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Adaptive Parameter Estimation With Convergence Analysis for the Prandtl–Ishlinskii Hysteresis Operator

Hysteresis is a nonlinear characteristic ubiquitously exhibited by smart material sensors and actuators, such as piezoelectric actuators and shape memory alloys. The Prandtl–Ishlinskii (PI) operator is widely used to describe hysteresis of smart material systems due to its simple structure and the existence of analytical inverse. A PI operator consists of a weighted superposition of play (backlash) operators. While adaptive estimation of the weights for PI operators has been reported in the literature, rigorous analysis of parameter convergence is lacking. In this article, we establish persistent excitation and thus parameter convergence for adaptive weight estimation under a rather modest condition on the input to the PI operator. The analysis is further supported via simulation, where a recursive least square (RLS) method is adopted for parameter estimation.
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Keywords: actuator, adaptive system, estimation, mechatronics

Introduction

Micro/nano drive technology is widely applied in the high-precision positioning field [1]. Smart actuators based on smart materials, such as piezoceramics [2], magnetically controlled shape memory alloys [3], and giant magnetostrictive materials [4], serve as the core part of high-accuracy positioning systems. The complex hysteresis exhibited by smart materials, however, severely challenges the control of these positioning systems. Modeling and parameter estimation of hysteresis in smart materials has been a subject of extensive interest [5,6].

Existing hysteresis models are mainly divided into two categories, which include physical models and phenomenological models. Phenomenological models are widely used, and they can be further classified into two categories depending on whether they are based on differential equations or operators. Differential equation-based models mainly include the backlash-like models [7], Duhem models [8,9], and Bouc–Wen models [10,11]. Operator-based models consist of weighted superposition of elementary hysteretic units, examples of which include Preisach model [12,13], Krasnosel'skii–Pokrovskii model [14,15], and Prandtl–Ishlinskii (PI) model [16–18]. Among these hysteresis models, the classical PI operator and its variants have been widely used in the modeling of smart material actuators. The main reason for the widespread use of the PI operator is that it possesses a simple structure and admits an analytical inverse, which provides convenience for the implementation of feed-forward inverse compensation control.

The PI operators typically consist of weighted combination of play operators, and the input–output curves of a PI operator are determined by both thresholds and their corresponding weights. Existing work on the modeling or control of the PI operator typically adopts predefined thresholds and identifies the values for the weights. For example, the particle swarm optimization algorithm was used offline to obtain the weight values of the rate-dependent PI model [19]. In addition, the gradient descent algorithm, the Levenberg–Marquardt method, the least squares algorithm, and their variants were used to identify the parameters of PI operators. For example, in Ref. [20], the Levenberg–Marquardt method was used to identify the parameters of a modified PI model for hysteresis of the pneumatic muscle actuator. In Ref. [21], the estimates of the weights of a PI operator were updated with an adaptive variable structure for stable nonlinear systems. In addition, the adaptive inverse compensation control method was proposed to address the hysteresis of the piezoelectric actuator, and the weights of the PI operator were adjusted adaptively [22]. Despite the extensive work on this topic, little work has been reported on rigorous analysis of parameter convergence conditions for adaptive estimation of PI operators.

In this article, we present a novel approach to examining the parameter convergence problem in adaptive identification of PI operators. Under a mild condition on the input to the PI operator, we establish that the outputs of constituent play operators are linearly independent and thus persistently exciting, which guarantees the parameter convergence under popular estimation schemes such as the recursive least square (RLS) algorithm and the gradient algorithm. Simulation results are further presented to support the analysis, where we show the parameter estimates converge to their true values under an RLS scheme.

The remainder of this article is organized as follows. The play operator and the PI operator are first introduced. Next, the convergence of parameter estimation is analyzed in detail. We then present the simulation results to support the analysis. Finally, concluding remarks are provided.

Prandtl–Ishlinskii Operator

The PI operators typically consist of weighted combination of play operators (sometimes stop operators are used instead). The input–output curve of the play operator is shown in Fig. 1. The mathematical expression of the play operator is given as follows:

$$\begin{aligned} x(t) &= H_r(u(t), x(t_{i-1}), r) \\ &= \max\{u(t) - r, \min\{u(t) + r, x(t_{i-1})\}\} \end{aligned} \tag{1}$$

where $u(t) \in C[0, T]$ represents the piecewise monotone input, $x(t) \in C[0, T]$ is the output, and $t \in [t_0, t_m]$, $t_0 \le t_1 \le \cdots \le t_{i-1} \le t \le t_i \le \cdots \le t_{m-1} \le t_m$, where t_is are times of input reversals and r is

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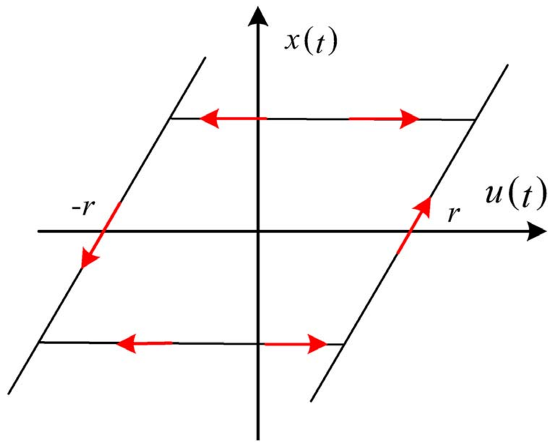

Fig. 1 The input–output curve of the play operator

the threshold of the play operator. The initial condition of the play operator is given by

$$x(t_0) = H_r(u(t_0), x(0), r) \\ = \max\{u(t_0) - r, \min\{u(t_0) + r, x(0)\}\} \tag{2}$$

where $x(0) \in \mathcal{R}$ is an initial condition, which does not have to be zero in actual applications. The PI operator is formed as a weighted sum of multiple plays:

$$y(t) = \sum_{j=1}^{n} \omega_j H_{r_j}(u(t), x_j(t_{i-1}), r_j) \\ = \sum_{j=1}^{n} \omega_j \max\{u(t) - r_j, \min\{u(t) + r_j, x_j(t_{i-1})\}\} \tag{3}$$

where $y(t)$ is the output of the PI operator, ω_j and r_j are the weight and threshold of the play operator, respectively, r_j satisfies that $0 = r_1 < \cdots < r_n < +\infty$, and n is the number of the play operators.

The threshold r influences the width of the hysteresis loop. Figure 2 shows the variations in the input–output behavior of the play operator with different threshold values under the input signal $u(t) = 5\sin(2\pi t)$. From Eq. (1), we can see that its output $x(t)$ depends on the play threshold in a complex nonlinear manner.

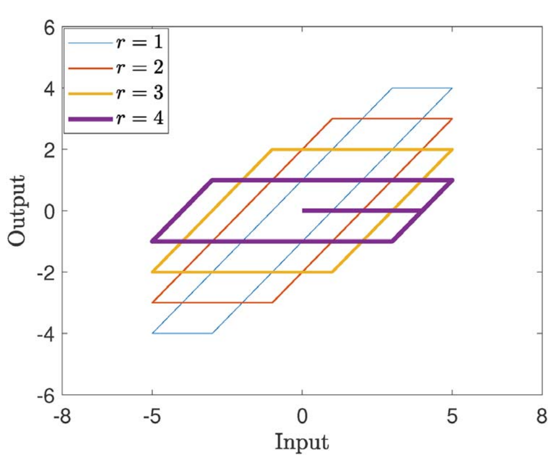

Fig. 2 Output of the play operator with different threshold values *r*

Convergence Analysis of the Parameter Estimation Algorithm

In this section, we present the technical analysis of algorithms for adaptive estimation of the weights of PI operators. To put the discussion in context, we will use the RLS estimation method as an example. The persistent excitation condition we will establish, however, will work equally well with other estimation schemes (such as the gradient algorithm) to ensure parameter convergence.

We rewrite the PI operator as follows:

$$y(t) = \sum_{j=1}^{n} \omega_j H_{r_j}(u(t), x_j(t_{i-1}), r_j) \\ = H_r^T(t) W(t) \tag{4}$$

where $W(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_n(t)]^T$. For ease of presentation, we write $H_{r_j}(u(t), x_j(t), r_j)$ as $H_{r_j}(t)$. Here, $H_r^T(t) = [H_{r_1}(t), H_{r_2}(t), \ldots, H_{r_n}(t)]$. With the RLS method, the estimate of the weights, denoted as $\hat{W}(t)$, evolves as follows:

$$\dot{\hat{W}}(t) = P(t)\epsilon H_r^T(t) \tag{5}$$

where $\epsilon = (H_r^T(t)W(t) - H_r^T(t)\hat{W}(t))/(m^2)$, $m = 1 + H_r^T(t)H_r(t)$, and $P(t)$ are determined by

$$\dot{P}(t) = -\frac{P(t)H_r(t)H_r^T(t)P(t)}{m^2}, \quad P(0) = P_0 \tag{6}$$

where P_0 is a positive definite matrix. The estimated output of the PI operator is expressed as follows:

$$\hat{y}(t) = H_r^T(t)\hat{W}(t) \tag{7}$$

The convergence of the RLS algorithm for the weights of the PI operator is analyzed under the persistent excitation condition. We first give the definition of the persistent excitation condition, and the assumption on the input $u(t)$ that we will show implies persistent excitation.

Definition 1. *A continuous vector function $H_r(t)$: $[0, \infty) \to \mathcal{R}^n$ is persistently exciting, if there exist positive constants T_0 and c_1, such that for every $t_0 > 0$:*

$$\frac{1}{T_0}\int_{t_0}^{t_0+T_0} H_r(t)H_r^T(t)dt \geq c_1 I_n \tag{8}$$

where I_n represents the n-dimensional identity matrix.

Assumption 1. *There exists $T_0 > 0$, such that, for any $t_0 > 0$, the following is true*:

$$\max_{t\in[t_0,t_0+T_0]} u(t) - \min_{t\in[t_0,t_0+T_0]} u(t) \geq 2r_{\max} \tag{9}$$

where $r_{\max}$ is the upper bound of the play thresholds.

Note that Assumption 1 does not require the input to be periodic, although a periodic signal with peak-to-peak magnitude larger than $2r_{\max}$ automatically satisfies this assumption. The assumption ensures that, within each T_0, for any play operator in the given PI operator, it will traverse different operating regimes (linearly increasing/decreasing envelopes and flat interiors) of the hysteresis loop as illustrated in Fig. 1. We will show that the assumption ensures the persistent excitation of the resulting vector $H_r(t)$, which is also the regressor vector in the adaptive estimation algorithm. Therefore, the adaptive estimation scheme results in parameter convergence. To show that, we first present the following lemma.

Lemma 1. *The outputs of the plays, $H_{r_i}(t)$, $i = 1, 2, \ldots, n$, with $t \in [t_0, t_0 + T_0]$, are linearly independent, if and only if $H_r(t) = [H_{r_1}(t), H_{r_2}(t), \ldots, H_{r_n}(t)]^T$ satisfies*

$$\int_{t_0}^{t_0+T_0} H_r(t)H_r^T(t)dt \geq c_1 I_n \tag{10}$$

for some $c_1 > 0$.

Proof. We use contradiction to prove this statement. Note that $H_{r_i}(t)$, $i = 1, 2, \ldots, n$ are continuous functions. First, we assume that $\int_{t_0}^{t_0+T_0} H_r(t)H_r^T(t)dt \geq c_1 I_n$ is true, but for $t \in [t_0,\ t_0 + T_0]$, $H_{r_i}(t)$'s are linearly dependent. According to the definition of the linear dependence, there exists a nonvanishing vector, $\Lambda = [\lambda_1,\ \lambda_2,\ \ldots,\ \lambda_n]^T$ that satisfies $H_r^T(t)\Lambda = 0$. Then, we can get

$$\begin{aligned}&\Lambda^T\left[\int_{t_0}^{t_0+T_0} H_r(t)H_r^T(t)dt\right]\Lambda \\ &= \int_{t_0}^{t_0+T_0} [\Lambda^T H_r(t)]^2 dt \\ &= 0\end{aligned} \tag{11}$$

which contracts Eq. (10). Next, we assume that $H_{r_i}(t)$'s are linearly independent, but Eq. (10) fails to hold. Since the integral in Eq. (10) is a symmetric and semipositive definite matrix, all of its eigenvalues are real and nonnegative. Since Eq. (10) does not hold, one can infer at least one of the eigenvalues is zero, which implies the existence of a nonvanishing vector Λ, such that

$$\begin{aligned}\Lambda^T\left[\int_{t_0}^{t_0+T_0} H_r(t)H_r^T(t)dt\right]\Lambda &= \int_{t_0}^{t_0+T_0} [\Lambda^T H_r(t)]^2 dt \\ &= 0\end{aligned} \tag{12}$$

Since $\Lambda^T H_r(t)$ is a continuous function, Eq. (12) implies $\Lambda^T H_r(t) \equiv 0$, for $t \in [t_0,\ t_0 + T_0]$, i.e., $H_{r_i}(t)$'s are linearly dependent, which is a contraction. ∎

With Lemma 1, we will just need to show that $H_{r_1}(t), H_{r_i}(t),\ \ldots, H_{r_n}(t)$ are linearly independent with an input $u(t)$ satisfying Assumption 1.

Theorem 1. *If the input $u(t)$ of the PI operator satisfies Assumption 1, the outputs of the plays $H_{r_i}(t)$ are linearly independent and thus persistently exciting*.

Proof. To facilitate the discussion, we illustrate the argument with the special case of a PI operator with three plays. The argument extends to the general case of n plays in a straightforward manner.

We again prove the statement by contradiction. We assume that $H_{r_1}(t)$, $H_{r_2}(t)$, and $H_{r_3}(t)$ are linearly dependent on $[t_0,\ t_0 + T_0]$, which means that $\exists\, \Lambda = [\lambda_1,\ \lambda_2,\ \lambda_3]^T \neq 0$, such that, for $t \in t_0,\ t_0 + T_0]$,

$$\lambda_1 H_{r_1}(t) + \lambda_2 H_{r_2}(t) + \lambda_3 H_{r_3}(t) = 0 \tag{13}$$

From Assumption 1, one can find an interval $[t_1,\ t_2] \subset [t_0,\ t_0 + T_0]$, such that at t_1, all plays operate at the interior flat regimes, and at t_2, all plays have exited the interior and operate on their linearly increasing envelopes (see Figs. 3 and 4 for illustration). Clearly, Eq. (13) is satisfied on $[t_1,\ t_2]$ since the latter is a subinterval of $[t_0,\ t_0 + T_0]$. We further divide $[t_1,\ t_2]$ into four segments, according to the output inflection points of these three play operators: τ_1 is the

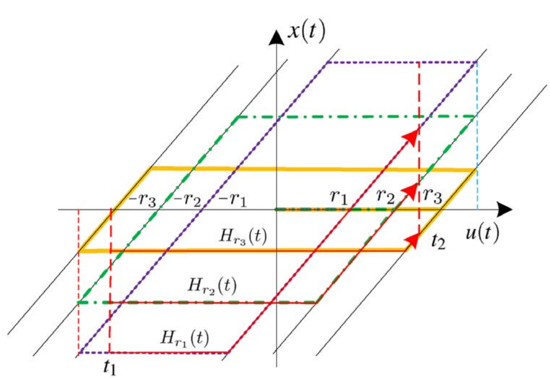

Fig. 3 Illustration for the proof of Theorem 1: the output of three plays with the input *u*(*t*)

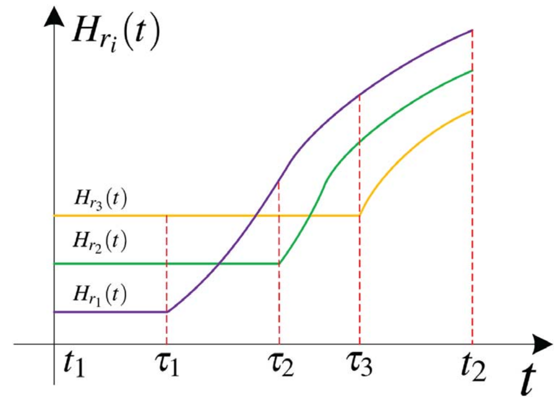

Fig. 4 Illustration for the proof of Theorem 1: the output of three plays with the time *t*

inflection point (time at which the output transitions from the interior flat regime to the linearly growing envelope on the $x - u$ plane) of H_{r_1}, τ_2 is the inflection point of H_{r_2}, and τ_3 is the inflection point of H_{r_3}. As shown in Fig. 4, by applying Eq. (13) to $t = \tau_1$, we have

$$\lambda_1 H_{r_1}(\tau_1) + \lambda_2 H_{r_2}(\tau_1) + \lambda_3 H_{r_3}(\tau_1) = 0 \tag{14}$$

Then, applying Eq. (13) to $t \in (\tau_1,\ \tau_2]$, we have

$$\lambda_1 H_{r_1}(t) + \lambda_2 H_{r_2}(\tau_1) + \lambda_3 H_{r_3}(\tau_1) = 0 \tag{15}$$

where we have used $H_{r_2}(t) \equiv H_{r_2}(\tau_1)$ and $H_{r_3}(t) \equiv H_{r_3}(\tau_1)$ (see Fig. 4). From Eqs. (14) and (15), it is evident that

$$\lambda_1 [H_{r_1}(\tau_1) - H_{r_1}(t)] = 0 \tag{16}$$

Since $H_{r_1}(\tau_1) = H_{r_1}(t)$ cannot be true for all $t \in (\tau_1,\ \tau_2]$, we get $\lambda_1 = 0$ from Eq. (16). Now proceeding in a similar manner and applying Eq. (13) to $t \in (\tau_2,\ \tau_3]$, we get

$$\lambda_1 H_{r_1}(t) + \lambda_2 H_{r_2}(t) + \lambda_3 H_{r_3}(\tau_2) = 0 \tag{17}$$

Since $\lambda_1 = 0$ and $H_{r_3}(\tau_1) = H_{r_3}(\tau_2)$, subtracting Eq. (17) from Eq. (15), we get

$$\lambda_2 [H_{r_2}(\tau_1) - H_{r_2}(t)] = 0 \tag{18}$$

Since $H_{r_2}(\tau_1) = H_{r_2}(t)$ cannot be true for all $t \in (\tau_2,\ \tau_3]$, we get $\lambda_2 = 0$ from Eq. (18). For $t \in (\tau_3,\ t_2]$, it is derived that $\lambda_3 H_{r_3}(t) = 0$ according to Eq. (13). Since $H_{r_3}(t)$ is strictly increasing in this time interval, we obtain $\lambda_3 = 0$. Therefore, we get $\Lambda = [0,\ 0,\ 0]$, which is in contradiction with $\Lambda \neq 0$. ∎

Simulation Results

This section shows the simulation results, where an example of a PI operator with three plays is used. The chosen input u in the simulation easily satisfies Assumption 1. First, we consider the input signal $u(t) = 10\sin(2\pi ft)$, where $f = 5$ Hz, $t \in [0,\ 1]$, and the sampling time is set to be 0.001 second. First, we consider the weight values $\omega_1 = 1$, $\omega_2 = 3$, and $\omega_2 = 5$, with threshold values $r_1 = 0$, $r_2 = 10/3$, and $r_3 = 20/3$. The simulation results are shown in Fig. 5. From Fig. 5, it can be seen that the parameter estimation process is convergent and the estimated errors $\tilde{\omega}_1(t)$, $\tilde{\omega}_2(t)$, and $\tilde{\omega}_3(t)$ approach zero.

We further consider another input that generates minor hysteresis loops, $u(t) = 10\sin(10\pi t) + 5\sin(2\pi t)$. The unknown weights are set to be $\omega_1 = 2$, $\omega_2 = 6$, and $\omega_3 = 8$, with play threshold values set to be $r_1 = 0$, $r_2 = 10/3$, and $r_3 = 20/3$. The simulation results are shown in Fig. 6. Again, one can see that the errors of weight parameter estimation go to 0, quickly.

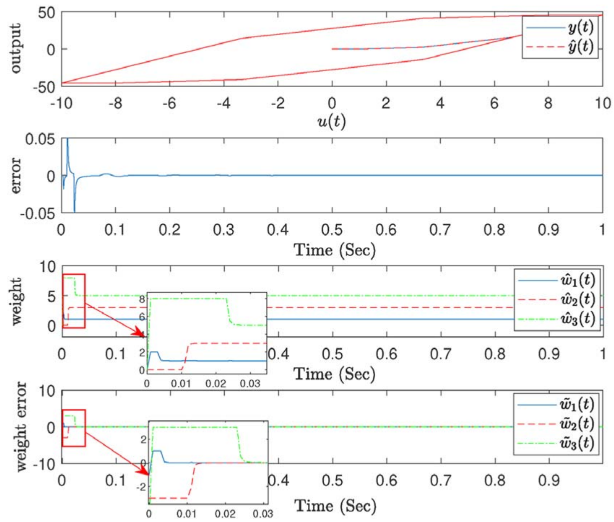

Fig. 5 Simulation results on weights estimation for a PI operator with the desired weights $\omega_1 = 1$, $\omega_2 = 3$, and $\omega_2 = 5$, with a sinusoidal input

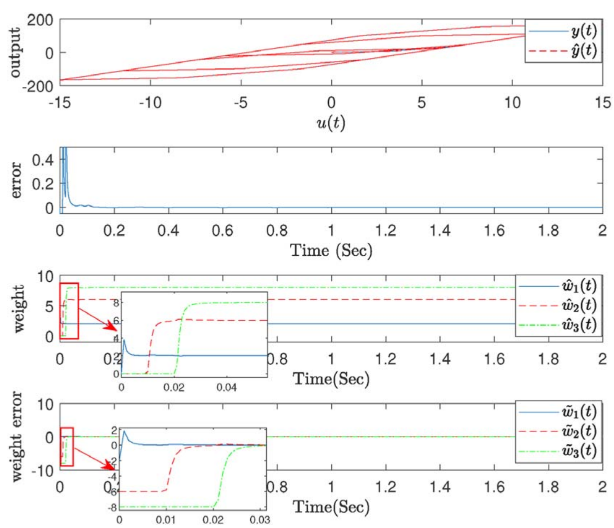

Fig. 6 Simulation results on weights estimation for a PI operator with three plays, with a composite input

Conclusion and Future Work

This study focused on the estimation of the weight parameters of the PI hysteresis operators. We considered the RLS method as an example for the estimation algorithm. The convergence of the weight parameters was analyzed by establishing a persistent excitation condition under a fairly modest condition on the hysteresis input. In particular, the condition only requires the input to have adequate variation across some interval T_0, which "excites" all play operators within the PI operator. Such a condition, unique to systems with hysteresis, is in sharp contrast to conditions for persistent excitation in linear systems, which typically involve the number of frequency components in the input. In some sense, the strong nonlinearity of the hysteresis operators have provided an edge when it comes to weight parameter estimation—the n-dimensional regressor vector, no matter how large n is, could be persistently exciting even under a sinusoidal input.

For future work, we will focus on extending the analytical approach to other operator-based models, for example, the Preisach and Krasnosel'skii–Pokrovskii model, and further validate the theoretical results with experiments involving smart material actuators.

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Conflict of Interest

There are no conflicts of interest.

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