International Journal of Theoretical Physics (2020) 59:1330–1337
https://doi.org/10.1007/s10773-020-04410-7

Many-Body Localization Transition in the Heisenberg Ising Chain

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Received: 18 October 2019 / Accepted: 4 February 2020 / Published online: 17 February 2020

Abstract
In this paper, we use exact matrix diagonalization to explore the many-body localization (MBL) transition of the Heisenberg Ising spin-1/2 chain with nearest neighbor couplings and disordered external fields. It demonstrates that the fidelity, magnetization and spin-spin space correlation can be used to characterize the many-body localization transition in this closed spin system which is also in agreement with previous analytical and numerical results. We test the properties for the middle third many-body eigenstates. It shows that for this model with random-field, the excited-state fidelity exhibits a pronounced drop at the transition and then gradually tends to be stable in the localized phase, the critical point and the final value of averaged fidelity are all size dependent. It demonstrates that disordered external fields drive the occurrence of the MBL transition. Moreover, we investigate the magnetization and spin-spin space correlation in this model to verify the conclusion we got and further explore the properties of ergodic phase and localized phase.

Keywords Many-body localization transition · Disorder system · Ising spin chain

1 Introduction

Much attention has been devoted to researching the properties of disordered systems with interactions over the past decades. Early seminal paper by Anderson in 1958 [1] has showed that the closed quantum system of single particles shows a complete absence of diffusion in

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both the ground state and excited states with sufficiently strong quenched disorder, and since then this proposition has attracted extensive attention and has ultimately led to the complete conclusion that non-interacting systems in one or two dimensions will be localized for arbitrary disorder, even for very small disorder, which was named Anderson localization [2, 3]. Generalizing Anderson localization concept to interacting quantum systems, Anderson also predicted that for sufficiently strong disorder, a closed interacting quantum system would also be localized which has been confirmed by Basko et al. with new arguments much more recently [4] which revived the idea of many-body localization (MBL). MBL at finite temperature serves as a counter example of the basic hypothesis of statistical physics, showing that interacting quantum systems fail to approach thermal equilibrium with sufficiently strong disorder. While many-body interaction often leads to the Anderson localization breakdown because of the new channels for energy or particle transport [5, 6].

There are many distinctions between the localized phase with stronger disorder and delocalized phase at lower disorder which is named ergodic phase. The eigenstates thermalization hypothesis(ETH) [7–10] indicates that the systems should reach local thermal equilibrium. On the contrary, one of the key features of MBL is the system remember its initial state instead of thermally equilibrate. MBL is a quantum phase transition that occurs at nonzero (or even infinite) temperature, where the system fail to quantum statistical mechanics equilibrium. Like the more familiar ground-state quantum-phase transitions, this transition is a sharp change in the properties of the many-body eigenstates of the Hamiltonian, while MBL transition at nonzero temperature appears to be only a dynamical phase transition [11]. These fundamental questions about the dynamics of MBL were experimentally proved since such systems can be produced and studied with strongly interaction, and this phenomenon have attracted growing attention because of the significant effect in quantum information [12–17]. Many studies [18–29] have confirmed that this novel dynamical phase transition can happen in the interacting disordered systems and explored many features of the MBL phase.

Generally a dramatic change in the state around the quantum critical point should result in a great difference of physics, which was used to distinguish between the ergodic regime and the MBL one. In this letter we mainly study the Heisenberg Ising model with disordered external field [30]. This transition is a quantum phase transition can occurring at nonzero (even infinite) temperature, in order to distinguish our research from ground-state phase transition we pay attention to the middle third of the eigenstates. Recently, generous of effort [31–43] has been devoted to the properties of fidelity, a popular concept in quantum critical phenomena. As a measure of similarity between states, fidelity can be used to signal any phase transition. It emerged from quantum-information science and plays an important role in quantum phase transitions (QPTs) [44]. In particular, the minimum of fidelity near a critical point has been studied in several models [45–47]. In this letter we mainly study the excited-state fidelity to characterize many-body localization transition. Following Ref. [31], the fidelity of the n-th excited state $|\Psi_n(\lambda)\rangle$ of the system is defined here as the overlap of the excited states with parameters λ and $\lambda + \delta\lambda$:

$$F_n(\lambda, \lambda + \delta\lambda) = \langle \Psi_n(\lambda) | \Psi_n(\lambda + \delta\lambda) \rangle, \tag{1}$$

2 Model Used for Numerics

As Anderson's original proposal many-body localization appears to occur for a wide variety of spin models. To investigate the MBL transition, we study a specific simple spin model, a one-dimensional Heisenberg Ising spin-1/2 chain with random fields along the z direction. Considering open boundary conditions, the Hamiltonian reads as follows:

$$H = \sum_{i}^{N-1} J S_i^z S_{i+1}^z + \sum_{i}^{N} h_i S_i^z, \tag{2}$$

N is the number of spins, S_i^z is the spin operator at the i-th qubit, and at each site i, the disorder realization of the static-random fields h_i are independent random variables with a probability distribution that is uniform in $[-h, h]$. For convenience of calculations, we take J=1. We will study the behavior of systems given in (2) by exact diagonalization to explore whether the MBL transition can occur in this model and then explore some properties of ergodic phase and localized phase. To test fidelity of excited states, for the small parameter perturbation δh_i for each site, we let $\delta h_i = \epsilon h_i$ ($\epsilon = 10^{-5}$). It is worth noting that the parameter perturbation δh_i for each site are also different random variables. Then, for each disorder realization, we find the many-body eigenstates $|\Psi_n\rangle$ that are in the middle one third of the energy-ordered list of all data. Our qualitative conclusions do not depend on the exact values of these parameters. We then compute the fidelity F_n for each eigenstate $|\Psi_n\rangle$. Averaging over all selected excited states and disorder realizations yields the mean value $\mathsf{E}[F]$. The numerical analyses were performed using standard libraries for exact matrix diagonalization. This model has two global conservation laws: one for the total energy and one for the total magnetization S^z along the z direction. The total S^z symmetry and parallel programming techniques were employed to make the computations feasible. For each disorder amplitude $|h|$, we used 10^4 disorder realizations for N=6 and N=8, 2000 realizations for N=10 and N=12, and 200 realizations for N=14 to obtain the data shown in this paper.

3 Results and Discussion

In Fig. 1, we plot the averaged excited-state fidelity $\mathsf{E}[F]$ as a function of the disorder strength h for system sizes from $N = 6$ to $N = 14$, for the energies in the middle one third of the spectrum. It shows that the MBL transition dose occur in this isolated Ising model with the increase of disorder strength. One can see the $\mathsf{E}[F]$ versus h show a sequential decline until the disorder strength reach the critical point, then tend to be stable gradually. Notably, the final value of $\mathsf{E}[F]_c$ is size dependent and decreases as the system size increases. As the pronounced data change shown in Fig. 1, the critical disorder strength h_c are all exact values for different system sizes, for N=6, h_c = 2.6, N=8, h_c = 3.1, N=*10*, h_c = 3.4, N=12, h_c = 3.6, N=14, h_c = 3.7. So we get the extent of the critical point $h_c \in$ [2.6,3.7] for the breakdown of egodic phase, which agree with the prediction in [18, 24]

There are many distinctions between the ergodic phase and the localized phase, which are caused by the differences of many-body eigenstates of the Hamiltonian. As is known that in the ergodic phase ($h < h_c$), the many-body eigenstates are thermal, so the isolate quantum system can relax to thermal equilibrium [9, 48, 49]. On the contrary, in the localized phase ($h > h_c$), the many-body eigenstates are not thermal and the isolate quantum system does not relax to thermal equilibrium [4]. So we can further confirm the occurrence of MBL transition by probing how thermal the many-body eigenstates appear to be. We study the local expectation value of the z component of spin, for each disorder realization, and identify the critical points h_c for the model given in (2).

$$m_i^{(n)} = \langle \Psi_n | s_i^z | \Psi_n \rangle, \tag{3}$$

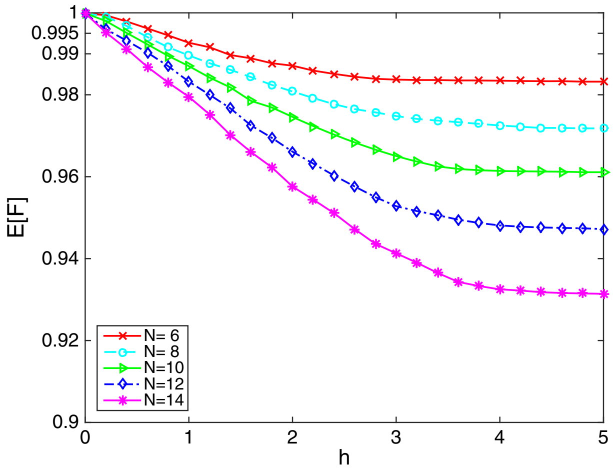

Fig. 1 Averaged fidelity as a function of the disorder strength h for system sizes from 6 to 14. The system sizes N are indicated in the legend. $\mathsf{E}[F]$ versus h show a sequential decline until the disorder strength reach the critical point and then become stable gradually. Notably, the final value of $\mathsf{E}[F]_c$ is size dependent and decreases as the number of system size increases

S_i^z is the spin operator at site i, $|\Psi_n\rangle$ is the eigenstate. For each site in each disorder realization, we compare these expectation values for eigenstates that are adjacent in energy; averaging over all selected excited states, disorder realizations, and sites yields the mean value of the difference $E[|m_i^{(n)} - m_i^{(n+1)}|]$. In the selected energy range, the difference $E[|m_i^{(n)} - m_i^{(n+1)}|]$ in energy density between adjacent states $|\Psi_n\rangle$ and $|\Psi_{n+1}\rangle$ is of order $\sqrt{N}2^{-N}$ and thus the difference is exponentially decreasing in N as N increases in the ergodic phase. So If these eigenstates are thermal, then they represent temperatures that differ only by this exponentially small amount; therefore, the expectation values of s_i^z for two such states should be the same for $N \to \infty$.

In Fig. 2, we plot the averaged difference $ln(E[|m_i^{(n)} - m_i^{(n+1)}|])$ as a function of the system size N for various values of the disorder amplitude h, for energies in the middle one third of the energy list. As expected, in the ergodic phase (at small h), the averaged differences $E[|m_i^{(n)} - m_i^{(n+1)}|]$ tend to vanish exponentially as N increases. Moreover, in the localized phase (at large h), the averaged differences $E[|m_i^{(n)} - m_i^{(n+1)}|]$ between adjacent eigenstates remain large as N increases; our work indicates that disordered external field does drive the occurrence of the MBL transition of this one-dimensional Heisenberg Ising spin-1/2 chain with nearest neighbor couplings. Accordingly, we obtain the span of the critical point $h_c \in (2.5, 3.7)$ for the many-body localized phase transition in this disordered Ising spin chain, which is consistent with the previous work [24, 47, 50]. Comparing Figs. 1 and 2, it shows that the behavior of the transition region for the excited-state fidelity is consistent with that of the difference $E[|m_i^{(n)} - m_i^{(n+1)}|]$. This observation indicates that the the MBL transition does indeed occur in this disordered Ising model.

To further explore the properties of ergodic phase and localized phase and verify the conclusion of previous work in our model, we next study the spin correlation on length scales. The correlation functions within a many-body eigenstate $|\Psi_n\rangle$ in the middle one third of the energy list of the Hamiltonian of sample α was given by

$$C_{n\alpha}^{zz}(ij) = \langle\Psi_n|s_i^z s_j^z|\Psi_n\rangle_\alpha - \langle\Psi_n|s_i^z|\Psi_n\rangle\langle\Psi_n|s_j^z|\Psi_n\rangle_\alpha, \quad (4)$$

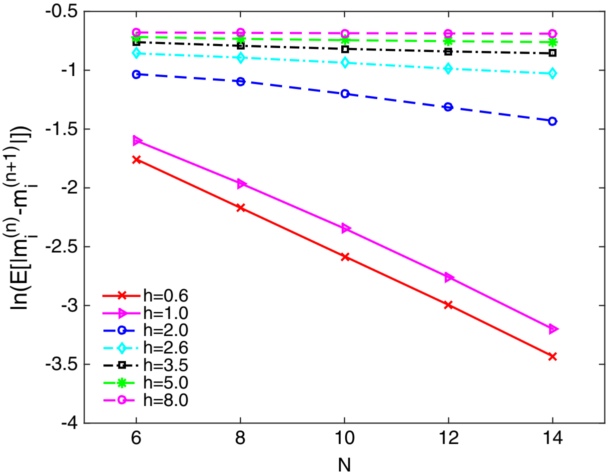

Fig. 2 Averaged difference as a function of the system sizes N from 6 to 14 for various values of the disorder amplitude h. The values of h are indicated in the legend. In the ergodic phase ($h < h_c$), where the eigenstates are thermal, the Averaged difference vanish exponentially in N as N increases. On the contrary, in the localized phase ($h > h_c$), the Averaged difference remain large

In Fig. 3 we plot the average value $[\ln|C^{zz}_{n\alpha}(i, i+d)|]$ as a function of the distance d for three representative values of h in the two phases and near the critical point. We also select the eigenstates in the middle one third of the energy list, averaging over all selected excited states, disorder realizations, and sites to get all the data. In the ergodic phase($h = 0.6$), These distant spins at sites i and j are entangled and correlated: if spin i is flipped, that quantum of spin is delocalized and may instead be at any of the other sites. In the localized

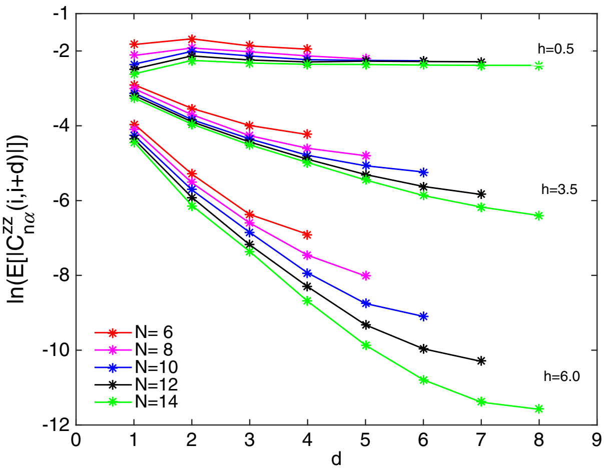

Fig. 3 The spin-spin correlations as a function of the distance d for three representative values of h. The system sizes N are indicated in the legend. In the localized phase (h=6.0), the correlations decrease exponentially with d, while in the ergodic phase (h=0.5) they are independent of d at large d. The intermediate behavior (h =3.5) is near the localization transition

phase ($h = 6.0$), the eigenstates are not thermal. If spin i is flipped within a single eigenstate that quantum of spin remains localized near site i with its amplitude for being at site j falling off exponentially with the distance. The data of Figs. 1–3 all data confirm the occurrence of MBL transition in the one-dimensional Heisenberg Ising spin-1/2 chain and show some of the differences between the ergodic phase and localized phase.

4 Summary

Disorder is an intrinsic property of all real systems, and the interplay between disorder and interaction constitutes the driving mechanism of the glass transition (metal-insulator transition); similarly, the transition from the ergodic to the many-body localized phase is a highly non-equilibrium phenomenon, but one that is poorly understood at present. In this paper, we use exact matrix diagonalization to explore the many-body localization (MBL) transition of the one-dimensional Heisenberg Ising spin-1/2 chain with nearest neighbor couplings and disordered external fields. In order to get some properties of the many-body eigenstates of our model near the critical point of the localization transition, we test the fidelity between two excited states related by a small parameter perturbation δh. The consequence is consistent with previous analysis and numerical results showing that the excited-state fidelity does characterize the MBL transition. The results show that for this model with random-field, the fidelity exhibits a sequential decline until the disorder strength reach the critical point, then tend to be stable gradually, and the critical disorder strength h_c are size dependent, the span is $h_c \in [2.6,3.7]$ for the breakdown of egodic phase. In order to further explore the properties of ergodic phase and localized phase, we also study the magnetization and spin spatial correlations. All the data confirm the occurrence of MBL transition in the one-dimensional Heisenberg Ising spin-1/2 chain and show some of the differences between the ergodic phase and localized phase. We hope that the present work provides a novel window into the remarkable phenomenon of many-body localization.

Acknowledgments This work is supported by "the Fundamental Research Funds for the Central Universities" (No. 2412019FZ037). T. T. H was also supported in part by the Government of China through CSC.

References

1. Anderson, P.W.: Absence of diffusion in certain random lattices. Phys. Rev. Lett. **109**, 1492 (1958)
2. Abrahams, E.: 50 Years of Anderson Localization. World Scientific Publishing, Singapore (2010)
3. Lee, P.A., Ramakrishnan, T.V.: Disordered electronic systems. Rev. Mod. Phys. **57**, 287 (1985)
4. Basko, D.M., Aleiner, I.L., Altshuler, B.L.: Metal-insulator transition in a weakly interacting many-electron system with localized single-particle states. Ann. Phys. (Amsterdam) **321**, 1126 (2006)
5. Fleishman, L., Anderson, P.W.: Interactions and the Anderson transition. Phys. Rev. B. **21**, 2366 (1980)
6. Gornyi, I.V., Mirlin, A.D., Polyakov, D.G.: Interacting electrons in disordered wires: Anderson localization and low-T transport. Phys. Rev. Lett. **95**, 206603 (2005)
7. Deutsch, J.M.: Quantum statistical mechanics in a closed system. Phys. Rev. A **43**, 2046 (1991)
8. Tasaki, H.: From quantum dynamics to the canonical distribution: General picture and a rigorous example. Phys. Rev. Lett. **80**, 1373 (1998)
9. Rigol, M., Dunjko, V., Olshanii, M.: Thermalization and its mechanism for generic isolated quantum systems. Nature (London) **452**, 854 (2008)
10. Srednicki, M.: Chaos and quantum thermalization. Phys. Rev. E. **50**, 888 (1994)
11. Oganesyan, V., Huse, D.A.: Localization of interacting fermions at high temperature. Phys. Rev. B **75**, 155111 (2007)

12. Quan, H.T., Song, Z., Liu, X.F., Zanardi, P., Sun, C.P.: Decay of Loschmidt echo enhanced by quantum criticality. Phys. Rev. Lett. **96**, 140604 (2006)
13. Polkovnikov, A., Sengupta, K., Silva, A., Vengalattore, M.: Nonequilibrium dynamics of closed interacting quantum systems. Rev. Mod. Phys. **83**, 863 (2011)
14. Burrell, C.K., Osborne, T.J.: Bounds on the speed of information propagation in disordered quantum spin chains. Phys. Rev. Lett **99**, 167201 (2007)
15. Zheng, Y., Yang, J., Shen, Z., et al.: Optically induced transparency in a micro-cavity. Light Sci Appl **5**, e16072 (2016). https://doi.org/10.1038/lsa.2016.72
16. Saeed, S., de Weerd, C., Stallinga, P., et al.: Carrier multiplication in germanium nanocrystals. Light Sci Appl **4**, e251 (2015). https://doi.org/10.1038/lsa.2015.24
17. Lai, Y., Lan, Y., Lu, T.: Strong light-matter interaction in ZnO microcavities. Light Sci Appl **2**, e76 (2013). https://doi.org/10.1038/lsa.2013.32
18. Pal, A., Huse, D.A.: Many-body localization phase transition. Phys. Rev. B **82**, 174411 (2010)
19. Schreiber, M., Hodgman, S.S., Bordia, P., Lüschen, H.P., Fischer, M.H., Vosk, R., Altman, E., Schneider, U., Bloch, I.: Observation of many-body localization of interacting fermions in a quasirandom optical lattice. Science **349**, 842 (2015)
20. Smith, J., Lee, A., Richerme, P., Neyenhuis, B., Hess, P.W., Hauke, P., Heyl, M., Huse, D.A., Monroe, C.: Many-body localization in a quantum simulator with programmable random disorder. Nat. Phys. **12**, 907 (2016)
21. Choi, J.-y., Hild, S., Zeiher, J.P., Schau, B., Rubio-Abadal, A., Yefsah, T., Khemani, V., Huse, D.A., Bloch, I., Gross, C.: Exploring the many-body localization transition in two dimensions. Science **352**, 1547 (2016)
22. Bordia, P., Lüschen, H.P., Hodgman, S.S., Schreiber, M., Bloch, I., Schneider, U.: Coupling identical one-dimensional many- body localized systems. Phys. Rev. Lett. **116**, 140401 (2016)
23. Canovi, E., Rossini, D., Fazio, R., Santoro, G.E., Silva, A.: Quantum quenches, thermalization, and many-body localization. Phys. Rev. B **83**, 094431 (2011)
24. De Luca, A., Scardicchio, A.: Ergodicity breaking in a model showing many-body localization. Europhys. Lett. **101**, 37003 (2013)
25. Kjall, J.A., Bardarson, J.H., Pollmann, F.: Many-body localization in a disordered quantum Ising chain. Phys. Rev. Lett. **113**, 107204 (2014)
26. Nandkishore, R., Huse, D.A.: Many-body localization and thermalization in quantum statistical mechanics. Ann. Rev. Condensed Matter Phys. **6**, 15–38 (2015)
27. Luitz, D.J., Laflorencie, N., Alet, F.: Many-body localization edge in the random-field Heisenberg chain. Phys. Rev. B **91**, 081103 (2015)
28. Goold, J. et al.: Total correlations of the diagonal ensemble herald the many-body localization transition. Phys. Rev. B **92**, 180202(R) (2015)
29. Lev, Y.B., Cohen, G., Reichman, D.R.: Absence of diffusion in an interacting system of spinless fermions on a one-dimensional disordered lattice. Phys. Rev. Lett. **114**, 00601 (2015)
30. Burin, A.L.: Phys. Rev. B. **92**, 104428 (2015)
31. Zanardi, P., Paunkovic, N.: Ground state overlap and quantum phase transitions. Phys. Rev. E **74**, 031123 (2006)
32. Garnerone, S., Jacobson, N.T., Haas, S., Zanardi, P.: Fidelity approach to the disordered quantum XY model. Phys. Rev. Lett. **102**, 057205 (2009)
33. Quan, H.T., Song, Z., Liu, X.F., Zanardi, P., Sun, C.P.: Decay of Loschmidt echo enhanced by quantum criticality. Phys. Rev. Lett. **96**, 140604 (2006)
34. Cozzini, M., Giorda, P., Zanardi, P.: Quantum phase transitions and quantum fidelity in free fermion graphs. Phys. Rev. B **75**, 014439 (2007)
35. Zanardi, P., Giorda, P., Cozzini, M.: Information-theoretic differential geometry of quantum phase transitions. Phys. Rev. Lett. **99**, 100603 (2007)
36. Zanardi, P., Quan, H.T., Wang, X., Sun, C.P.: Mixed-state fidelity and quantum criticality at finite temperature. Phys. Rev. A. **75**, 032109 (2007)
37. Buonsante, P., Vezzani, A.: Ground-state fidelity and bipartite entanglement in the Bose–Hubbard model. Phys. Rev. Lett. **98**, 110601 (2007)
38. Campos Venuti, L., Zanardi, P.: Quantum critical scaling of the geometric tensors. Phys. Rev. Lett. **99**, 095701 (2007)
39. Gu, S.J., Kwok, H.M., Ning, W.Q., Lin, H.Q.: Fidelity susceptibility, scaling, and universality in quantum critical phenomena. Phys. Rev. B **77**, 245109 (2008)
40. Paunkovic, N., Sacramento, P.D., Nogueira, P., Vieira, V.R., Dugaev, V.K.: Fidelity between partial states as a signature of quantum phase transitions. Phys. Rev. A **77**, 052302 (2008)

41. Siegle, T., Schierle, S., Kraemmer, S., et al.: Photonic molecules with a tunable inter-cavity gap. Light Sci. Appl. **6**, e16224 (2017). https://doi.org/10.1038/lsa.2016.224
42. Yang, J., Xu, R., Pei, J., et al.: Optical tuning of exciton and trion emissions in monolayer phosphorene. Light Sci Appl **4**, e312 (2015). https://doi.org/10.1038/lsa.2015.85
43. Chen, L., Lei, J., Romero, J.: Quantum digital spiral imaging. Light Sci Appl **3**, e153 (2014). https://doi.org/10.1038/lsa.2014.34
44. Rams, M.M., Zwolak, M., Damski, B.: A quantum phase transition in a quantum external field: Superposing two magnetic phases. Sci. Rep. **2**, 655 (2012)
45. Rams, M.M., Damski, B.: Quantum fidelity in the thermodynamic limit. Phys. Rev. Lett. **106**, 055701 (2011)
46. Albuquerque, A.F., Alet, F., Sire, C., Capponi, S.: Quantum critical scaling of fidelity susceptibility. Phys. Rev. B. **81**, 064418 (2010)
47. Pal, A., Huse, D.A.: Many-body localization phase transition. Phys. Rev. B **82**, 174411 (2010)
48. Deutsch, M.: Quantum statistical mechanics in a closed system. Phys. Rev. A **43**, 2046 (1991)
49. Tasaki, H.: From quantum dynamics to the canonical distribution: General picture and a rigorous example. Phys, Rev. Lett. **80**, 1373 (1998)
50. Hu, T., Xue, K., Li, X., Zhang, Y., Ren, H.: Fidelity of the diagonal ensemble signals the many-body localizationtransition. Phys. Rev. E. **94**, 052119 (2016)