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The Behavior of Many-body Localization in the Periodically Driven Heisenberg XXX Model

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Abstract

In this paper, we use exact matrix diagonalization to research property of the many-body localization (MBL) in the disordered Heisenberg XXX model with periodic driving. We get the periodic time-dependent external field by trigonometric function, which is added to periodically drive this model. It is demonstrated that the fidelity of eigenstate is able to capture quantum criticality underlying many-body physics (Zhou et al. Phys. Rev. Lett. **100**, 080601, 2008, Zhou and Barjaktarevi J. Phys. A: Math. Theor. **41**, 412001 2008), which can be used to characterize the many-body localization transition in this closed spin system (Zanardi and Paunkovic Phys. Rev. E **74**, 031123, 2006). We obtain the fidelity for high-energy many-body eigenstates, namely, the excited state fidelity, which shows the phase transition of periodically driven Heisenberg XXX chain with different disordered external field strengths and different system sizes. It is demonstrated that when Heisenberg XXX system is in a very small disorder, periodic driving can cause the occurrence of a transition from ergodic phase to MBL phase. In contrast to the HS model which has global two-body interaction, which we have studied recently with the same situation, there is no MBL phase transition when we drive the HS model in ergodic phase with periodic driving. It also shows that for the strong disordered Heisenberg XXX system, there will exist a critical driving period Tc, when driving period T is higher than Tc, the system will undergo a transition from localized phase to ergodic phase and the MBL phase will be broken. Furthermore, we discover that the size of the system and the strength of disorder will affect the critical point of driving period and the magnitude of the phase change. For the same system, the critical point increases as the strength of disorder increases. We also explore the non-disorder system of HS model with the same driving to explore the properties of MBL, it shows that under periodic driving, the non-disordered HS system has the quantum phase transition rather than MBL phase transition. This illustrates the important role of disorder on MBL.

Keywords Many-body localization · Heisenberg XXX mode · Periodic driving

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1 Introduction

The concept of Anderson localization was first put forward by Anderson in 1958 [4, 5], when he solved the issue of diffusion in single-particles disordered systems, and has drawn a complete conclusion that for all non-interacting systems in one and two dimensions the extended states are always changed be localized as long as there exists a arbitrary disorder [6, 7]. Then in 2006, Basko et al. [8] used a disturbance method to calculate whether there existed Anderson localization when short-range interactions were added and come to some definite conclusions to revive this idea of many-body localization (MBL). Many successful recent studies [9–22] have investigated and confirmed the phenomenon of MBL and many features of the MBL phase have been explored. For closed disordered systems, the system is in the ergodic phase when it is in a disordered external field of relatively low disorder strength, and in the localized phase when the disorder is strong.

Substantial attention has been devoted to researching the role of disorder on systems over the past few decades, and the exploration of the dynamic behaviour of driven non-equilibrium quantum system has only become a main focus of research in the last decade [23–30]. Studying the dynamical mechanisms of period-driven MBL systems provides a kinetic approach for conducting research related to solid-state and cold atoms systems [31]. A typical example is the kick-rotor [32], which induces a dynamical Anderson localization and a shift in behaviour between chaos and order. Recent studies of periodically driven many-body systems with local interactions were solved In Ref. [33], which put forward two different kinetic assumptions. One is that the system maintains the absorption of energy and continually heats up to infinite temperature (e.g., using time-averaged to deal with problems). The other is that dynamically locates at a certain energy (e.g., in the form of kicked rotor) [34].

For a period-driven system, its Hamiltonian quantities are periodic functions of time, i.e., H(t+T)=H(t). Based on the Floquet operator, one can get a time-independent Floquet Hamiltonian $\hat{\mathrm{H}}_{\mathrm{F}}$, which determines the time evolution of the system, $\hat{\mathrm{F}} = e^{-i\mathrm{H}_{\mathrm{F}}T}$. The Floquet theorem indicates that the Floquet operator $\hat{\mathrm{F}}$ is the periodic unitary operator after the the integration of the evolution operator over one period, which can be expressed as

$$\hat{F} = \mathcal{T}\exp\left\{-i\int_0^T H(t)dt\right\} \tag{1}$$

Here $\mathcal{T}$ exp is the time-ordered exponential, which signifies that the later times in the integral always appear on the left. The eigenstates of $\hat{F}$ completely determine the evolution of the system. In the eigenstates of $\hat{F}$, its one-period form can be obtained as

$$\hat{F} = e^{-i\hat{H}_F T/\hbar} = \sum_{\mathrm{n}=1}^{\mathcal{M}} e^{-i\theta_n} \left|\phi_n\right\rangle \left\langle\phi_n\right| \tag{2}$$

where $|\phi_n\rangle$ and $e^{-i\theta_n}$ are the eigenstates and eigenvalues of $\hat{F}$. It is concluded that, the eigenstate of $\hat{H}_F$ is also as $|\phi_n\rangle$.

$$\hat{H}_F = \sum_{\mathrm{n}=1}^{\mathcal{M}} \left|\phi_n\right\rangle \varepsilon_n \left\langle\phi_n\right| \tag{3}$$

where ε_n are the Floquet quasienergies. In a short period of time, the Magnus expansion [35, 36] is convergent, leading to effective time-independent many-body (Floquet) Hamiltonian quantities with local energy. In this context, the system maintains the memory about

$\hat{H}_F$ and the system is at a finite temperature relative to $\hat{H}_F$ after numerous periods. Conversely, if the system heats up to infinite temperature for a long time, the Magnus expansion will not converge. This expansion breaks the thermodynamic limit and results in the delocalization transition of the system [33, 34]. Therefore, in this work, we investigate the MBL transition in the disordered couplings Heisenberg XXX chain driven periodically by time-dependent perturbations. And then the many-body localization property of the system is further explored through the phase transformation in the model.

2 Model Used for Numerics

In the last decade, a great deal of research has focused on driving simple Hamiltonian, e.g., in a pair of coupled quantum rotors, the diffusion behavior is restored and the dynamical Anderson localization is constrained [37, 38]. Furthermore, in 2014, D. Huse et al. gave a formal theory of the effective Hamiltonian H_{MBL} based on local integration of motion by using entanglement area-law. And we obtained a valid MBL theory [39]. The process of the many-body localization phase transition is driven by a combination of internal interactions of the many-body system and disordered external fields. For concreteness, we focus on the one-dimensional Heisenberg XXX chain, the Hamiltonian of the Heisenberg XXX spin chain with random fields in the z direction is given by

$$H = J \sum_{i=1}^{L-1} \mathbf{S_i} \cdot \mathbf{S}_{i+1} + \sum_{i=1}^{L} h_i S_i^z \tag{4}$$

We restrict our calculations to $J = 1$, and where fields h_i are independent random variables with a probability distribution that is uniform in [-h, h]. h is the disorder strength of the disordered external field and h_i is the disordered realization of the random external field at each lattice point. S_i is the spin operator at the i-th qubit. The one-dimensional spin Heisenberg XXX model that we are studying is driven by a time-periodic field in the trigonometric form, described by the following Hamiltonian:

$$H(t) = H + V_0 \cos \omega t \sum_{i=1}^{L} S_i^z \tag{5}$$

3 Results and Discussion

In recent years, the application of fidelity, a concept in quantum information theory, to the study of quantum phase transitions (QPTs) of systems has become a very vigorous research topic [40–46]. It has been demonstrated that fidelity plays an essential role in QPTs [47, 48] and the fidelity could quantify QPT of all quantum many-body systems. Fidelity is a pure geometrical quantity, an obvious superiority of the fidelity is that it can characterize the QPT without a priori knowledge of order parameter and symmetry breaking, whether the internal sequence is a traditional symmetry-breaking sequence or a novel topological sequence [49, 50]

In the present work, we will focus on the features of the fidelity for the model (8). Following Ref. [3], The ground-state fidelity per lattice site is defined as the overlap of the first ground-state with parameter λ and λ+δλ, that is,

$$F_0(\lambda, \lambda + \delta\lambda) = |\langle \psi_0(\lambda) \mid \psi_0(\lambda + \delta\lambda)\rangle| \tag{6}$$

Analogously, we get the definition of the fidelity of the n-th excited state $\psi_n(\lambda)$ as the overlap between $|\psi_n(\lambda)\rangle$ and $|\psi_n(\lambda+\delta\lambda)\rangle$; while $\delta\lambda$ is a small shift of this field, with the following forms: $\delta\lambda = \epsilon\lambda$, let $\epsilon = 10^{-3}$.

$$F_n(\lambda, \lambda+\delta\lambda) = |\langle \psi_n(\lambda) \mid \psi_n(\lambda+\delta\lambda)\rangle| \tag{7}$$

It has been shown that [51] not only the ground state fidelity but also the excited state fidelity is a latently valuable quantity. Then for each disordered implementation, we select the many-body eigenstate $|\psi_n\rangle$ in the excited state in the middle segment of the energy ordered list of all data. Because the excited state is in the higher energy state, it is more convincing for the occurrence of the localized phase transition. We next calculate the fidelity F_n for each eigenstate $|\psi_n\rangle$. The average $\mathsf{E}[F]$ was obtained by averaging over all chosen excited states and disordered realizations. The standard libraries for exact matrix diagonalization are adopted for numerical analyses. For each disorder amplitude h, We used 10000 disorder realizations for N=6, 1000 disorder realizations for N=8 and N=10, 100 disorder realizations for N=12 to yield the data illustrated in this article.

In Fig. 1, we plot the averaged excited-state fidelity $\mathsf{E}[F]$ as function of the driving period T with different system sizes from N=6 to N=12 when the disorder strength h=0.5, for the energies in the middle one third of the spectrum. Then, we observe the curve changes of different many-body systems for the same disorder strength h. The results show that the MBL phase transition occurs in this isolated Heisenberg XXX model as the driving period

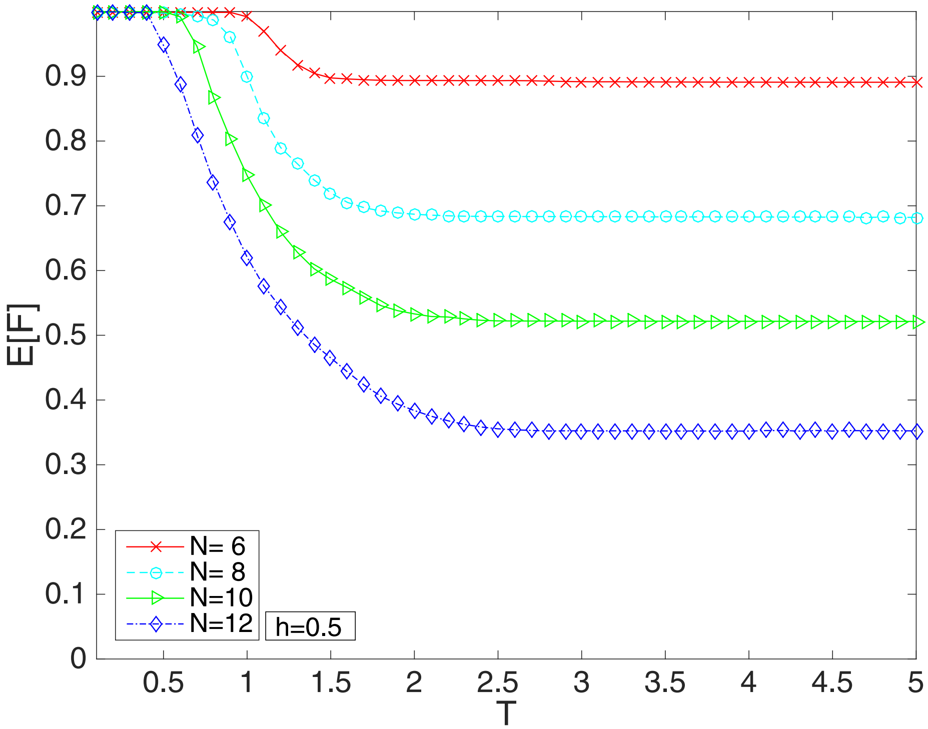

Fig. 1 Averaged fidelity $\mathsf{E}[F]$ as a function of the driving period T for the value of disorder strength h=0.5. The system sizes N are indicated in the legend

T increases. We can see that E[F] tends to decrease as the drive period T increases, eventually reach saturation at a nearly stable value. When h=0.5, the Heisenberg XXX system is in the ergodic phase, it shows that the periodic driving induce the phase transitions from the ergodic phase to MBL phase. It just because a small oscillation occurs near the critical point of a physical system with some symmetry, and by choosing one of all possible bifurcations, the symmetry of this physical system is broken, i.e., a symmetry-breaking occurs. This behaviour of a system with interactions that preserve the memory of the initial state information can effectively avoid thermalisation. This is one of the motivations for studying the MBL phase, with the aim of avoiding thermalisation of the system. This is one of our motivations for studying the MBL phase, which is to prevent thermalisation of the system. According to Fig. 1, one can get the crtiical point T_c, for N=6, $T_c \rightarrow 1.5$; N=8, $T_c \rightarrow 2.1$; N=10, $T_c \rightarrow 2.5$; N=12, $T_c \rightarrow 3.0$. So we obtain $T_c \in [1.5, 3]$ for the breakdown of ergodic phase, which agrees with the prediction in [9, 52]. E[F] decays the fastest when N=12. And when N=6, E[F] decays very little versus driving period T no matter h is large or small. By comparing, one can get that the size of the system will affect the critical point of the phase transition. The larger the system, the larger the critical driving period.

In Fig. 2a and b, we plot the average excited state fidelity E[F] as function of driving period T for different system size with disorder strengths h = 3.5 and h = 10 respectively. The location of the critical point at which the many-body localization phase transition occurs varies for systems of different sizes, as does the change curve. It is worth noting that the critical point depends on the size of the system and decreases as the size of the system increases. Many body localization systems driven by different disorder strengths h and we could see manifest differences among the data in the three figures. One can obtain the approximate critical driving period Tc for different system size N. In Fig. 2a, and b, we increase the disorder strength as h =3.5 and h=10 indicating a more obvious trend of phase transition. For h =3.5, $T_c \in [1.6, 3.3]$. For h =10, $T_c \in [1.8, 3.9]$. It indicates that the many-body local system undergoes a phase transition from localized phase to ergodic phase. Compared to the three figures, the higher the disorder strength, the larger the critical driving period Tc is. This because the more particles in the system, the more complex the interaction between the two particles and the more difficult the phase transition from localized phase to ergodic phase.

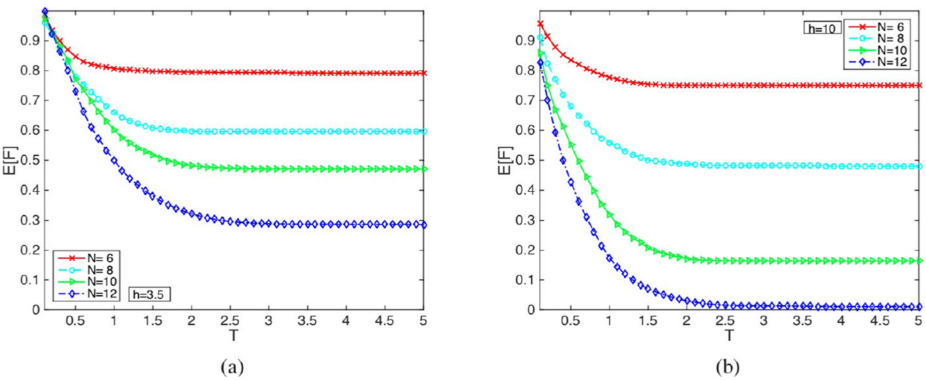

Fig. 2 **a** Average fidelity E[F] as a function of the driving period T for system sizes N from 6 to 12. The value of disorder strength h=3.5. The system sizes N are indicated in the legend. **b** Averaged fidelity E[F] as a function of the driving periods T for the value of disorder strength h=10. E[F] decays as driving periods T increase

In order to identify the correlation between the disorder strength h and the critical driving strength, we select N = 8 to plot the variation of the average fidelity E[F] versus driving period T for different disorder strenth h. In Fig. 3, E[F] decays as driving period T increases, it decays the fastest when h = 10. At the critical point, E[F] varies very slightly with the increase in driving period. The greater the variation of E[F] is for the larger disorder strength. Comparing the three curves in Fig. 3, the larger the disorder strength h, the larger the corresponding critical point T_c is. For h = 3.5, $T_c \to 2.1$, h = 5, $T_c \to 2.3$, h = 10, $T_c \to 2.5$. One can see that as the disorder strength increases, the localized property of the system become more stable, requiring a greater critical driving period.

To further study the property of MBL phase, we also investigate the non-disordered system when h=0.5. Here we let the external field be constant, not disordered. We then perturb the non-disordered system by the same periodic driving. In Fig. 4, we plot the average excited-state fidelity as a function of driving period T for the system N=8 to see if it can drive the phase transition to occur. In Fig. 4, one can see the E[F] versus T show a sharp decrease at T=1.6 towards a minimum to 0 and then a sudden rise approaching to 1. As the data change of Fig. 4, it shows that here the phase transition is a sharp transition which is unlike the MBL transition. It indicates that the non-disordered system has the quantum-phase transition rather than MBL transition. This illustrates that disorder plays an important role on MBL transitions.

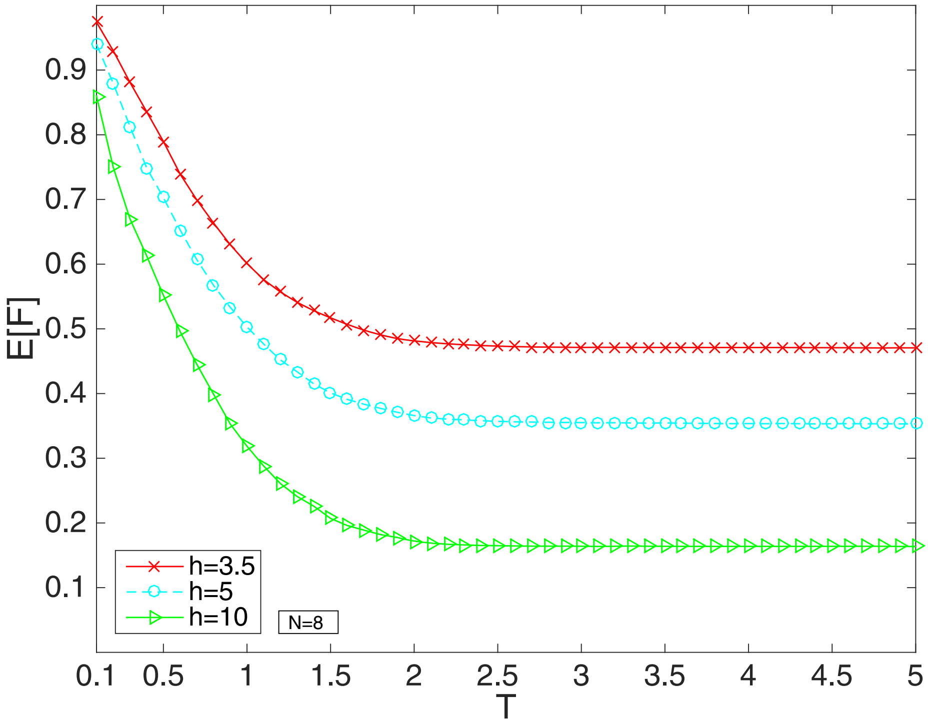

Fig. 3 Averaged fidelity E[F] as a function of the driving period T for the different values of disorder strength h from small to large. The values of disorder strength h are indicated in the legend. The size of system is N=8. E[F] decays as the driving periods increase, the drop gets sharper as disorder strength h increases

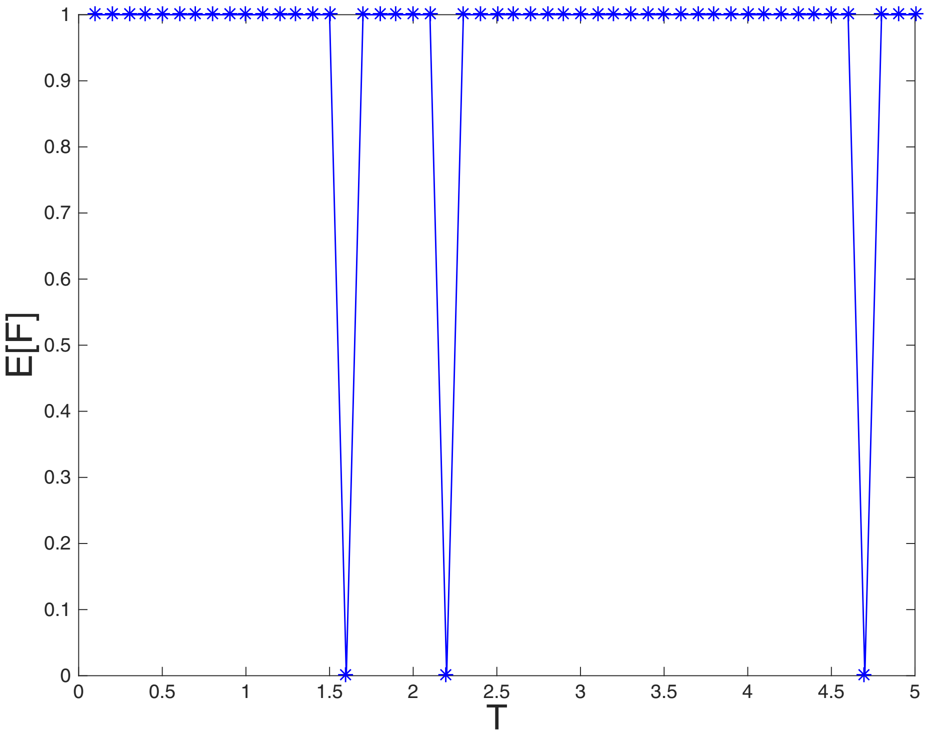

Fig. 4 Averaged fidelity E[F] as a function of the driving periods T for a system of size N=8. E[F] versus T show an sharp decrease at critical point towards a minimum to 0 and then a sudden rise approaching to 1. There are three phase transformation sudden change points in the diagram

Finally, in order to further study the properties of ergodic phase under periodic driving, we study the disordered Haldane-Shastry (HS) model with global two-body interactions for comparison and plot Fig. 5. It is found that there is no significant change in the trend of the curves under the same type of periodic driving. At this situation the system is still in the ergodic phase and doesn't have the MBL phase transition. It illustrates that interaction has important influence on the MBL. The stronger the interaction of the system, the more difficult it is to undergo a MBL phase transition.

4 Summary

In this paper, We extend Anderson localization in disordered systems to MBL interacting quantum systems at finite temperature. According to the conditions and properties of localized phase transition of interacting many-body system with static disordered external field, we study the influence of periodic driving on the properties of MBL in the periodically driven Heisenberg XXX model. Here we drive the Heisenberg XXX model periodically with the time-periodic field formed by the trigonometric functions. We explore whether periodic driving can cause phase transitions in Heisenberg XXX chains with nearest-neighbour coupling and disordered external fields using the exact matrix diagonalization. In order to

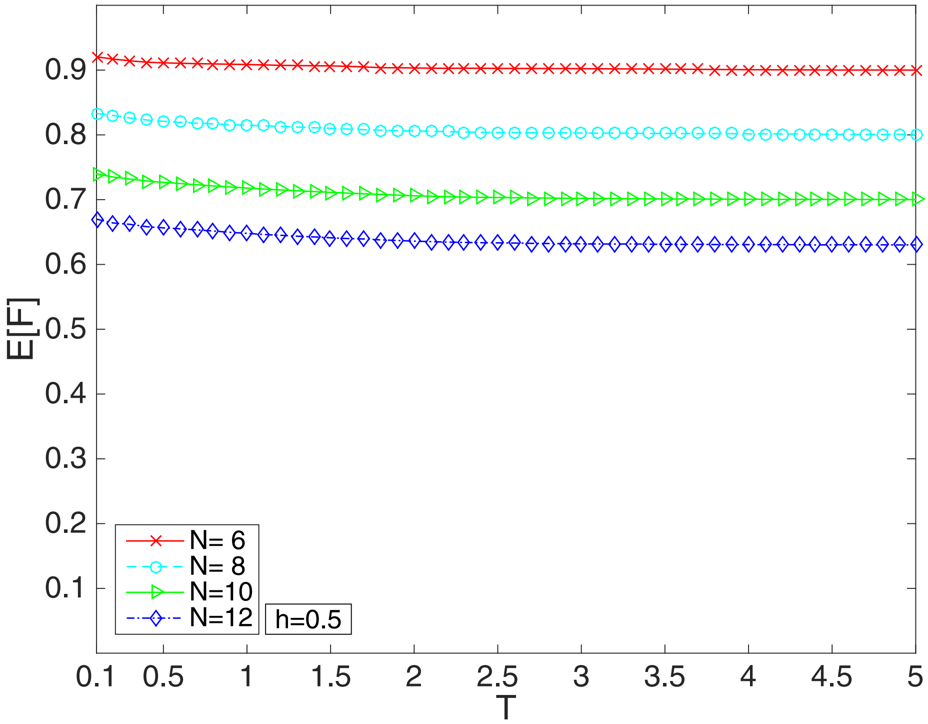

Fig. 5 Average fidelity E[F] as a function of the driving periods T for the value of disorder strength h=0.5 in the Haldane-Shastry (HS). The system sizes N are indicated in the legend and we used 1000 disorder realizations for each N

obtain some properties of the many-body eigenstates of the model in the vicinity of the localization leap critical point, we define the concept of fidelity of the n-th excited state. The results are consistent with previous analytical and numerical results, indicating that the excited-state fidelity does characterize the MBL transition. When the system is in weak disorder, it will drive the transition from the ergodic phase to the MBL phase. Conversely, for sufficiently strong disorder, the localized system will undergo a delocalized phase transition and MBL phase will be broken. The size of the system and the strength of disorder will affect the critical point of the phase transition and the magnitude of the phase transition. For non-disordered systems with an external field of equal strength, a sharp quantum phase occurs under the same periodic driving, unlike the MBL phase transition. To further illustrate the properties of many-body localization in the periodical driving, we investigate the phase transition properties of of the Haldane-Shastry (HS) model under the same drive. It is found that for the disordered two-body HS model which has long-range interactions in the ergodic phase, no phase transition occurs under the periodic driving. It illustrates that the interaction and disorder has important influence on the properties of localization. The stronger the interaction of the system, the more difficult it is for a phase transition to occur. We hope that the present work will contribute to a better understanding of the properties of MBL, and be helpful to explore unexpected and potentially useful properties in further research.

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Author Contributions Taotao H, Hui Zhao contributed the idea. Taotao Hu, Hui Zhao, Haoyue Li performed the calculations and prepared the figures. Hui Zhao wrote the main manuscript. Taotao Hu, Kang Xue, Xiaodan Li, Shuangyuan Ni, Jiali Zhang and Hang Ren checked the calculations and improved the manuscript. All authors contributed to discussions and reviewed the manuscript.

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Availability of data and material We guarantee that all data and materials support our published claims and comply with field standards.

Declarations

Conflict of Interests The authors declare that they have no competing interests.

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