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Long period grating in a liquid crystal waveguide

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ABSTRACT

This paper introduces, for the first time to the best of our knowledge we theoretically report the long period grating (LPG) in the liquid crystal (LC) layer of planar optical waveguide. LPG has the ability to couple the light between core and cladding modes of waveguide. LPG can be used for the development of band rejection filter, gain flattening in erbium-doped fiber amplifiers (EDFA) and for sensing applications. We have shown the realization of proposed LPG in temperature sensing. First, we theoretically analyzed the propagation of light through a 6-layer planar optical waveguide by analyzing its modal field profiles and effective index profile of fundamental mode TE_0 and two higher order modes TE_1 and TE_2. Then we investigated in details the phase-matching curves of the long period grating, which supervise the association between the grating period and the resonance wavelength. Using phase matching curve to choose grating period, we have studied the transmission spectrum of the PLG which display evident elimination of bands at individual wavelengths, noted as the resonance wavelengths. High sensitivity of LC to the environmental temperature makes it a good choice for temperature sensing. The performance of LPG has been studied for the temperature ranging from 306 K to 329 K. The novelty of this structure is the realization of LPG in the LC, which is a key component of display devices. Hence, our structure can provide freedom to use this in variety of pragmatic applications.

1. Introduction

Long period fiber gratings (LPFGs) have been extensively explored for their applications as gain flatteners of EDFA [1–4], wavelength filters [5–10], broadband add/drop multiplexers [11], dispersion controllers [12,13] and various kind of optical sensors [14–20]. LPFG is having the ability to couple light between the core and the cladding modes of an optical fiber and it results into a dip at individual wavelength (resonance wavelength) in the transmission spectrum of the fiber. LPF can be created by a regular modulation in core refractive index along the fiber length with a few hundred microns of period. Commonly the modulation has order of 10^{-4} and LPG length of 1–3 cm. Since LPG has larger grating period and consequently easy to fabricate. Point-by-point and amplitude masking are the promising technique to fabricate LPG. The geometry and material compulsion of the fibers enforce substantial limitations on their practical purposes. To overcome these compulsions, the introduction of LPGs into the waveguide has been studied [21,22], due to the flexibility in choosing waveguide parameters and possibility to fabricate LPWGs in different shapes and sizes with different materials. This study exposed the significance of cladding layer for the coupling of core mode with the cladding mode and revealed the effects of cladding refractive index and thickness on transmission spectrum of LPWG. So contrary to fibre, waveguide has

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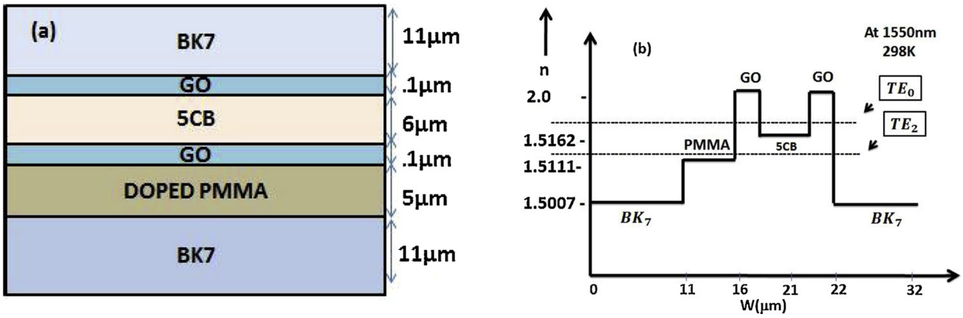

Fig. 1. (a). Layer structure of the proposed waveguide (b) Refractive index profile of proposed structure.

more flexibility to choose the geometry and materials, hence has a lot of applications in integrated optics. The performance of polymer waveguides as a widely tunable LPWG filters have been established [23–26], these filters beat the recorded tunable LPFG filters [27,28]. Since then lots of remarkable applications of LPWGs have been reported. Particularly, tunable wavelength filters subjected to the LPWGs are appropriate for functional wavelength division multiplexing systems exploiting optical planar integrated circuits. There are materials such as silicon, $LiNbo_3$, polymer and glass useful for the fabrication of the LPG based devices. Exploiting electro-optic properties of $LiNbo_3$, fabrication of LPG on $LiNbo_3$ substrate for the application of mode switch has been reported [23]. Nowadays, LC has become suitable material choice for the waveguide applications. Various properties such as large electro-optic effect, high transmission at visible as well as IR wavelength, high birefringence and low voltage requirement make LC suitable material for integrated optical devices. The refractive index of LC is found to be strongly dependent on temperature [29]. LPG is sensitive to different physical parameters and an important one is temperature, as discussed by V. Bhatia et al. in 1996 [30]. Our structure provides the temperature sensing as an addition feature of LPG. LPG has been realized in the LC layer, which is made of 5CB-4-Cyano-4'-pentylbiphen. To get the desired output and following coupled mode theory, we have coupled the core mode (fundamental mode TE_0) with the cladding mode (second higher order mode, TE_2). The shift in the resonance wavelength with the variation of temperature (306 K–329 K) has been studied for different grating periods. At the end, we have calculated the sensitivity of the LPG with temperature.

2. Proposed structure and analysis

We propose a planar optical waveguide structure geometry, consists of a thick glass substrate (BK_7), LC (5CB), polymethyl methacrylate (PMMA) doped with 20% concentration of quinoid molecules and two identical layers of graphene oxide (GO), which is shown in Fig. 1(a) and the corresponding refractive index profile (RI) is shown in Fig. 1 (b). Here, the substrate layer is 11 μm thick having refractive index n_d and liquid crystal layer has the refractive index n_{lc} with thickness of 6 μm [31]. Doped PMMA has the refractive index n_p with thickness of 5 μm [32]. Both GO layers (0.2 mg/ml) are identical with the refractive index n_g and thickness of 100 nm and has a good transmission characteristic in the near infrared range. Hence, we are using GO as the transparent conducting films (TCFs) [33,34]. Moreover, due to its excellent electrical conductivity, optical transparency and mechanical properties make GO a good choice to replace the existing and more expensive indium tin oxide (ITO) as TCFs. LC layer is sandwiched between two layers of GO, where $n_g > n_{lc} > n_p, n_{ex}$. We now hypothesize that primarily, the waveguide sustains only the fundamental (TE_0 and TM_0) mode with $n_{lc} < N_0 < n_g$, where N_0 stands for the mode index, and LPG is ingrained in the liquid crystal layer. Although at 1550 nm waveguide supports many modes depending on the width of LC layer. We can see the fundamental (TE_0) and one higher order mode (TE_2) at the 1550 nm these two modes are necessary for coupling, Fig. 1(b).

The LPG is responsible for light coupling directly from the fundamental mode to the cladding (TE_m *and* TM_m) modes whose mode indices N_m (m = 1, 2, 3…) are smaller than n_{lc}, *i. e.* $.n_p < N_m < n_{lc}$. In order to allow the LPG to function, it is imperative to create a whole set of discrete cladding modes by introducing a new cladding layer. In our calculations, we have followed the coupled-mode theory, which stands for the representation of the total field in the waveguide expressed as a superposition of the fields of guided and the cladding modes. We are considering the TE modes only, expressing the total field Ψ as

$$\psi = \frac{1}{2}([A(z)E_0(x)e^{i(\omega t-\beta_0 z)}] + [B(z)E_m(x)e^{i(\omega t-\beta_m z)}]) + cc \quad (1)$$

Where, $E_0(x)$ and $E_m(x)$ (real functions) stand for power normalized fields of the guided and cladding mode respectively, obtained by solving the eigenvalue equations of the six-layer waveguide. With A(z) and B(z) being the corresponding z-dependent amplitude coefficients, and β_0 and β_m are the respective propagation constants at optical frequency. It is found that the total field Ψ satisfies the following scalar wave equation:

$$\frac{\partial^2\psi}{dx^2} + \frac{\partial^2\psi}{dz^2} + k_0^2\,[n^2(x) + \Delta n^2(x, z)]\psi = 0 \quad (2)$$

Where, $\Delta n^2(x, z) = \Delta n_0^2\, sin\left(\frac{2\pi}{\wedge}\right)z$ stands for sinusoidal index perturbation in the z-direction and $\triangle n^2{}_0$ is representing the amplitude of perturbation.

Here $k_0 = \frac{2\pi}{\lambda}$, stands for the free space wavenumber with λ is representing the free space wavelength. On substituting Eq. (1) into Eq. (2) and using the slowly varying envelop approximation. We eventually obtain the two coupled mode equations [35].

$$\frac{dA}{dz} = kBe^{(i\Gamma z)} \quad (3)$$

$$\frac{dB}{dz} = \frac{dB}{dz} - kAe^{(-i\Gamma z)} \quad (4)$$

Where, $\Gamma = \beta_0 - \beta_m - \frac{2}{\Lambda}$stands for the phase mismatch; $k = (k_0 \frac{\triangle n_0^2}{8c\mu_0})\eta$ represents the coupling coefficient and c denotes the speed of light in vacuum and μ_0 is the permeability and $\eta = \int_0^{dc} E_m E_0 dx$ is the overlap integral measuring the spatial overlap between the guided and cladding mode fields in the guiding film region. Eqs. (3) and (4) can be solved analytically and the variation of power in the guided mode with the propagation distance is represented by

$$\frac{P(z)}{P(0)} = \left(1 - \frac{k^2}{\gamma^2} sin^2(\gamma z)\right) = A(z)^2 \quad (5)$$

Where, $\gamma = k^2 - \Gamma^2$

By using Eq. (5) study of variation of the transmitted power with wavelength and grating parameters for a given waveguide can be obtained. In general, the maximum amount of light coupling takes place at wavelengths corresponding to $\Gamma = 0$, called resonance wavelengths λ_0 [36].

$$\lambda_0 = (N_0 - N_m)\bigwedge \quad (6)$$

Where, $N_0 = \frac{\beta_0}{k_0}$ and $N_m = \frac{\beta_m}{k_0}$ $(m = 0, 1, 3\ldots)$ are calculated at λ_0. Eq. (6) represents the phase matching condition of the grating.

3. Simulation results and discussion

3.1. Modal field profile of waveguide and effective index

The modal field profiles of the proposed six-layer waveguide have been shown in Fig. 2 at the wavelength of 1550 nm. The waveguide supports up to 6 modes. Out of these modes we are considering the coupling between the lowest order mode (guided mode, confined to GO layer) and the second higher order mode TE_2 (confined in the LC layer at low wavelengths and in the doped PMMA layer at high wavelengths).

Fig. 2(a) and (b) show the fundamental guided mode TE_0 and the first order mode TE_1 with both the modes having effective index values in the GO layers. Due to the presence of two GO layers in the waveguide structure, two peaks can be observed clearly, since the refractive index of GO is higher than the rest of the layers of waveguide. The dominating field of the TE_0 mode exists within the GO layer because effective index value confinement is in graphene oxide layer. For TE_2 mode, the effective index is confined in the LC layer, hence, maximum field distribution is in the liquid LC layer itself for $TE_{2,}$ which is shown in Fig. 2(c).

Fig. 3(a) and (b) showing the effective index variation with the wavelength for the guided mode TE_0 and one higher order mode TE_1 respectively. The effective index value for TE_0 is between 1.566 and 1.576 while for TE_1 it is in between 1.532 and 1.554. Effective index value for TE_0 and TE_1 within the range of core refractive index value. So, we choose next higher TE_2 for the coupling as it is necessary condition for couple mode theory and long period grating to have one core mode with one cladding mode

3.2. Phase matching curve and transmission spectrum of waveguide

We start our observation with existing relationship between the resonance wavelength and the grating period for the proposed waveguide structure. Refractive indexes of 5CB and GO are taken from [31,34]. Usually refractive index of PMMA at 1550 nm is 1.478 but in our case, we increase refractive index of PMMA up to 1.511 by doping of 20% solution of quinoid molecules [32]. Thickness of various layers are set to be as: 5CB (6 µm), GO layers of 100 nm [34], doped PMMA (5 µm), BK_7 substrate (11 µm).

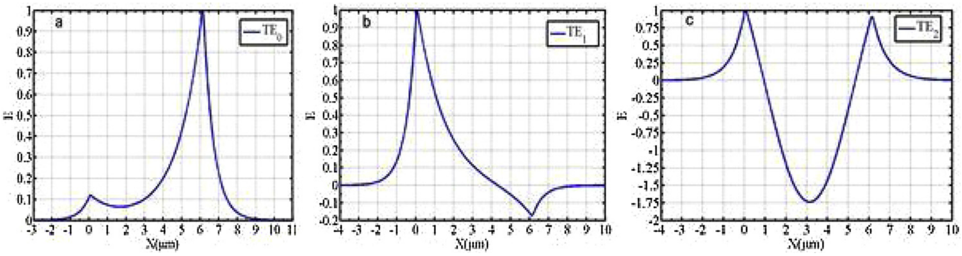

Fig. 2. Modal profile of waveguide for the first three fundamental modes at 1550 nm (a) TE_0 (b) TE_1 (c) TE.

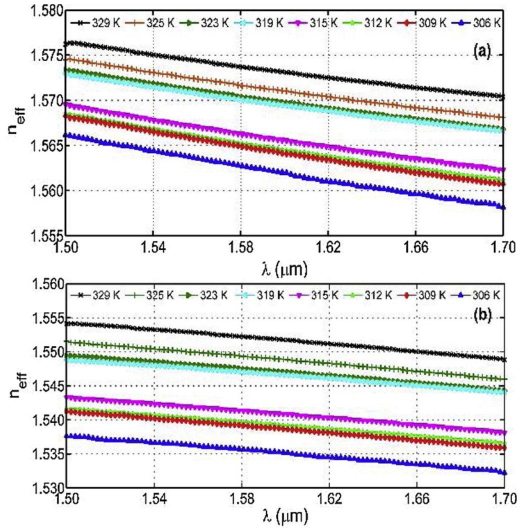

Fig. 3. Effective indexes as a function of wavelength for proposed waveguide for fundamental (a) TE_0 and second order (b) TE1 modes at different temperatures.

Grating length $L = 3.469$ mm and index modulation $(\frac{\triangle n_0^2}{2n_f})$ of 2×10^{-4}.

In our calculations, the refractive index perturbation is considered to restrict within the LC layer (5CB) only. The qualitative nature of the results remains unchanged with any alteration in additional index modulation in other regions which though affects the coupling efficiency. The curves in Fig. 4 are referred to as the phase-matching curves and they are obtained from the phase-matching condition represented in Eq. (6). The phase-matching curve enables us to make a choice of grating period in filtering out a certain wavelength from the transmission spectrum. On increasing or decreasing the cladding thickness, the corresponding modes aided by waveguide increase or decrease.

A distinct feature of the LPWG is that it can couple light to substrate modes. Since the substrate is thick, the substrate modes encompass practically a continuum of effective mode indices. As a result, the coupling efficiency is low and no clear and distinct rejection band can be observed and produced. For the generation of distinct rejection bands, it is important to create discrete cladding modes by introducing a low-index cladding, attached with the guiding core. Obviously, the thickness and the refractive index of the cladding tend to have significant effects on the mode indices and consequently on the resonance wavelength [21]. The transmission spectrum of the grating period of waveguide strongly depends on the criteria in which the phase mismatch between the interacting modes alters with the wavelength. Phase mismatch between the TE_0 guided mode and TE_2 cladding mode, $\Delta\beta = \beta_0 - \beta_m$ as a function of wavelength shown in Fig. 5.

The transmission spectrum of waveguide with $\wedge = 62.32\,\mu m$ at 306 K is shown in the Fig. 6. It can be observed in transmission spectrum that the resonance wavelength is 1.644 μm.

Now we have observed the transmission spectrum of waveguide by changing temperature, varying from 306 K to 329 K. It is observed that on increasing the temperature, the resonance wavelength is shifted towards the lower wavelength side. The transmission spectra of the waveguide with the grating period of $\wedge = 65\,\mu m$ and $\wedge = 68.50\,\mu m$ for TE_2 are presented in Figs. 7 and 8 respectively. The transmission spectrum of the grating relies potentially on the nature in which the phase mismatch between the interacting modes changes with the wavelength.

The main reason of shifting of resonance wavelength with increase in temperature is due to the significant change in refractive index of LC. Basically, LC is an intermediate state between the solid and liquid. The substances which have arrangement like solids but freedom of motion like liquid are called liquid crystals. The Nematic liquid crystal has positional orientation aligned in a certain direction, interrupted only by external factors like temperature. Similar to solids, LC also shows diffraction. When light passes through a LC, it appears colored due to their property of diffraction. Now, if the temperature changes, the LC molecules will show

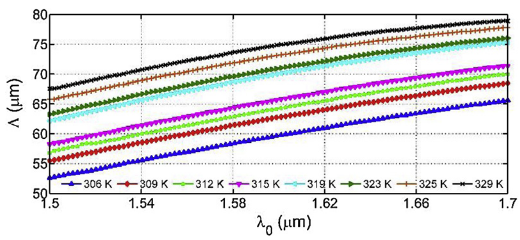

Fig. 4. Phase matching curves for LPWG for TE_0 - TE_2 modes as a function of wavelength for different temperatures. d_{lc}= 6 μm, $d_g = 100\,nm$.

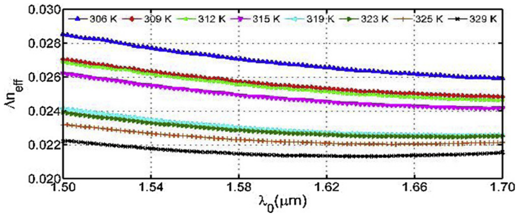

Fig. 5. Change in effective refractive index of TE_0 - TE_2 modes as a function of resonance wavelength at different temperatures. d_{lc}= 6 μm.$d_g = 100\,nm$.

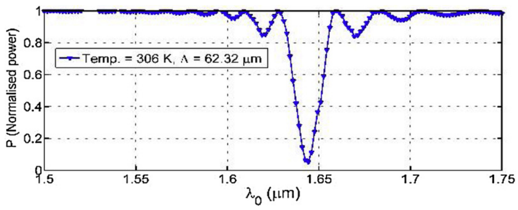

Fig. 6. Transmission spectrum of waveguide at 306 K with period of 62.32 μm for TE_2 - TE_0 modes.

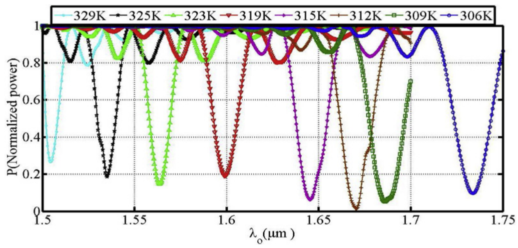

Fig. 7. Transmission spectrum of a 3.469 mm long waveguide having grating period of $\wedge$ = 65 μm for TE_2 - TE_0 as a function of wavelength for the temperature range of.306 K–329 K.

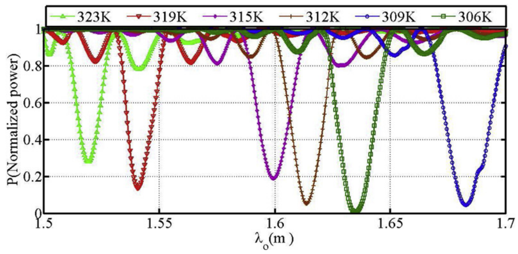

Fig. 8. Transmission spectrum of a 3.469 mm long waveguide having grating period of $\wedge$ = 68.50 μm for TE_2 - TE_0 as a function of wavelength for the temperature range of.306 K–329 K.

different positional orientations with different refractive index values. Due to this change in refractive index value of LC the diffraction condition will get changed. Therefore the color of the deflected light will change accordingly. With this observation, it can be inferred that one can utilize the property of LC of changing refractive index with temperature.

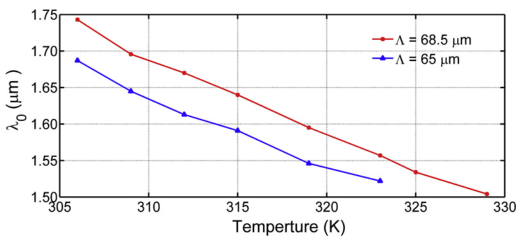

Fig. 9. Change in position of resonant peak wavelength in the LPG transmission spectra as a function of temperature.

Fig. 9 show the wavelength shift as a function of temperature and corresponds to two different grating periods. We conclude that if we choose higher grating period, we will have higher shift in resonance wavelength. So, by increasing grating period we can increase the sensitivity of the device. We finally calculate the sensitivity of device 10.43 nm/K for the grating period of $\wedge = 68.50\,\mu m$. For the grating period of $\wedge = 65\,\mu m$ we get sensitivity 9.70 nm/K which is a little lower than the period of $\wedge = 68.50\,\mu m$. So, we can draw a conclusion from here that sensitivity can be increase by increasing grating period.

4. Conclusion

In this paper, we have theoretically introduced LPG in LC layer of planar optical waveguide and analyzed the response of LPWG by observing its transmission spectrum and showing LPG application in temperature sensing. Contrary to fiber where choice of materials and its parameters are fixed, fabrication of the waveguides can be done in the various forms with various materials. Additionally, we can imagine a lot of new applications of waveguide-based LPG by utilizing several material systems accessible for production of waveguide. The resonance wavelength shift in the transmission spectrum of LPG for ambient temperature ranging from 306 K to 329 K has been observed. Phase matching curve is useful to choose grating period. Using phase matching curve, we chose two different gating periods of 65 μm and 68.50 μm and analyze the transmission curve and corresponding sensitivity for the temperature ranging from 306 K to 329 K in both the cases. We also inferred that the sensitivity of LPG can be increased significantly by increasing grating period. All the simulation work has been carried out using MATLAB.

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