighbors in time  $O(n^2)$ , where  $n$  is the number of jobs.
- Hybrid genetic algorithm (HGA) [95]. HGA is a hybrid GA, which is used to solve the no wait FSSP with completion period as the goal.

**Table 9** | Comparison on car and Rec benchmarks.

Problem	$n * m$	$c^*$	HGA	HBSA	DBA	PSOMA	PSONVS	CWA
Car01	11*5	7038	BRE	0	0	0	0	<b>0</b>
			ARE	0	0	0	0	<b>0</b>
			WRE	–	0	0	0	<b>0</b>
Car02	13*4	7166	BRE	0	0	0	0	<b>0</b>
			ARE	0	0	0	0	<b>0</b>
			WRE	–	0	0	0	<b>0</b>
Car03	12*5	7312	BRE	0	0	0	0	<b>0</b>
			ARE	0	0.06	0.397	0	<b>0</b>
			WRE	–	1.19	1.109	0	<b>1.189</b>
Car04	14*4	8003	BRE	0	0	0	0	<b>0</b>
			ARE	0	0	0	0	<b>0</b>
			WRE	–	0	0	0	<b>0</b>
Car05	10*6	7720	BRE	0	0	0	0	<b>0</b>
			ARE	0	0	0.018	0.018	<b>0.039</b>
			WRE	–	0	0.375	0.375	<b>0.389</b>
Car06	8*9	8505	BRE	0	0	0	0	<b>0</b>
			ARE	0.04	0	0	0.076	<b>0.076</b>
			WRE	–	0	0	0.764	<b>0.764</b>
Car07	7*7	6590	BRE	0	0	0	0	<b>0</b>
			ARE	0	0	0	0	<b>0</b>
			WRE	–	0	0	0	<b>0</b>
Car08	8*8	8366	BRE	0	0	0	0	<b>0</b>
			ARE	0	0	0	0	<b>0</b>
			WRE	–	0	0	0	<b>0</b>
REC01	20*5	1247	BRE	0	0	0	0	<b>0.16</b>
			ARE	0.14	0.14	0.08	0.144	<b>0.168</b>
			WRE	–	0.16	0.16	0.16	<b>0.321</b>
REC03	20*5	1109	BRE	0	0	0	0	<b>0</b>
			ARE	0.09	0.08	0.081	0.189	<b>0.158</b>
			WRE	–	0.18	0.18	0.721	<b>0.18</b>
REC05	20*5	1242	BRE	0	0.24	0.242	0.242	<b>0.242</b>
			ARE	0.29	0.24	0.242	0.249	<b>0.249</b>
			WRE	–	0.24	0.242	0.402	<b>0.42</b>
REC07	20*10	1566	BRE	0	0	0	0	<b>0.702</b>
			ARE	0.69	0.46	0.575	0.986	<b>1.095</b>
			WRE	–	1.15	1.149	1.149	<b>1.405</b>
REC09	20*10	1537	BRE	0	0	0	0	<b>0</b>
			ARE	0.64	0.07	0.638	0.621	<b>0.651</b>
			WRE	–	0.65	2.407	1.691	<b>1.366</b>
REC11	20*10	1431	BRE	0	0	0	0	<b>0.071</b>
			ARE	1.1	0	1.167	0.129	<b>1.153</b>
			WRE	–	0	2.655	0.978	<b>2.656</b>
REC13	20*15	1930	BRE	0.36	0.1	0.415	0.259	<b>1.036</b>
			ARE	1.68	0.53	1.461	0.893	<b>1.79</b>
			WRE	–	1.14	3.782	1.502	<b>2.643</b>
REC15	20*15	1950	BRE	0.56	0.05	0.154	0.051	<b>0.769</b>
			ARE	1.12	0.64	1.226	0.628	<b>1.487</b>
			WRE	–	1.18	2.103	1.076	<b>2.256</b>
REC17	20*15	1902	BRE	0.95	0	0.368	0	<b>0.999</b>
			ARE	2.32	1	1.277	1.33	<b>2.453</b>
			WRE	–	2.16	2.154	2.155	<b>3.365</b>
REC19	30*10	2093	BRE	0.62	0.29	0.573	0.43	<b>1.529</b>
			ARE	1.32	0.81	0.929	1.313	<b>2.099</b>
			WRE	–	1.29	2.023	2.102	<b>2.532</b>
REC21	30*10	2017	BRE	1.44	0.69	1.438	1.437	<b>1.487</b>
			ARE	1.57	1.5	1.671	1.596	<b>1.671</b>
			WRE	–	1.83	2.231	<b>1.636</b>	<b>2.033</b>
REC23	30*10	2011	BRE	0.4	0.45	0.796	0.596	<b>1.343</b>
			ARE	0.87	1.28	1.173	1.31	<b>2.106</b>
			WRE	–	3.08	2.381	2.038	<b>2.884</b>
REC27	30*15	2373	BRE	1.1	0.25	1.011	1.348	<b>1.728</b>
			ARE	1.83	1.27	1.419	1.605	<b>2.463</b>
			WRE	–	2.57	2.298	2.402	<b>3.203</b>
REC29	30*15	2287	BRE	1.4	0.57	1.049	1.442	<b>1.968</b>
			ARE	2.7	1.42	2.58	1.888	<b>3.109</b>
			WRE	–	2.97	3.935	2.492	<b>4.067</b>
REC31	50*10	3045	BRE	0.43	0.43	2.299	1.51	<b>2.594</b>
			ARE	1.34	1.91	3.392	2.254	<b>3.232</b>
			WRE	–	2.66	4.532	2.692	<b>4.237</b>

(Continued)

**Table 9** | Comparison on car and Rec benchmarks. (Continued)

Problem	$n * m$	$c^*$	HGA	HBSA	DBA	PSOMA	Psovns	CWA
REC33	50*10	3114	BRE	0	0	0.61	0	0.835
			ARE	0.78	0.59	0.728	0.645	1.007
			WRE	–	1.28	1.734	0.834	1.477
REC35	50*10	3277	BRE	0	0	0	0	0
			ARE	0	0	0.037	0	0.038
			WRE	–	0	0.092	0	0.092
REC39	75*20	5087	BRE	2.2	0.9	2.28	1.553	2.85
			ARE	2.79	1.88	3.851	2.426	3.371
			WRE	–	3.38	5.347	2.83	3.951
REC41	75*20	4960	BRE	3.64	1.69	3.81	2.641	4.173
			ARE	4.92	2.72	5.095	3.684	4.867
			WRE	–	3.55	6.532	4.052	5.585

**Table 10** | Comparison on Heller benchmark.

Instance	$n * m$	Known Best	NEH	SG	DSOMA	CWA
Hel1	20*10	516	518	<b>515</b>	631	<b>515</b>
Hel2	100*10	136	141	137	150	<b>135</b>

- Tabu-mechanism improved iterated greedy (TMIIG) [104]. The authors have modified the IG algorithm by utilizing a Tabu-based reconstruction strategy to enhance its exploration ability. A powerful neighborhood search method that involves insert, swap, and double-insert moves is then applied to obtain better solutions from the reconstructed solution in the previous step.
- Discrete water wave optimization algorithm (DWWO) [105]. In refraction operator, a crossover strategy is employed by DWWO to avoid the algorithm falling into local optima.

The **bold** in Table 11 represents the best value of each instance. It is obvious that the best value of each instance is obtained by CWA, so the performance of CWA is better than the other six algorithms.

Table 11 shows the results of comparisons with other six methods on Taillard benchmark based on the best scheduling and this table gives the value of scheduling time directly. Because the scale of Taillard benchmark is relatively large, so we can see that when the size of the instance is relatively small, CWA can achieve the optimal scheduling, but with the increment of the size of the instance, it is difficult to achieve the optimal scheduling. Even if the optimal scheduling value is not reached, it is close to this value, and among the seven algorithms, the maximum completion time of CWA is the smallest.

Figure 9 shows the average error (AE) [59] of best scheduling, which is defined as follows:

$$AE = \frac{1}{d} * \sum_{i=1}^k \frac{z^* - z_i}{z^*} * 100 \quad (19)$$

where  $z^*$  indicates the best scheduling time for the known instances.  $z_i$  is the scheduling time of CWA in the  $i^{th}$  instance of best, mean, and worst scheduling. In this benchmark,  $d$  is equal to 120, representing 120 instances. CWA obtained the minimum value of all algorithms, which is 0.85.

From Figure 9, the performance of DSOMA is second only to CWA, but the AE of DSOMA is 24 times that of CWA. The performance of CWA is much better than other algorithms.

Figure 10 shows the degree of improvement of CWA of best scheduling relative to other algorithms, called improvement percentage (IP) [59], which is defined as follows:

$$IP = \frac{m_{CWA} - m_x}{m_{CWA}} \quad (20)$$

where  $m_{cwa}$  and  $m_x$  respectively indicate the completion time of CWA and the completion time of other algorithms in these three schedulers.

In Figure 10, the IP of HGA, DOOW, TMIIG, and IIGA are comparatively similar, which indicates that their performance on Taillard benchmark is comparatively close. What's more, CWA outperforms HGA and DOOW to the greatest extent and IG\_RIS to the smallest extent, but it outperforms all these algorithms.

Table 12 compares the average scheduling time of the Taillard benchmark, as compared with the following algorithm:

- Nawaz-Enscore-Ham (NEH) [90]
- Tabu-mechanism improved iterated greedy (TMIIG) [104]
- Improved iterated greedy algorithm (IIGA) [103]
- Hybrid genetic algorithm (HGA) [95]
- Discrete water wave optimization algorithm (DWWO) [105]

Tables 8 and 11 represent the average of the algorithm. This shows that the CWA's scheduling results can achieve a greater degree of optimization in different instances. Similarly, the proposed CWA method is superior to all other algorithms in mean scheduling. It can achieve the minimum scheduling time for each instance.

Figure 11 is AE of all algorithms in mean scheduling. The maximum value of IIGA is 46.84, and the minimum value of CWA is 1.65, that means they are 28 times different. It can be seen from the diagram that the optimization range of CWA is very large, also shown in Figure 12. At the same time, it can be seen from Figure 12, the performance of NEH algorithm is the worst in Taillard instance, while the performance of the other four algorithms is comparatively

**Table 11** | Comparison on Taillard benchmark based on the best scheduling.

Instance	<i>n*m</i>	Known Best	DSOMA	IG_RIS	IIGA	HGA	TMIIG	DOOW	CWA
Ta001	20*5	1278	1374	–	1486	1449	1486	1486	1278
Ta002		1359	1408	–	1528	1460	1528	1528	1359
Ta003		1081	1280	–	1460	1386	1460	1460	1081
Ta004		1293	1448	–	1588	1521	1588	1588	1293
Ta005		1235	1341	–	1449	1403	1449	1449	1235
Ta006		1195	1363	–	1481	1430	1481	1481	1195
Ta007		1239	1381	–	1483	1461	1483	1483	1239
Ta008		1206	1379	–	1482	1433	1482	1482	1206
Ta009		1230	1373	–	1469	1398	1469	1469	1230
Ta010		1108	1283	–	1377	1324	1377	1377	1108
Ta011	20*10	1582	1698	–	2044	1955	2044	2044	1586
Ta012		1659	1833	–	2166	2123	2166	2166	1671
Ta013		1496	1676	–	1940	1912	1940	1940	1512
Ta014		1377	1546	–	1811	1782	1811	1811	1386
Ta015		1419	1617	–	1933	1933	1933	1933	1424
Ta016		1397	1590	–	1892	1827	1892	1892	1397
Ta017		1484	1622	–	1963	1944	1963	1963	1484
Ta018		1538	1731	–	2057	2006	2057	2057	1560
Ta019		1593	1747	–	1973	1908	1973	1973	1611
Ta020		1591	1782	–	2051	2001	2051	2051	1608
Ta021	20*20	2297	2436	–	2973	2912	2973	2973	2318
Ta022		2099	2234	–	2852	2780	2852	2852	2122
Ta023		2326	2479	–	3013	2922	3013	3013	2345
Ta024		2223	2348	–	3001	2967	3001	3001	2237
Ta025		2291	2435	–	3003	2953	3003	3003	2296
Ta026		2226	2383	–	2998	2908	2998	2998	2235
Ta027		2273	2390	–	3052	2970	3052	3052	2293
Ta028		2200	2328	–	2839	2763	2839	2839	2216
Ta029		2237	2363	–	3009	2972	3009	3009	2245
Ta030		2178	2323	–	2979	2919	2979	2979	2195
Ta031	50*5	2724	3033	2974	3161	3127	3161	3170	2724
Ta032		2834	3045	3171	3432	3438	3440	3441	2836
Ta033		2621	3036	2988	3211	3182	3213	3218	2622
Ta034		2751	3011	3113	3339	3289	3343	3349	2751
Ta035		2863	3128	3140	3356	3315	3361	3376	2863
Ta036		2829	3166	3158	3347	3324	3346	3352	2829
Ta037		2725	3021	3005	3231	3183	3234	3243	2725
Ta038		2683	3063	3040	3235	3243	3241	3239	2700
Ta039		2552	2908	2889	3072	3059	3075	3078	2554
Ta040		2782	3120	3094	3317	3301	3322	3330	2782
Ta041	50*10	2991	3638	3605	4274	4251	4274	4274	3025
Ta042		2867	3511	3470	4177	4139	4179	4180	2905
Ta043		2839	3492	3465	4099	4083	4099	4099	2888
Ta044		3063	3672	3649	4399	4480	4399	4407	3063
Ta045		2976	3633	3614	4322	4316	4324	4324	3006
Ta046		3006	3621	3574	4289	4282	4290	4294	3020
Ta047		3093	3704	3667	4420	4376	4420	4420	3115
Ta048		3037	3572	3549	4318	4304	4321	4323	3042
Ta049		2897	3541	3510	4155	4162	4158	4155	2904
Ta050		3065	3624	3603	4283	4232	4286	4286	3099
Ta051	50*20	3850	4511	4484	6129	6138	6129	6129	3934
Ta052		3704	4288	4262	5725	5721	5725	5725	3797
Ta053		3640	4289	4261	5862	5847	5873	5862	3717
Ta054		3720	4378	4338	5788	5781	5789	5789	3790
Ta055		3610	4271	4249	5886	5891	5886	5886	3715
Ta056		3681	4202	4271	5863	5875	5874	5871	3785
Ta057		3704	4315	4291	5962	5937	5968	5969	3781
Ta058		3691	4326	4298	5926	5919	5940	5926	3797
Ta059		3743	4329	4304	5876	5839	5876	5876	3852
Ta060		3756	4422	4398	5958	5935	5959	5958	3812
Ta061	100*5	5493	6151	6038	6397	6492	6397	6433	5493
Ta062		5268	6064	5933	6234	6353	6246	6268	5268
Ta063		5175	6003	5837	6121	6148	6133	6162	5175
Ta064		5014	5786	5661	6026	6080	6028	6055	5023
Ta065		5250	6021	5873	6200	6254	6206	6221	5250
Ta066		5135	5869	5732	6074	6177	6088	6121	5135
Ta067		5246	6004	5890	6247	6257	6254	6311	5247
Ta068		5094	5924	5785	6130	6225	6150	6197	5099
Ta069		5448	6154	6029	6370	6443	6391	6418	5453
Ta070		5322	6186	6049	6381	6441	6396	6404	5328

(Continued)

**Table 11** | Comparison on Taillard benchmark based on the best scheduling. (Continued)

Instance	<i>n*m</i>	Known Best	DSOMA	IG_RIS	IIGA	HGA	TMIIG	DOOW	CWA
Ta071	100*10	5770	7042	6896	8077	8115	8080	8093	5785
Ta072		5349	6813	6622	7880	7986	7888	7891	5391
Ta073		5676	6943	6766	8028	8057	8042	8047	5691
Ta074		5781	7198	7037	8348	8327	8350	8364	5830
Ta075		5467	6815	6690	7958	7991	7967	7966	5497
Ta076		5303	6685	6517	7801	7823	7808	7791	5308
Ta077		5595	6827	6684	7866	7915	7880	7881	5638
Ta078		5617	6874	6729	7913	7939	7912	7924	5660
Ta079		5871	6092	6908	8161	8226	8164	8152	5840
Ta080		5845	6990	6832	8114	8186	8130	8126	5860
Ta081	100*20	6202	7854	7683	10700	10745	10722	10727	6357
Ta082		6183	7910	7739	10594	10655	10611	10604	6314
Ta083		6271	7825	7697	10611	10672	10629	10624	6356
Ta084		6269	7902	7730	10607	10630	10615	10615	6359
Ta085		6314	7901	7694	10539	10548	10563	10551	6421
Ta086		6364	7921	7745	10690	10700	10684	10680	6468
Ta087		6268	8051	7848	10825	10827	10832	10824	6406
Ta088		6401	8025	7879	10839	10863	10846	10839	6548
Ta089		6275	7969	7771	10723	10751	10763	10745	6393
Ta090		6434	8036	7818	10798	10794	10797	10787	6538
Ta091	200*10	10862	13507	13100	15319	15739	15377	15418	10885
Ta092		10480	13458	13048	15085	15534	15167	15252	10551
Ta093		10922	13521	13135	15376	15755	15416	15412	10948
Ta094		10889	13686	13112	15200	15842	15250	15304	10893
Ta095		10524	13547	13097	15209	15692	15268	15277	10537
Ta096		10326	13247	12869	15109	15622	15163	15222	10375
Ta097		10854	13910	13351	15395	15877	15441	15459	10868
Ta098		10730	13830	13225	15237	15733	15295	15307	10792
Ta099		10438	13410	13036	15100	15573	15155	15186	10465
Ta100		10675	13744	13119	15340	15803	15382	15378	10690
Ta101	200*20	11195	15027	14484	19681	20148	19723	19724	11355
Ta102		11203	15211	14690	20096	20539	20127	20091	11438
Ta103		11281	15247	14776	19913	20511	19973	19989	11517
Ta104		11275	15174	14694	19928	20461	19997	19940	11504
Ta105		11259	15047	14547	19843	20339	19900	19875	11397
Ta106		11176	15212	14734	19942	20501	20003	19980	11366
Ta107		11360	15168	14744	20112	20680	20145	20119	11544
Ta108		11334	15247	14763	20056	20614	20053	20053	11537
Ta109		11192	15136	14643	19918	20300	19998	19932	11406
Ta110		11288	15243	14703	19935	20437	19932	19916	11551
Ta111	500*20	26059	37064	35372	46689	49095	47046	46871	26483
Ta112		26520	37419	35743	47275	49461	47630	47294	26890
Ta113		26371	37059	35452	46544	48777	46977	46846	26717
Ta114		26456	37014	35687	46899	49283	47328	47185	26768
Ta115		26334	36894	35417	46741	48950	47238	47037	26611
Ta116		26477	37372	35747	46941	49533	47553	47166	26767
Ta117		26389	36998	35395	46509	48943	46944	46794	26639
Ta118		26560	36944	35568	46873	49277	47346	47127	26873
Ta119		26005	36862	35304	46743	49207	47205	47025	26282
Ta120		26457	37098	35643	46847	49092	47374	47105	26735

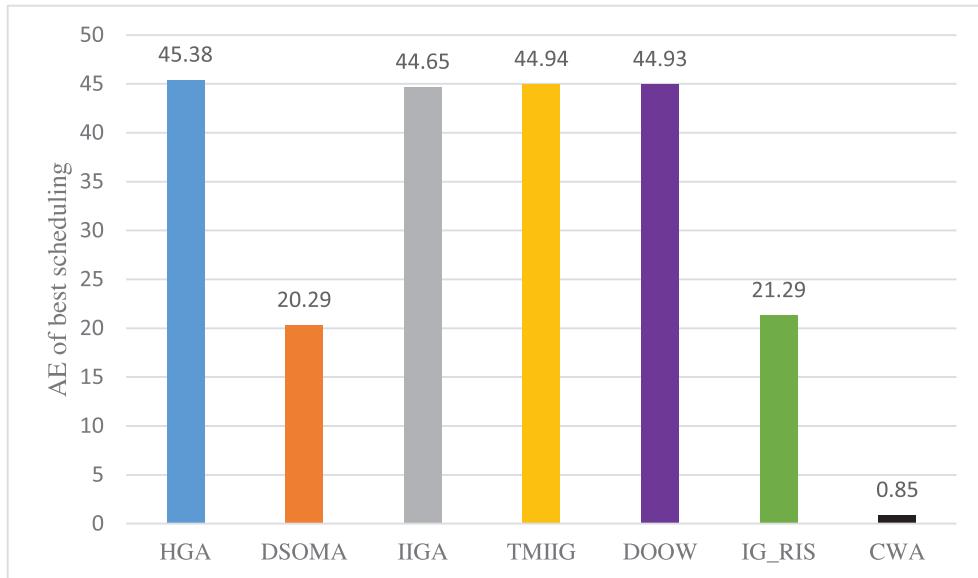
similar. It shows that CWA improves other algorithms to a similar degree, and the degree of improvement is very large.

Table 13 shows the worst scheduling of Taillard benchmark instances. Comparing CWA with HGA [95], the results show that CWA is always better than HGA, including smaller or larger instances.

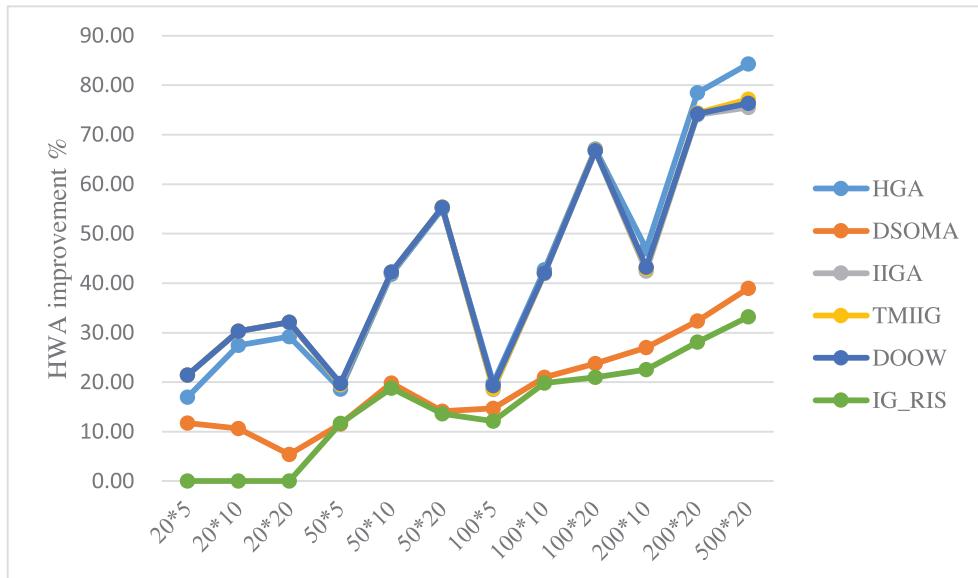
Figure 13 shows the improvement degree of CWA relative to HGA, and CWA is significantly improved. With the increment of instance size, the optimization degree of CWA increases, and reached 85.7% when the instance is 500\*20. The experimental results show that the optimization of CWA is very successful.

## 7. CONCLUSIONS AND FUTURE WORK

This paper proposed a new CWA which combined chaotic mapping, inserted local search, and cross selection operator. The chaotic map stabilizes the random factors in the algorithm and increases the stability and optimization of the algorithm. What's more, these strategies also make the algorithm avoid falling into the local optimum through local search with small probability and circular cross selection operator. NEH is used to initialize the population so that a better result can be obtained in the initialization stage. In the process of initializing population, LRV rule is used to transform the continuous search agent into discrete job sequence,



**Figure 9** | Average error (AE) of best scheduling.



**Figure 10** | Improvement percentage (IP) of chaotic whale algorithm (CWA) based on the best scheduling.

which optimizes the rationality of job generation. The experimental results show that it is feasible to use chaos strategy to optimize whale algorithm.

Compared with other scheduling algorithms, CWA has other advantages, such as fewer variables to be adjusted, faster convergence speed, and more stability. Because the parameters have a great influence on the performance of the algorithm, its stability is improved when chaotic mapping is used to make the parameters tend to be controllable [106]. However, the algorithm still has uncertainty, which needs further optimization and improvement. In future, except the methods used in the current paper, some of the most representative metaheuristic algorithms [107], such as BA [108–110], biogeography-based optimization (BBO) [111,112], ACO [113], cuckoo search (CS) [114,115], earthworm optimization algorithm (EWA) [116], elephant herding optimization (EHO)

[117,118], moth search (MS) algorithm [119], firefly algorithm (FA) [120], ABC [121–123], harmony search (HS) [124], monarch butterfly optimization (MBO) [125,126], PSO [127,128], genetic programming [129], CS [84,130,131] and more recently, the KH algorithm [81,132–135].

In this paper, CWA is tested on Carlier, Reeves, Heller, and Tail-lard benchmarks. Since only these four groups of benchmark functions have been tested, another limitation is the lack of research on broader dimensions. But from the results of the experiment, the proposed CWA algorithm has better stability and optimization results. In the current work, we only apply CWA to PFSSP. In the future, the algorithm will be used to solve other engineering problems. We are ready to apply this algorithm to the differential distributed job shop scheduling system, and use it to study big data of ocean.

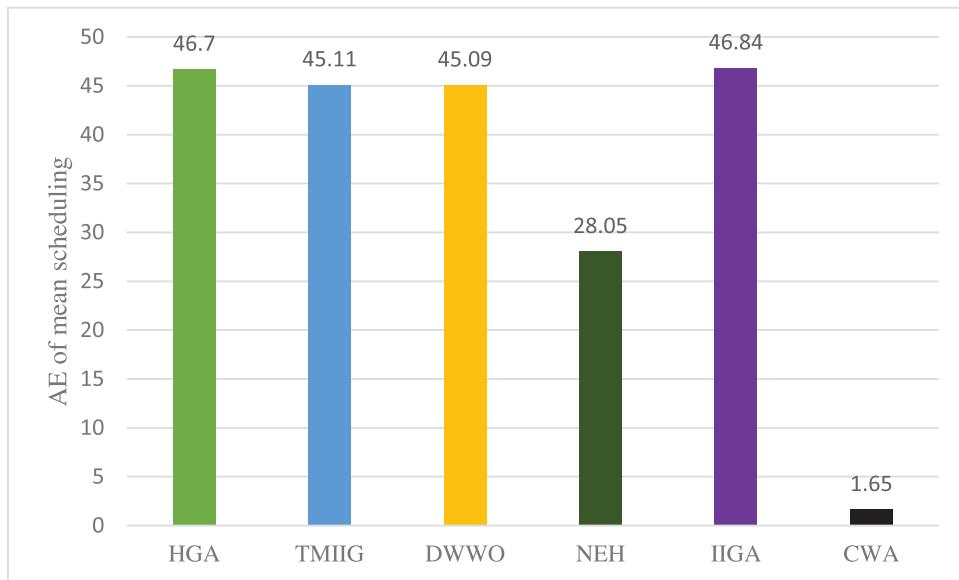
**Table 12** | Comparison on Taillard benchmark based on the mean scheduling.

Instance	$n*m$	Known Best	NEH	TMIIG	IIGA	HGA	DWWO	CWA
Ta001	20*5	1278	1299	1486	1486	1472.7	1486	1278
Ta002		1359	1365	1528	1528	1477.6	1528	1359
Ta003		1081	1132	1460	1460	1406.8	1460	1081
Ta004		1293	1329	1588	1588	1546.4	1588	1293
Ta005		1235	1305	1449	1449	1426.6	1449	1235
Ta006		1195	1251	1481	1481	1446.6	1481	1195
Ta007		1239	1251	1483	1483	1479.4	1483	1239
Ta008		1206	1215	1482	1482	1459.2	1482	1206
Ta009		1230	1284	1469	1469	1409.2	1469	1230
Ta010		1108	1127	1377	1377	1345.5	1377	1108
Ta011	20*10	1582	1681	2044	2044	1972.6	2044	1604
Ta012		1659	1766	2166	2166	2154.6	2166	1691
Ta013		1496	1562	1940.4	1940	1931.1	1940	1530.5
Ta014		1377	1416	1811	1811	1794.3	1811	1409.35
Ta015		1419	1502	1933	1933	1934.4	1933	1450.75
Ta016		1397	1456	1892	1892	1850	1892	1423.05
Ta017		1484	1531	1963	1963	1951.5	1963	1500.35
Ta018		1538	1626	2057	2058.6	2038.5	2057	1577.4
Ta019		1593	1639	1973	1973	1953.9	1973	1620.5
Ta020		1591	1656	2051	2051	2019.3	2051	1633.05
Ta021	20*20	2297	2443	2973	2973	2938.5	2973	2346
Ta022		2099	2134	2852	2852	2814.2	2852	2146.9
Ta023		2326	2414	3019.4	3019.4	2962.4	3014.3	2381.15
Ta024		2223	2257	3001	3001	2982.5	3001	2273.5
Ta025		2291	2370	3003	3003	2995.1	3003	2340.6
Ta026		2226	2349	2998	2998	2932.8	2998	2243.9
Ta027		2273	2383	3052	3052	3004.8	3052	2316.75
Ta028		2200	2249	2839	2839	2789.5	2839	2242.05
Ta029		2237	2306	3009	3009	3005.3	3009	2282.55
Ta030		2178	2257	2979	2979	2956.3	2979	2242.9
Ta031	50*5	2724	2729	3162.4	3222	3198.3	3171.8	2720.6
Ta032		2834	2882	3441	3474.4	3453.1	3444.5	2837.9
Ta033		2621	2650	3216	3262.8	3242.5	3231.8	2624.5
Ta034		2751	2782	3346.6	3374.8	3349.8	3350.8	2759.35
Ta035		2863	2868	3364.6	3406	3369.1	3376.7	2863.65
Ta036		2829	2835	3347.6	3382.2	3356.4	3356.2	2833.75
Ta037		2725	2806	3235.8	3267.6	3246.4	3245.3	2725
Ta038		2683	2700	3242.4	3275.2	3264.9	3240.3	2694.2
Ta039		2552	2606	3078.8	3123.2	3088.2	3086.8	2570.25
Ta040		2782	2801	3327.2	3377.2	3341.5	3336.5	2785.3
Ta041	50*10	2991	3175	4276.8	4311.2	4289.1	4281.5	3043.55
Ta042		2867	3073	4185	4201	4193.5	4184.5	2918.55
Ta043		2839	2994	4107.6	4124	4108.6	4105.5	2916.7
Ta044		3063	3218	4405	4439.2	4517.4	4405.7	3072.45
Ta045		2976	3186	4330.4	4347	4333.2	4324.7	3025.95
Ta046		3006	3148	4297.2	4330	4301.6	4295.2	3050.1
Ta047		3093	3277	4429.6	4441.4	4412.6	4420	3145.65
Ta048		3037	3170	4327	4357.6	4331.9	4323.3	3053.55
Ta049		2897	3025	4164.2	4194.8	4173	4161.7	2931.55
Ta050		3065	3267	4286.2	4301	4279	4286.2	3115
Ta051	50*20	3850	4006	6139.6	6154.6	6149.7	6138.5	3966.67
Ta052		3704	3958	5741	5762.2	5751.5	5733.5	3829
Ta053		3640	3866	5882.4	5907	5884.4	5865.5	3760
Ta054		3720	3953	5791.4	5802.6	5804.7	5790.7	3837.15
Ta055		3610	3872	5899.4	5930.6	5909.7	5893.5	3759.82
Ta056		3681	3861	5883.4	5912.6	5890.6	5874.3	3820.91
Ta057		3704	3927	5974	6012	5974.2	5974	3814
Ta058		3691	3914	5945.4	5970.2	5951	5930.5	3842.58
Ta059		3743	3970	5883.2	5900.6	5873.5	5876	3882.91
Ta060		3756	4036	5959	5982	5963.5	5958.8	3872
Ta061	100*5	5493	5514	6413.4	6586.4	6557.2	6438.3	5493
Ta062		5268	5284	6252.2	6428.2	6409.4	6285.5	5279.27
Ta063		5175	5222	6135.8	6292.8	6260.4	6164.2	5186.27
Ta064		5014	5023	6031.4	6184.8	6159	6050.7	5023

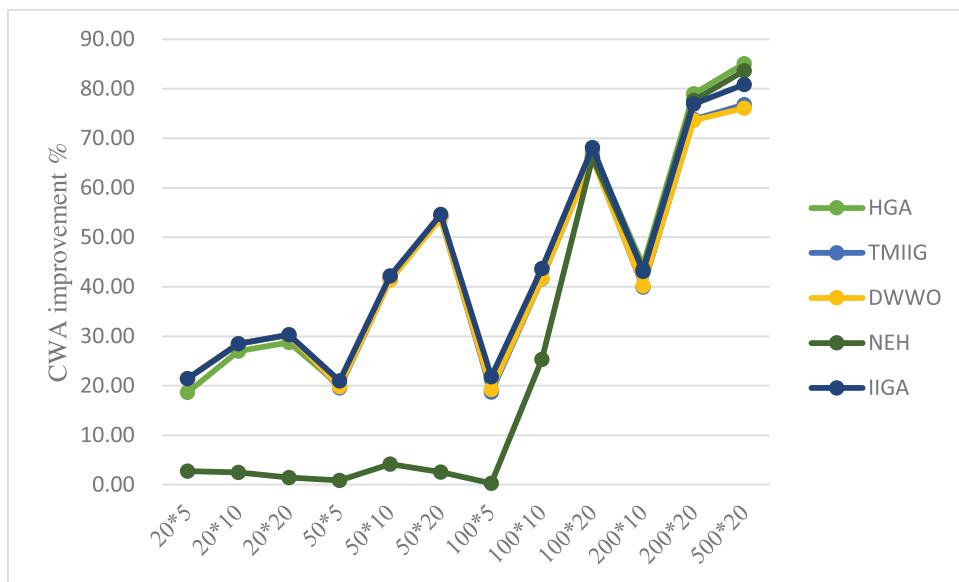
(Continued)

**Table 12** | Comparison on Taillard benchmark based on the mean scheduling. (Continued)

Instance	<i>n*m</i>	Known Best	NEH	TMIIG	IIGA	HGA	DWWO	CWA
Ta065		5250	5261	6217.8	6358.4	6325.6	6223.3	<b>5254.4</b>
Ta066		5135	5154	6096	6269.2	6225.7	6122.7	<b>5137.8</b>
Ta067		5246	5282	6263.4	6442.2	6409	6308.8	<b>5258.6</b>
Ta068		5094	5140	6156.6	6338.6	6308.6	6210.8	<b>5124.9</b>
Ta069		5448	5489	6395	6557.4	6516.3	6422	<b>5465.4</b>
Ta070		5322	5336	6401.6	6561	6542.5	6427.2	<b>5327.3</b>
Ta071	100*10	5770	5897	8094.2	8224.6	8173.5	8100.8	<b>5798.4</b>
Ta072		5349	5466	7909	8061.4	8048.1	7938.3	<b>5403.3</b>
Ta073		5676	5747	8054.4	8152	8142.2	8051	<b>5699.2</b>
Ta074		5781	5924	8362.4	8484.2	8437.5	8378.2	<b>5860</b>
Ta075		5467	7991	7981.2	8087.6	8046.4	7980	<b>5539.8</b>
Ta076		5303	7823	7821.6	7930.4	7883.7	7809	<b>5325.6</b>
Ta077		5595	7915	7887.2	7983.8	8007	7889	<b>5676.3</b>
Ta078		5617	7939	7924.8	8051.4	8049.2	7931.7	<b>5688</b>
Ta079		5871	8226	8172	8312.6	8290.4	8183.8	<b>5902.3</b>
Ta080		5845	8186	8148.8	8263.6	8255.3	8141	<b>5898.7</b>
Ta081	100*20	6202	10745	10782.4	10887	10826.6	10744	<b>6370</b>
Ta082		6183	10655	10623	10755.6	10762.5	10608.7	<b>6350</b>
Ta083		6271	10672	10650	10774.6	10740.4	10637	<b>6390.9</b>
Ta084		6269	10630	10647.2	10765.8	10679.7	10629.8	<b>6379.5</b>
Ta085		6314	10548	10579.8	10690.6	10658.2	10568.3	<b>6449.9</b>
Ta086		6364	10700	10696.2	10819.4	10753	10690	<b>6503.2</b>
Ta087		6268	10827	10849.2	10997.2	10913.6	10843.2	<b>6489.3</b>
Ta088		6401	10863	10862	11011.6	10905.2	10850.2	<b>6571.9</b>
Ta089		6275	10751	10780.4	10875	10859	10758	<b>6433</b>
Ta090		6434	10794	10817.2	10944.6	10928.8	10808.5	<b>6594.5</b>
Ta091	200*10	10862	15397.4	15751.4	15843.5	15449.3		<b>10913.95</b>
Ta092		10480	15534	15203	15572.2	15645.3	15281.3	<b>10894.8</b>
Ta093		10922	15755	15433.2	15762.4	15882.4	15454.5	<b>11002</b>
Ta094		10889	15842	15279.8	15625	15927	15304.7	<b>13054.8</b>
Ta095		10524	15692	15280.8	15626.2	15763.8	15295.8	<b>10651.2</b>
Ta096		10326	15622	15189.6	15557	15669.9	15235.5	<b>10447.91</b>
Ta097		10854	15877	15476.6	15842	15962.3	15508.3	<b>10889.4</b>
Ta098		10730	15733	15319.2	15685.6	15833.2	15340	<b>10805.7</b>
Ta099		10438	15573	15185.2	15525.8	15626.2	15240.2	<b>10485.4</b>
Ta100		10675	15803	15410.8	15754.4	15869.1	15412.2	<b>10723.3</b>
Ta101	200*20	11195	20148	19770.8	20127.8	20331.3	19736.7	<b>11401</b>
Ta102		11203	20539	20148.8	20536.6	20763.5	20118.8	<b>11500.5</b>
Ta103		11281	20511	19995.2	20355.6	20583.8	19999	<b>11585.6</b>
Ta104		11275	20461	20032	20349.6	20594.6	19967.2	<b>11552.1</b>
Ta105		11259	20339	19936	20331.4	20544.4	19923.8	<b>11450.3</b>
Ta106		11176	20501	20050.6	20407	20661.9	20011.3	<b>11436.4</b>
Ta107		11360	20680	20167.4	20539.2	20804.3	20147.5	<b>11586.2</b>
Ta108		11334	20614	20095.6	20449.8	20665.7	20084	<b>11585.7</b>
Ta109		11192	20300	20022.2	20358.2	20574.9	19975.5	<b>11457.3</b>
Ta110		11288	20437	19968.6	20284	20587.6	19946.3	<b>11603.45</b>
Ta111	500*20	26059	49095	47147.2	48253.4	49289.4	46965.5	<b>26550.1</b>
Ta112		26520	49461	47692.2	48699	49948.2	47429.8	<b>27020.9</b>
Ta113		26371	48777	47049.6	48116.4	49139	46878.7	<b>26802.2</b>
Ta114		26456	49283	47375.8	48513.2	49657.8	47232.8	<b>26845.3</b>
Ta115		26334	48950	47280	48345.8	49423.1	47099.2	<b>26659.55</b>
Ta116		26477	49533	47591.2	48701.6	49848.4	47331.5	<b>26833.8</b>
Ta117		26389	48943	47087.2	48186	49366.5	46866.3	<b>26722.3</b>
Ta118		26560	49277	47421.4	48468.2	49675.8	47248.7	<b>26990.3</b>
Ta119		26005	49207	47248.6	48297.2	49429.3	47083.5	<b>26436.7</b>
Ta120		26457	49092	47402.4	48483.4	49545.6	47183.2	<b>26803</b>



**Figure 11** | Average error (AE) of mean scheduling.



**Figure 12** | Improvement percentage (IP) of chaotic whale algorithm (CWA) based on the mean scheduling.

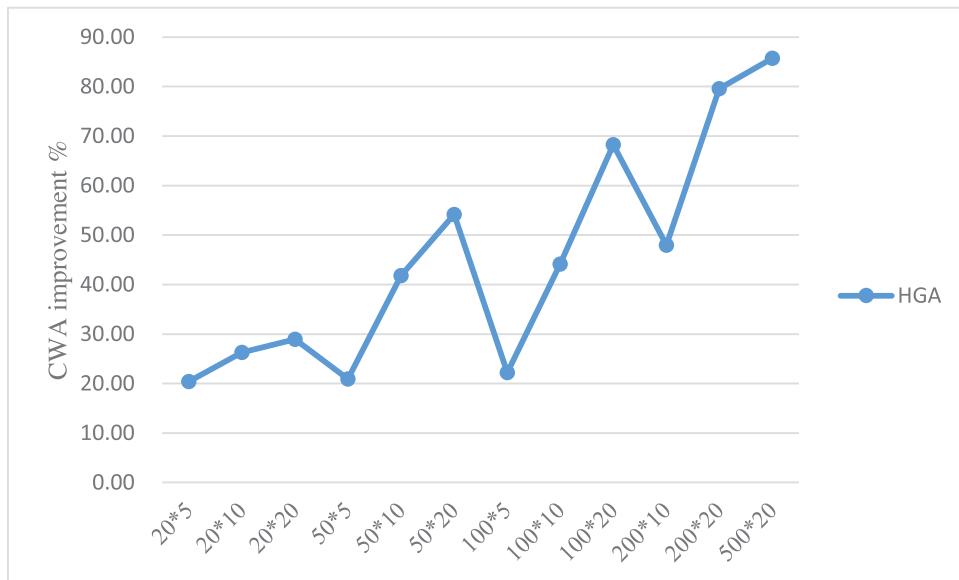
**Table 13** | Comparison on Taillard benchmark based on the worst scheduling.

Instance	n*m	Known Best	HGA	CWA	Instance	n*m	Known Best	HGA	CWA
Ta001	20*5	1278	1485	<b>1278</b>	Ta061	100*5	5493	6594	<b>5493</b>
Ta002		1359	1505	<b>1359</b>	Ta062		5268	6469	<b>5284</b>
Ta003		1081	1431	<b>1081</b>	Ta063		5175	6320	<b>5193</b>
Ta004		1293	1573	<b>1293</b>	Ta064		5014	6198	<b>5023</b>
Ta005		1235	1445	<b>1235</b>	Ta065		5250	6397	<b>5265</b>
Ta006		1195	1471	<b>1195</b>	Ta066		5135	6296	<b>5139</b>
Ta007		1239	1496	<b>1239</b>	Ta067		5246	6460	<b>5266</b>
Ta008		1206	1475	<b>1206</b>	Ta068		5094	6359	<b>5123</b>
Ta009		1230	1429	<b>1230</b>	Ta069		5448	6561	<b>5465</b>
Ta010		1108	1368	<b>1108</b>	Ta070		5322	6592	<b>5335</b>

(Continued)

**Table 13** | Comparison on Taillard benchmark based on the worst scheduling. (Continued)

Instance	<i>n*m</i>	Known Best	HGA	CWA	Instance	<i>n*m</i>	Known Best	HGA	CWA
Ta011	20*10	1582	1998	<b>1622</b>	Ta071	100*10	5770	8230	<b>5800</b>
Ta012		1659	2166	<b>1692</b>	Ta072		5349	8101	<b>5424</b>
Ta013		1496	1942	<b>1534</b>	Ta073		5676	8215	<b>5740</b>
Ta014		1377	1811	<b>1434</b>	Ta074		5781	8505	<b>5860</b>
Ta015		1419	1947	<b>1467</b>	Ta075		5467	8096	<b>5525</b>
Ta016		1397	1879	<b>1539</b>	Ta076		5303	7947	<b>5328</b>
Ta017		1484	1971	<b>1504</b>	Ta077		5595	8081	<b>5692</b>
Ta018		1538	2066	<b>1571</b>	Ta078		5617	8105	<b>5694</b>
Ta019		1593	1973	<b>1646</b>	Ta079		5871	8349	<b>5940</b>
Ta020		1591	2032	<b>1674</b>	Ta080		5845	8340	<b>5903</b>
Ta021	20*20	2297	2972	<b>2362</b>	Ta081	100*20	6202	10932	<b>6435</b>
Ta022		2099	2835	<b>2160</b>	Ta082		6183	10847	<b>6373</b>
Ta023		2326	2984	<b>2372</b>	Ta083		6271	10821	<b>6374</b>
Ta024		2223	2994	<b>2316</b>	Ta084		6269	10797	<b>6408</b>
Ta025		2291	3017	<b>2338</b>	Ta085		6314	10777	<b>6460</b>
Ta026		2226	2964	<b>2263</b>	Ta086		6364	10853	<b>6535</b>
Ta027		2273	3028	<b>2353</b>	Ta087		6268	11031	<b>6568</b>
Ta028		2200	2826	<b>2267</b>	Ta088		6401	10992	<b>6593</b>
Ta029		2237	3009	<b>2299</b>	Ta089		6275	10963	<b>6482</b>
Ta030		2178	2979	<b>2246</b>	Ta090		6434	11067	<b>6620</b>
Ta031	50*5	2724	3229	<b>2735</b>	Ta091	200*10	10862	15916	<b>10950</b>
Ta032		2834	3475	<b>2838</b>	Ta092		10480	15764	<b>10570</b>
Ta033		2621	3277	<b>2624</b>	Ta093		10922	16026	<b>11045</b>
Ta034		2751	3384	<b>2762</b>	Ta094		10889	16111	<b>10911</b>
Ta035		2863	3404	<b>2864</b>	Ta095		10524	15829	<b>10553</b>
Ta036		2829	3377	<b>2835</b>	Ta096		10326	15731	<b>10575</b>
Ta037		2725	3280	<b>2725</b>	Ta097		10854	16029	<b>10908</b>
Ta038		2683	3288	<b>2705</b>	Ta098		10730	15933	<b>10803</b>
Ta039		2552	3121	<b>2577</b>	Ta099		10438	15759	<b>10492</b>
Ta040		2782	3383	<b>2814</b>	Ta100		10675	15934	<b>10727</b>
Ta041	50*10	2991	4306	<b>3054</b>	Ta101	200*20	11195	20458	<b>11461</b>
Ta042		2867	4235	<b>2923</b>	Ta102		11203	20889	<b>11496</b>
Ta043		2839	4124	<b>2953</b>	Ta103		11281	20636	<b>11609</b>
Ta044		3063	4549	<b>3071</b>	Ta104		11275	20753	<b>11638</b>
Ta045		2976	4367	<b>3051</b>	Ta105		11259	20601	<b>11466</b>
Ta046		3006	4332	<b>3083</b>	Ta106		11176	20780	<b>11442</b>
Ta047		3093	4444	<b>3165</b>	Ta107		11360	20915	<b>11627</b>
Ta048		3037	4347	<b>3076</b>	Ta108		11334	20814	<b>11601</b>
Ta049		2897	4188	<b>2965</b>	Ta109		11192	20757	<b>11501</b>
Ta050		3065	4304	<b>3120</b>	Ta110		11288	20712	<b>11626</b>
Ta051	50*20	3850	6172	<b>3984</b>	Ta111	500*20	26059	49580	<b>26582</b>
Ta052		3704	5790	<b>3859</b>	Ta112		26520	50354	<b>27161</b>
Ta053		3640	5929	<b>3831</b>	Ta113		26371	49399	<b>26832</b>
Ta054		3720	5827	<b>3864</b>	Ta114		26456	50004	<b>26838</b>
Ta055		3610	5950	<b>3794</b>	Ta115		26334	49847	<b>26716</b>
Ta056		3681	5911	<b>3803</b>	Ta116		26477	50046	<b>27048</b>
Ta057		3704	6001	<b>3798</b>	Ta117		26389	49591	<b>26832</b>
Ta058		3691	5971	<b>3880</b>	Ta118		26560	49942	<b>27020</b>
Ta059		3743	5899	<b>3826</b>	Ta119		26005	49697	<b>26513</b>
Ta060		3756	5979	<b>3922</b>	Ta120		26457	50002	<b>26886</b>



**Figure 13** | Improvement percentage (IP) achieved by chaotic whale algorithm (CWA) for hybrid genetic algorithm (HGA) on the worst scheduling.

## CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

## AUTHORS' CONTRIBUTIONS

Conceptualization, J.L. and L.W.; methodology, L.G. and H.H.; software, Y.L. and L.W.; validation, J.L.; writing-original draft preparation, J.L. and C.L.; writing-review and editing, L.G.; All authors have read and agreed to the published version of the manuscript.

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## REFERENCES

- [1] J. Deng, L. Wang, A competitive memetic algorithm for multi-objective distributed permutation flow shop scheduling problem, *Swarm Evol. Comput.* 32 (2017), 121–131.
- [2] M.R. Garey, D.S. Johnson, R. Sethi, The complexity of flowshop and jobshop scheduling, *Math. Oper. Res.* 1 (1976), 117–129.
- [3] H.-Y. Sang, Q.-K. Pan, P.-Y. Duan, J.-Q. Li, An effective discrete invasive weed optimization algorithm for lot-streaming flowshop scheduling problems, *J. Intell. Manuf.* 29 (2015), 1337–1349.
- [4] H.-Y. Sang, Q.-K. Pan, J.-Q. Li, P. Wang, Y.-Y. Han, K.-Z. Gao, P. Duan, Effective invasive weed optimization algorithms for distributed assembly permutation flowshop problem with total flowtime criterion, *Swarm Evol. Comput.* 44 (2019), 64–73.
- [5] Q.-K. Pan, H.-Y. Sang, J.-H. Duan, L. Gao, An improved fruit fly optimization algorithm for continuous function optimization problems, *Knowl. Based Syst.* 62 (2014), 69–83.
- [6] D. Gao, G.-G. Wang, W. Pedrycz, Solving fuzzy job-shop scheduling problem using DE algorithm improved by a selection mechanism, *IEEE Trans. Fuzzy Syst.* 28 (2020), 3265–3275.
- [7] A.K. Sangaiah, M.Y. Suraki, M. Sadeghilalimi, S.M. Bozorgi, A.A.R. Hosseiniabadi, J. Wang, A new meta-heuristic algorithm for solving the flexible dynamic job-shop problem with parallel machines, *Symmetry* 11 (2019), 165.
- [8] M. Li, D. Xiao, Y. Zhang, H. Nan, Reversible data hiding in encrypted images using cross division and additive homomorphism, *Signal Process. Image Commun.* 39 (2015), 234–248.
- [9] M. Li, Y. Guo, J. Huang, Y. Li, Cryptanalysis of a chaotic image encryption scheme based on permutation-diffusion structure, *Signal Process. Image Commun.* 62 (2018), 164–172.
- [10] H. Fan, M. Li, D. Liu, E. Zhang, Cryptanalysis of a colour image encryption using chaotic APFM nonlinear adaptive filter, *Signal Process.* 143 (2018), 28–41.
- [11] Y. Zhang, D. Gong, Y. Hu, W. Zhang, Feature selection algorithm based on bare bones particle swarm optimization, *Neurocomputing* 148 (2015), 150–157.
- [12] Y. Zhang, X.-F. Song, D.-W. Gong, A return-cost-based binary firefly algorithm for feature selection, *Inf. Sci.* 418–419 (2017), 561–574.
- [13] W. Mao, J. He, J. Tang, Y. Li, Predicting remaining useful life of rolling bearings based on deep feature representation and long short-term memory neural network, *Adv. Mech. Eng.* 10 (2018), 1–18.
- [14] M. Jian, K.-M. Lam, J. Dong, Facial-feature detection and localization based on a hierarchical scheme, *Inf. Sci.* 262 (2014), 1–14.

- [15] L. Fan, S. Xu, D. Liu, Y. Ru, Semi-supervised community detection based on distance dynamics, *IEEE Access.* 6 (2018), 37261–37271.
- [16] A.K. Sangaiah, D.V. Medhane, T. Han, M.S. Hossain, G. Muhammad, Enforcing position-based confidentiality with machine learning paradigm through mobile edge computing in real-time industrial informatics, *IEEE Trans. Ind. Inf.* 15 (2019), 4189–4196.
- [17] G.-G. Wang, H.E. Chu, S. Mirjalili, Three-dimensional path planning for UCAV using an improved bat algorithm, *Aerospace Sci. Technol.* 49 (2016), 231–238.
- [18] G. Wang, L. Guo, H. Duan, L. Liu, H. Wang, M. Shao, Path planning for uninhabited combat aerial vehicle using hybrid meta-heuristic DE/BBO algorithm, *Adv. Sci. Eng. Med.* 4 (2012), 550–564.
- [19] G.-G. Wang, X. Cai, Z. Cui, G. Min, J. Chen, High performance computing for cyber physical social systems by using evolutionary multi-objective optimization algorithm, *IEEE Trans. Emerg. Topics Comput.* 8 (2020), 20–30.
- [20] Z. Cui, B. Sun, G.-G. Wang, Y. Xue, J. Chen, A novel oriented cuckoo search algorithm to improve DV-Hop performance for cyber-physical systems, *J. Parallel Distrib. Comput.* 103 (2017), 42–52.
- [21] M. Jian, K.-M. Lam, J. Dong, Illumination-insensitive texture discrimination based on illumination compensation and enhancement, *Inf. Sci.* 269 (2014), 60–72.
- [22] G.-G. Wang, L. Guo, H. Duan, L. Liu, H. Wang, The model and algorithm for the target threat assessment based on Elman\_AdaBoost strong predictor, *Acta Electronica Sinica.* 40 (2012), 901–906.
- [23] M. Jian, K.M. Lam, J. Dong, L. Shen, Visual-patch-attention-aware saliency detection, *IEEE Trans. Cybern.* 45 (2015), 1575–86.
- [24] G.-G. Wang, M. Lu, Y.-Q. Dong, X.-J. Zhao, Self-adaptive extreme learning machine, *Neural Comput. Appl.* 27 (2016), 291–303.
- [25] W. Mao, Y. Zheng, X. Mu, J. Zhao, Uncertainty evaluation and model selection of extreme learning machine based on Riemannian metric, *Neural Comput. Appl.* 24 (2013), 1613–1625.
- [26] G. Liu, J. Zou, Level set evolution with sparsity constraint for object extraction, *IET Image Process.* 12 (2018), 1413–1422.
- [27] K. Liu, D. Gong, F. Meng, H. Chen, G.-G. Wang, Gesture segmentation based on a two-phase estimation of distribution algorithm, *Inf. Sci.* 394–395 (2017), 88–105.
- [28] R.P. Parouha, K.N. Das, Economic load dispatch using memory based differential evolution, *Int. J. Bio-Inspir. Comput.* 11 (2018), 159–170.
- [29] R.M. Rizk-Allah, R.A. El-Sehiemy, G.-G. Wang, A novel parallel hurricane optimization algorithm for secure emission/economic load dispatch solution, *Appl. Soft Comput.* 63 (2018), 206–222.
- [30] R.M. Rizk-Allah, R.A. El-Sehiemy, S. Deb, G.-G. Wang, A novel fruit fly framework for multi-objective shape design of tubular linear synchronous motor, *J. Supercomputing.* 73 (2017), 1235–1256.
- [31] J.-H. Yi, L.-N. Xing, G.-G. Wang, J. Dong, A.V. Vasilakos, A.H. Alavi, L. Wang, Behavior of crossover operators in NSGA-III for large-scale optimization problems, *Inf. Sci.* 509 (2020), 470–487.
- [32] J.-H. Yi, S. Deb, J. Dong, A.H. Alavi, G.-G. Wang, An improved NSGA-III Algorithm with adaptive mutation operator for big data optimization problems, *Future Gener. Comput. Syst.* 88 (2018), 571–585.
- [33] H. Garg, An efficient biogeography based optimization algorithm for solving reliability optimization problems, *Swarm Evol. Comput.* 24 (2015), 1–10.
- [34] G. Liu, M. Deng, Parametric active contour based on sparse decomposition for multi-objects extraction, *Signal Process.* 148 (2018), 314–321.
- [35] Y. Han, F. He, Y. Chen, Y. Liu, H. Yu, SiRNA silencing efficacy prediction based on a deep architecture, *BMC Genom.* 19 (2018), 669.
- [36] J. Sun, Z. Miao, D. Gong, X.-J. Zeng, J. Li, G.-G. Wang, Interval multi-objective optimization with memetic algorithms, *IEEE Trans. Cybern.* 50 (2020), 3444–3457.
- [37] Y. Zhang, G.-G. Wang, K. Li, W.-C. Yeh, M. Jian, J. Dong, Enhancing MOEA/D with information feedback models for large-scale many-objective optimization, *Inf. Sci.* 522 (2020), 1–16.
- [38] K. Srikanth, L.K. Panwar, B.K. Panigrahi, E. Herrera-Viedma, A.K. Sangaiah, G.-G. Wang, Meta-heuristic framework: quantum inspired binary grey wolf optimizer for unit commitment problem, *Comput. Electr. Eng.* 70 (2018), 243–260.
- [39] A.K. Sangaiah, G.-B. Bian, S.M. Bozorgi, M.Y. Suraki, A.A.R. Hosseiniabadi, M.B. Shareh, A novel quality-of-service-aware web services composition using biogeography-based optimization algorithm, *Soft Comput.* 24 (2020), 8125–8137.
- [40] S. Chen, R. Chen, G.-G. Wang, J. Gao, A.K. Sangaiah, An adaptive large neighborhood search heuristic for dynamic vehicle routing problems, *Comput. Electr. Eng.* 67 (2018), 596–607.
- [41] Y. Feng, G.-G. Wang, Binary moth search algorithm for discounted {0–1} knapsack problem, *IEEE Access.* 6 (2018), 10708–10719.
- [42] Y. Feng, G.-G. Wang, L. Wang, Solving randomized time-varying knapsack problems by a novel global firefly algorithm, *Eng. Comput.* 34 (2018), 621–635.
- [43] M. Abdel-Basset, Y. Zhou, An elite opposition-flower pollination algorithm for a 0–1 knapsack problem, *Int. J. Bio-Inspir. Comput.* 11 (2018), 46–53.
- [44] J.-H. Yi, J. Wang, G.-G. Wang, Improved probabilistic neural networks with self-adaptive strategies for transformer fault diagnosis problem, *Adv. Mech. Eng.* 8 (2016), 1–13.
- [45] W. Mao, J. He, Y. Li, Y. Yan, Bearing fault diagnosis with auto-encoder extreme learning machine: a comparative study, *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.* 231 (2016), 1560–1578.
- [46] W. Mao, W. Feng, X. Liang, A novel deep output kernel learning method for bearing fault structural diagnosis, *Mech. Syst. Signal Process.* 117 (2019), 293–318.
- [47] H. Duan, W. Zhao, G. Wang, X. Feng, Test-sheet composition using analytic hierarchy process and hybrid metaheuristic algorithm TS/BBO, *Math. Probl. Eng.* 2012 (2012), 1–22.
- [48] M. Dorigo, V. Maniezzo, A. Colorni, Ant system: optimization by a colony of cooperating agents, *IEEE Trans. Syst. Man Cybern. Part B Cybern.* 26 (1996), 29–41.
- [49] O. Engin, A. Güçlü, A new hybrid ant colony optimization algorithm for solving the no-wait flow shop scheduling problems, *Appl. Soft Comput.* 72 (2018), 166–176.
- [50] N. Sharma, H. Sharma, A. Sharma, Bee froth artificial bee colony algorithm for job-shop scheduling problem, *Appl. Soft Comput.* 68 (2018), 507–524.

- [51] D.E. Goldberg, Genetic algorithms in search, Optim. Mach. Learn. 1 (1989), 1.
- [52] A.A.R. Hosseiniabadi, J. Vahidi, B. Saemi, A.K. Sangaiah, M. Elhoseny, Extended genetic algorithm for solving open-shop scheduling problem, Soft Comput. 23 (2019), 5099–5116.
- [53] D. Gong, Y. Han, J. Sun, A novel hybrid multi-objective artificial bee colony algorithm for blocking lot-streaming flow shop scheduling problems, Knowl. Based Syst. 148 (2018), 115–130.
- [54] C.-L. Wei, G.-G. Wang, Hybrid annealing krill herd and quantum-behaved particle swarm optimization, Mathematics. 8 (2020), 1403.
- [55] H. Garg, A hybrid GA-GSA algorithm for optimizing the performance of an industrial system by utilizing uncertain data, in: P. Vasant (Ed.), Handbook of Research on Artificial Intelligence Techniques and Algorithms, IGI Global, Hershey, 2015, pp. 620–654.
- [56] H. Garg, A hybrid PSO-GA algorithm for constrained optimization problems, Appl. Math. Comput. 274 (2016), 292–305.
- [57] H. Garg, A hybrid GSA-GA algorithm for constrained optimization problems, Inf. Sci. 478 (2019), 499–523.
- [58] S. Mirjalili, A. Lewis, The whale optimization algorithm, Adv. Eng. Softw. 95 (2016), 51–67.
- [59] M. Abdel-Basset, G. Manogaran, D. El-Shahat, S. Mirjalili, A hybrid whale optimization algorithm based on local search strategy for the permutation flow shop scheduling problem, Future Gener. Comput. Syst. 85 (2018), 129–145.
- [60] J. Wang, P. Du, T. Niu, W. Yang, A novel hybrid system based on a new proposed algorithm—multi-objective whale optimization algorithm for wind speed forecasting, Appl. Energy. 208 (2017), 344–360.
- [61] M.A. El Aziz, A.A. Ewees, A.E. Hassani, Whale optimization algorithm and moth-flame optimization for multilevel thresholding image segmentation, Expert Syst. Appl. 83 (2017), 242–256.
- [62] M. Nazari-Heris, M. Mehdinejad, B. Mohammadi-Ivatloo, G. Babamalek-Gharehpetian, Combined heat and power economic dispatch problem solution by implementation of whale optimization method, Neural Comput. Appl. 31 (2017), 421–436.
- [63] I. Aljarah, H. Faris, S. Mirjalili, Optimizing connection weights in neural networks using the whale optimization algorithm, Soft Comput. 22 (2018), 1–15.
- [64] S.M. Johnson, Optimal two- and three-stage production schedules with setup times included, Nav. Res. Logist. Q. 1 (1954), 61–68.
- [65] D.A. Rossit, F. Tohmé, M. Frutos, The non-permutation flowshop scheduling problem: a literature review, Omega. 77 (2018), 143–153.
- [66] Y. Han, D. Gong, Y. Jin, Q. Pan, Evolutionary multiobjective blocking lot-streaming flow shop scheduling with machine breakdowns, IEEE Trans. Cybern. 49 (2019), 184–197.
- [67] R. Ruiz, Q.-K. Pan, B. Naderi, Iterated Greedy methods for the distributed permutation flowshop scheduling problem, Omega. 83 (2019), 213–222.
- [68] W. Liu, Y. Jin, M. Price, A new improved NEH heuristic for permutation flowshop scheduling problems, Int. J. Prod. Econ. 193 (2017), 21–30.
- [69] J.M. Framinan, R. Leisten, C. Rajendran, Different initial sequences for the heuristic of Nawaz, Enscore and Ham to minimize makespan, idletime or flowtime in the static permutation flowshop sequencing problem, Int. J. Prod. Res. 41 (2010), 121–148.
- [70] V. Santucci, M. Baoletti, A. Milani, Algebraic differential evolution algorithm for the permutation flowshop scheduling problem with total flowtime criterion, IEEE Trans. Evol. Comput. 20 (2016), 682–694.
- [71] J.C. Bansal, A. Gopal, A.K. Nagar, Stability analysis of artificial bee colony optimization algorithm, Swarm Evol. Comput. 41 (2018), 9–19.
- [72] J. Li, Q. Pan, P. Duan, An improved artificial bee colony algorithm for solving hybrid flexible flowshop with dynamic operation skipping, IEEE Trans. Cybern. 46 (2016), 1311–1324.
- [73] A. Sioud, C. Gagné, Enhanced migrating birds optimization algorithm for the permutation flow shop problem with sequence dependent setup times, Eur. J. Oper. Res. 264 (2018), 66–73.
- [74] F. Pagnozzi, T. Stützle, Automatic design of hybrid stochastic local search algorithms for permutation flowshop problems, Eur. J. Oper. Res. 276 (2019), 409–421.
- [75] G. Wang, Y. Tan, Improving metaheuristic algorithms with information feedback models, IEEE Trans. Cybern. 49 (2019), 542–555.
- [76] A.J. Benavides, M. Ritt, Two simple and effective heuristics for minimizing the makespan in non-permutation flow shops, Comput. Oper. Res. 66 (2016), 160–169.
- [77] H. Gao, S. Kwong, B. Fan, R. Wang, A hybrid particle-swarm tabu search algorithm for solving job shop scheduling problems, IEEE Trans. Ind. Informat. 10 (2014), 2044–2054.
- [78] J. Wang, L. Wang, A knowledge-based cooperative algorithm for energy-efficient scheduling of distributed flow-shop, IEEE Trans. Syst. Man Cybern. Syst. 50 (2020), 1805–1819.
- [79] S.T.W. Kwong, H. Gao, C. Pun, Y. Shi, An improved artificial bee colony algorithm with its application to metallographic image segmentation, IEEE Trans. Ind. Inf. 15 (2019), 1853–1865.
- [80] Y. Fu, J. Ding, H. Wang, J. Wang, Two-objective stochastic flowshop scheduling with deteriorating and learning effect in Industry 4.0-based manufacturing system, Appl. Soft Comput. 68 (2018), 847–855.
- [81] G.-G. Wang, L. Guo, A.H. Gandomi, G.-S. Hao, H. Wang, Chaotic krill herd algorithm, Inf. Sci. 274 (2014), 17–34.
- [82] H. Mokhtari, A. Noroozi, An efficient chaotic based PSO for earliness/tardiness optimization in a batch processing flow shop scheduling problem, J. Intell. Manuf. 29 (2018), 1063–1081.
- [83] B. Alatas, Chaotic bee colony algorithms for global numerical optimization, Expert Syst. Appl. 37 (2010), 5682–5687.
- [84] G.-G. Wang, S. Deb, A.H. Gandomi, Z. Zhang, A.H. Alavi, Chaotic cuckoo search, Soft Comput. 20 (2016), 3349–3362.
- [85] E.Y. Xie, C. Li, S. Yu, J. Lü, On the cryptanalysis of Fridrich's chaotic image encryption scheme, Signal Process. 132 (2017), 150–154.
- [86] D. Abbasinezhad-Mood, M. Nikooghadam, Efficient anonymous password-authenticated key exchange protocol to read isolated smart meters by utilization of extended chebyshev chaotic maps, IEEE Trans. Ind. Inf. 14 (2018), 4815–4828.
- [87] L. Xu, Z. Li, J. Li, W. Hua, A novel bit-level image encryption algorithm based on chaotic maps, Opt. Laser Eng. 78 (2016), 17–25.
- [88] A.H. Gandomi, X.S. Yang, S. Talatahari, A.H. Alavi, Firefly algorithm with chaos, Commun. Nonlinear Sci. Numer. Simul. 18 (2013), 89–98.

- [89] X. Li, M. Yin, An opposition-based differential evolution algorithm for permutation flow shop scheduling based on diversity measure, *Adv. Eng. Softw.* 55 (2013), 10–31.
- [90] M. Nawaz, E.E. Enscore, I. Ham, A heuristic algorithm for the m-machine, n-job flow-shop sequencing problem, *Omega*. 11 (1983), 91–95.
- [91] Z. Xie, C. Zhang, X. Shao, W. Lin, H. Zhu, An effective hybrid teaching–learning-based optimization algorithm for permutation flow shop scheduling problem, *Adv. Eng. Softw.* 77 (2014), 35–47.
- [92] C.R. Reeves, A genetic algorithm for flowshop sequencing, *Comput. Oper. Res.* 22 (1995), 5–13.
- [93] J. Heller, Some numerical experiments for an  $M \times J$  flow shop and its decision-theoretical aspects, *Oper. Res.* 8 (1960), 178–184.
- [94] E. Taillard, Benchmarks for basic scheduling problems, *Eur. J. Oper. Res.* 64 (1993), 278–285.
- [95] L.-Y. Tseng, Y.-T. Lin, A hybrid genetic algorithm for no-wait flowshop scheduling problem, *Int. J. Prod. Econ.* 128 (2010), 144–152.
- [96] Q. Lin, L. Gao, X. Li, C. Zhang, A hybrid backtracking search algorithm for permutation flow-shop scheduling problem, *Comput. Ind. Eng.* 85 (2015), 437–446.
- [97] Q. Luo, Y. Zhou, J. Xie, M. Ma, L. Li, Discrete bat algorithm for optimal problem of permutation flow shop scheduling, *Sci. World J.* 2014 (2014), 630280.
- [98] B. Liu, L. Wang, Y. Jin, An effective PSO-based memetic algorithm for flow shop scheduling, *IEEE Trans. Syst. Man Cybern. Part B Cybern.* 37 (2007), 18–27.
- [99] M.F. Tasgetiren, Y.-C. Liang, M. Sevkli, G. Gencyilmaz, A particle swarm optimization algorithm for makespan and total flowtime minimization in the permutation flowshop sequencing problem, *Eur. J. Oper. Res.* 177 (2007), 1930–1947.
- [100] D. Davendra, M. Bialic-Davendra, Scheduling flow shops with blocking using a discrete self-organising migrating algorithm, *Int. J. Prod. Res.* 51 (2013), 2200–2218.
- [101] G.I. Zobolas, C.D. Tarantilis, G. Ioannou, Minimizing makespan in permutation flow shop scheduling problems using a hybrid metaheuristic algorithm, *Comput. Oper. Res.* 36 (2009), 1249–1267.
- [102] M.F. Tasgetiren, D. Kizilay, Q.-K. Pan, P.N. Suganthan, Iterated greedy algorithms for the blocking flowshop scheduling problem with makespan criterion, *Comput. Oper. Res.* 77 (2017), 111–126.
- [103] Q.-K. Pan, L. Wang, B.-H. Zhao, An improved iterated greedy algorithm for the no-wait flow shop scheduling problem with makespan criterion, *Int. J. Adv. Manuf. Technol.* 38 (2008), 778–786.
- [104] J.-Y. Ding, S. Song, J.n.d. Gupta, R. Zhang, R. Chiong, C. Wu, An improved iterated greedy algorithm with a Tabu-based reconstruction strategy for the no-wait flowshop scheduling problem, *Appl. Soft Comput.* 30 (2015), 604–613.
- [105] F. Zhao, H. Liu, Y. Zhang, W. Ma, C. Zhang, A discrete water wave optimization algorithm for no-wait flow shop scheduling problem, *Expert Syst. Appl.* 91 (2018), 347–363.
- [106] A.H. Gandomi, G.J. Yun, X.-S. Yang, S. Talatahari, Chaos-enhanced accelerated particle swarm optimization, *Commun. Nonlinear Sci. Numer. Simul.* 18 (2013), 327–340.
- [107] G.-G. Wang, Y. Tan, Improving metaheuristic algorithms with information feedback models, *IEEE Trans. Cybern.* 49 (2019), 542–555.
- [108] X.S. Yang, A.H. Gandomi, Bat algorithm: a novel approach for global engineering optimization, *Eng. Comput.* 29 (2012), 464–483.
- [109] G. Wang, L. Guo, A novel hybrid bat algorithm with harmony search for global numerical optimization, *J. Appl. Math.* 2013 (2013), 21.
- [110] H. Yu, N. Zhao, P. Wang, H. Chen, C. Li, Chaos-enhanced synchronized bat optimizer, *Appl. Math. Modell.* 77 (2020), 1201–1215.
- [111] D. Simon, Biogeography-based optimization, *IEEE Trans. Evol. Comput.* 12 (2008), 702–713.
- [112] G. Wang, L. Guo, H. Duan, L. Liu, H. Wang, Dynamic deployment of wireless sensor networks by biogeography based optimization algorithm, *J. Sensor Actuator Netw.* 1 (2012), 86–96.
- [113] M. Dorigo, T. Stutzle, *Ant Colony Optimization*, MIT Press, Cambridge, MA, USA, 2004.
- [114] J. Li, Y.-X. Li, S.-S. Tian, J. Zou, Dynamic cuckoo search algorithm based on Taguchi opposition-based search, *Int. J. Bio-Inspir. Comput.* 13 (2019), 59–69.
- [115] X.S. Yang, S. Deb, Engineering optimisation by cuckoo search, *Int. J. Math. Model. Numer. Optim.* 1 (2010), 330–343.
- [116] G.-G. Wang, S. Deb, L. dos Santos Coelho, Earthworm optimization algorithm: a bio-inspired metaheuristic algorithm for global optimization problems, *Int. J. Bio-Inspir. Comput.* 12 (2018), 1–22.
- [117] G.-G. Wang, S. Deb, L. dos Santos Coelho, Elephant herding optimization, in 2015 3rd International Symposium on Computational and Business Intelligence (ISCBI 2015), IEEE, Luleå, Sweden, 2015, pp. 1–5.
- [118] G.-G. Wang, S. Deb, X.-Z. Gao, L. dos Santos Coelho, A new metaheuristic optimization algorithm motivated by elephant herding behavior, *Int. J. Bio-Inspir. Comput.* 8 (2016), 394–409.
- [119] G.-G. Wang, Moth search algorithm: a bio-inspired metaheuristic algorithm for global optimization problems, *Memet. Comput.* 10 (2018), 151–164.
- [120] X.S. Yang, Firefly algorithm, stochastic test functions and design optimisation, *Int. J. Bio-Inspir. Comput.* 2 (2010), 78–84.
- [121] H. Wang, J.-H. Yi, An improved optimization method based on krill herd and artificial bee colony with information exchange, *Memet. Comput.* 10 (2018), 177–198.
- [122] D. Karaboga, B. Basturk, A powerful and efficient algorithm for numerical function optimization: Artificial Bee Colony (ABC) algorithm, *J. Global Optim.* 39 (2007), 459–471.
- [123] F. Liu, Y. Sun, G.-G. Wang, T. Wu, An artificial bee colony algorithm based on dynamic penalty and chaos search for constrained optimization problems, *Arab. J. Sci. Eng.* 43 (2018), 7189–7208.
- [124] Z.W. Geem, J.H. Kim, G.V. Loganathan, A new heuristic optimization algorithm: harmony search, *Simulation*. 76 (2001), 60–68.
- [125] G.-G. Wang, S. Deb, Z. Cui, Monarch butterfly optimization, *Neural Comput. Appl.* 31 (2019), 1995–2014.
- [126] G.-G. Wang, S. Deb, X. Zhao, Z. Cui, A new monarch butterfly optimization with an improved crossover operator, *Oper. Res. Int. J.* 18 (2018), 731–755.
- [127] J. Kennedy, R. Eberhart, Particle swarm optimization, in Proceeding of the IEEE International Conference on Neural Networks, IEEE, Perth, Australia, 1995, pp. 1942–1948.

- [128] Y. Sun, L. Jiao, X. Deng, R. Wang, Dynamic network structured immune particle swarm optimisation with small-world topology, *Int. J. Bio-Inspir. Comput.* 9 (2017), 93–105.
- [129] A.H. Gandomi, A.H. Alavi, Multi-stage genetic programming: a new strategy to nonlinear system modeling, *Inf. Sci.* 181 (2011), 5227–5239.
- [130] A.H. Gandomi, X.-S. Yang, A.H. Alavi, Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems, *Eng. Comput.* 29 (2013), 17–35.
- [131] G.-G. Wang, A.H. Gandomi, X. Zhao, H.E. Chu, Hybridizing harmony search algorithm with cuckoo search for global numerical optimization, *Soft Comput.* 20 (2016), 273–285.
- [132] A.H. Gandomi, A.H. Alavi, Krill herd: a new bio-inspired optimization algorithm, *Commun. Nonlinear Sci. Numer. Simul.* 17 (2012), 4831–4845.
- [133] G.-G. Wang, A.H. Gandomi, A.H. Alavi, An effective krill herd algorithm with migration operator in biogeography-based optimization, *Appl. Math. Modell.* 38 (2014), 2454–2462.
- [134] G.-G. Wang, A.H. Gandomi, A.H. Alavi, S. Deb, A multi-stage krill herd algorithm for global numerical optimization, *Int. J. Artif. Intell. Tools.* 25 (2016), 1550030.
- [135] G.-G. Wang, A.H. Gandomi, A.H. Alavi, D. Gong, A comprehensive review of krill herd algorithm: variants, hybrids and applications, *Artif. Intell. Rev.* 51 (2019), 119–148.