Received: 5 September 2019 | Revised: 27 February 2020 | Accepted: 15 April 2020

DOI: 10.1002/asjc.2370

REGULAR PAPER

WILEY

Frequency domain subspace identification of fractional order systems using time domain data with outliers

Zongyang Li[1] | Yiheng Wei[1] | Jiachang Wang[1] | Yongting Deng[2] | Jianli Wang[2] | Yong Wang[1]

[1]Department of Automation, University of Science and Technology of China, Anhui, Hefei, 230027, China

[2]Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun, 130033, China

Correspondence
Yong Wang, Department of Automation, University of Science and Technology of China, Hefei 230027, Anhui, China.
Email: yongwang@ustc.edu.cn

Funding information
National Natural Science Foundation of China, Grant/Award Number: 61573332 and 61601431; Natural Science Foundation of Anhui Province, Grant/Award Number: 1708085QF141

Abstract

This paper focuses on the identification of the multiple-input multiple-output commensurate fractional order systems. Different from the assumption of known frequency domain data and prior information about noise in the general frequency domain identification algorithm, the reliable frequency domain data is measured in this paper by designing special excitation signals. Then, in order to suppress the impact of noise, the data used in algorithm are truncated from the low frequency band. In addition, this paper considers the case where the sampling data are disturbed by outliers, and a matrix decomposition method with threshold value is developed to eliminate the influence of outliers. After that, the parameters of the system to be identified are estimated by the frequency domain subspace identification method. The validity of the proposed method is demonstrated by an illustrative numerical example.

KEYWORDS

fractional order systems, frequency response function, matrix decomposition, subspace identification method, system identification

1 | INTRODUCTION

Fractional calculus is an extension of the classical calculus to arbitrary order. Due to the complexity of calculation, it had always been a pure mathematical concept for a long time. With the persistent research and the development of computer technology, fractional calculus has gradually infiltrated into practical applications in recent years. Numerous research has shown that many physical phenomena can be described more concisely and precisely by fractional order models, such as electrochemical processes [1], heat conduction [2,3], viscoelastic systems [4] and velocity servo system [5]. Therefore, system modeling with the fractional order model has attracted great attention.

In general, it is difficult to build a fractional order model with mechanism analysis. The reason is that most of the existing physical laws are based on classical calculus, and it is hard to find a widely accepted physical interpretation for fractional calculus [6]. Fortunately, using the input and output data records, system identification does well in estimating the model of a dynamical system. Thus, the system identification technique for a fractional order system (FOS) has attracted the attention of many researchers and engineers. For example, system identification [7], system modeling [8] and parameters identification [9,10]. As a non-iterative scheme working best for the identification of multiple-input multiple-output (MIMO) systems, the subspace identification method has received much attention from the control community, and it has been developed both in the time domain and in the frequency domain. Many related works can be found in [11,12] and the references therein.

The subspace identification of FOSs started in the last decade. It is a salient issue to seek the fractional order derivative of signals for time domain algorithms. To solve this problem, some methods, such as state variable filter [13], Poisson moment function [14], modulation function [15], and block pulse functions [16] are adopted in the time domain identification method. Nevertheless, there is still a large computational complexity when calculating the fractional order derivative of time domain signals. However, if the calculation is converted into the frequency domain, the differential in the time domain will be transformed into a product in the frequency domain. Besides, the data needed in frequency domain identification algorithms is less, as long as it covers the bandwidth of systems. These merits make the frequency domain subspace identification method (FDSIM) more advantageous in identification of FOSs. However, all the existing studies based on FDSIM considered the case that the data in frequency domain and even the prior information of the noise correlation are given, which is a severe restriction on the application of this algorithm, because it is the time domain signals that are measured, and the information about noise correlation is always hard to obtain in most practical cases.

For the reasons discussed above, it is still necessary to consider how to get the data in frequency domain and how to handle the signals contaminated by noise or other unexpected information. In the frequency domain identification of the integer order system, some research [17, 18] about how to obtain the frequency response function (FRF) from time domain sampling signals have received satisfactory results, which provides a reference for the measurement of the FOS's FRF. As for the processing of noise, many optimization methods based the nuclear norm or rank minimization [19,20] are prevalent to deal with this issue. Hu approximately estimated the noise by taking the noise model into account [21], and this approach achieves good results in the time domain subspace identification of integer order systems. While the estimated noise is not exactly coincident with the real noise, it can not improve the quality of frequency data very well. Besides, to avoid spectrum leakage when acquiring FRF, it is necessary to increase the number of sampling points in time domain. It means that the computational complexity will be intensely increased, and the efficiency will also be greatly reduced when solving optimization problem. What's more, the presence of outliers makes the measured FRF much worse, and the method mentioned above does not work well.

Motivated by the above discussions, the work in this paper considers the identification problem of a MIMO commensurate FOS in the presence of white Gauss noise and outliers. The main contributions of this paper can be summarized as follows:

- A special excitation signal is designed, which is helpful for getting the system's frequency domain response data;
- The measurement noise without prior information is considered, whose effect is effectively eliminated by intercepting the data in low frequency band;
- The influence of the outliers in measured signals is considered, and an innovation matrix decomposition method with threshold value is adopted to overcome this affect.

It can be seen from the above that the entire frequency domain identification procedures in this work are based on the time domain sampling data, which consummate the frequency domain identification framework. In addition, it should be pointed out that the algorithm in this paper is also applicable to the situation where the commensurate order is unknown.

The remainder of this paper is organized as follows. Section 2 gives fundamental knowledge about fractional order derivative and a brief introduction about FOS, and the issue studied in this paper is described in the end of this section. In Section 3, an excitation signal is designed to acquired the FRF. Apart from this, the method to eliminate outliers and the entire procedures of subspace identification in frequency domain are demonstrated in this part. To illustrate the effectiveness of proposed approach, the numerical simulation is contained in Section 4. Some conclusions are drawn in Section 5.

2 | PRELIMINARIES

In this section, some fundamental knowledge about fractional calculus and a description of FOS will be provided. The issues studied in this paper will also be introduced at the end.

2.1 | Fractional order derivative and FOSs

As a universalization of integer order derivative, there are three widely used definitions of fractional order derivative: Riemann-Liouville definition, Caputo definition and Grünwald-Letnikov definition. This paper adopts the Caputo definition, and other definitions can be found in [1,22].

The Caputo derivative can be expressed as

$$ {}_{t_0}\mathscr{D}_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} \mathrm{d}\tau, \tag{1} $$

where $m-1<\alpha<m\,(m\in\mathbb{N}_+)$, $\Gamma(\cdot)$ is the Euler's Gamma function, and $f(t)$ is a smooth function. For simplicity, this paper substitutes the notation $\mathscr{D}^\alpha$ for ${}_{t_0}\mathscr{D}_t^\alpha$, when $t_0=0$. In this case, the Laplace transform of (1) can be written as

$$\mathscr{L}\{\mathscr{D}^\alpha f(t)\}=s^\alpha F(s)-\sum_{k=0}^{l-1}s^{\alpha-l-1}f^{(k)}(0), \tag{2}$$

where $l=\lceil\alpha\rceil$, and $\lceil\cdot\rceil$ is the ceiling function (see [23] for more property on Laplace transform of Caputo derivative).

The integer order system is generally described by the state space equation and the transfer function, which is still valid in FOSs. By means of the fractional derivative defined above, the state space equation of a FOS can be expressed as

$$\begin{cases}\mathscr{D}^\alpha x(t) &= Ax(t)+Bu(t),\\ y(t) &= Cx(t)+Du(t),\end{cases} \tag{3}$$

where the matrices A,B,C,D have appropriate dimensions. Under the zeros initial conditions, the relationship between the system matrices and the transfer function can be obtained by using (2), which is shown as below:

$$\begin{aligned}G(s)&=C(s^\alpha I-A)^{-1}B+D\\ &=\frac{\sum_{j=0}^{n}b_j s^{j\alpha}}{s^{n\alpha}+\sum_{i=0}^{n-1}a_i s^{i\alpha}},\end{aligned} \tag{4}$$

where n is the dimension of system matrix A. When the system's input has τ seconds delay, its transfer function can be obtained though multiplying 4) by $e^{-\tau s}$.

2.2 | Problem description

Consider the following MIMO commensurate fractional order liner time-invariant system:

$$\begin{cases}\mathscr{D}^\alpha x(t)=Ax(t)+Bu(t),\\ z(t)=Cx(t)+Du(t),\\ y(t)=z(t)+\varepsilon(t)+v(t),\end{cases} \tag{5}$$

where $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times r}$, $C\in\mathbb{R}^{r\times n}$, $D\in\mathbb{R}^{r\times m}$ are the constant but unknown matrices needing to be determined, $u(t)\in\mathbb{R}^m$, $y(t)\in\mathbb{R}^r$ are the input and output vectors, $\varepsilon(t)$ is the additive white Gauss noise and $v(t)$ signifies zero or outliers. Some reasonable assumptions are given as follows:

- The system (5) is stable;
- The system (5) is observable;
- The dimension of system matrix A is known.

This paper aims to use the time domain sampling data to identify the unknown parameters of the system with the help of FDSIM, and simultaneously, to overcome the effects of Gauss noise and outliers.

3 | MAIN RESULTS

The main results will be given in this section. Firstly, the excitation signal is designed for frequency response function measurement. Then, the method of outlier detection is introduced, and the subspace identification algorithm is provided at the end of this section.

3.1 | Excitation signals and FRF measurement

Generally, the excitation signal used for system identification needs to meet sufficient excitation conditions. In other words, the signal used for identification should have sufficient bandwidth relative to the system to be identified. There may be many input signals available [24], such as white noise, pseudo random binary sequence, multisine, etc. As for the FRF measurement, it is more important to choose an appropriate excitation signal. A suitable signal should have a minor crest factor/time factor [18] except for the sufficient spectral width. An excitation signals named multisine is widely used in the FRF measurements.

The multisine is a sum of harmonically related sine waves as follows:

$$u(t)=\sum_{k=1}^{L}A_k\cos(2\pi kT_sf_0t+\phi_k), \tag{6}$$

where T_s is the sample period, and f_0 is the fundamental frequency. As for ϕ_k , there are three different multisine forms corresponding to different phase selections:

- Schroeder multisine: $\phi_k=\frac{-k(k-1)\pi}{L}$, where ϕ_k is called Schroeder phase;
- random multisine: $\phi_k\sim U(0,2\pi)$, and $U(0,2\pi)$ means uniform distribution in internal $(0,2\pi)$;
- impulse multisine: $\phi_k=-\sigma\omega_k$, where $\sigma=0.3$, $\omega_k=\frac{2\pi k}{NT_s}$ and N is the number of sampling points.

In addition, the input $u(t)$ in [8] can also be obtained from the signal's spectrum and more information about this excitation can be found in [18].

When the input and output data are obtained, it seems that the FRF could be acquired directly from these time domain signals using the following formula:

$$G(j\omega_k)=\frac{Y(j\omega_k)}{U(j\omega_k)}, \tag{7}$$

where $Y(j\omega_k)$ can be obtained by discrete Fourier transform (DFT) as follows:

$$Y(j\omega_k)=\frac{1}{\sqrt{N}}\sum_{i=0}^{N-1}y(i)e^{-j2\pi ki/N}, \tag{8}$$

N indicates the number of sampling points, and $U(j\omega_k)$ is defined in the same way. However, it turns out that this direct approach does not yield reliable frequency domain

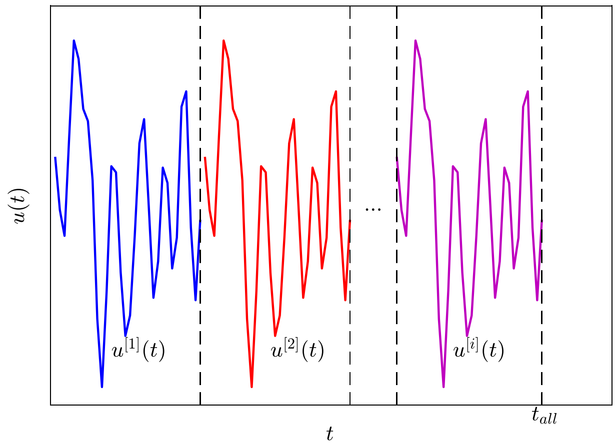


FIGURE 1 A sketch of the periodic excitation signal [Color figure can be viewed at wileyonlinelibrary.com]

data. The reason is that these time domain signals used directly are not the steady-state response.

In order to obtain accurate FRF without spectrum leakage errors, it is necessary to excite the system with a special processing excitation signal. A practical and easy approach is to utilize an integer number of periods $u(t)$, which means the entire excitation signal $u(t_{all})$ contains several complete signal $u(t)$. It can be interpreted more intuitively from Figure 1. It is clear that each period of input will correspond to a period of output, and the last period of output $y^{[i]}(t)$ corresponding $u^{[i]}(t)$ can be treated as the relatively accurate steady-state response. Finally, these part of signal in time domain is used to obtain the FRF by (7) and (8). Also, it is helpful to take the average of DFT of the signals in the last few periods, which is always more effective in the presence of noise. It is noted that periodic excitation signals are necessary to obtain response data in the frequency domain, otherwise, accurate frequency domain data cannot be obtained using a single excitation signal, and furthermore, these data cannot be used for parameter estimation.

3.2 | Outliers detection

When the time domain sampling data have outliers, the measured FRF is often not precise, especially when the magnitude and the number of outliers are both larger. The next discussion is intended to address this issue.

It is well known that the Hankel matrix formed by the output without the useless information is a low rank one, therefore, the presence of outliers will increase the rank of the output martrix. This low rank property can be used to detect the outliers. In this section, the problem of outliers detection is summed up as a matrix decomposition problem, and a method will be introduced to eliminate the impact of outliers on measured FRF with the help of matrix norm.

Firstly, it is essential to establish a Hankel matrix using the sampling data in time domain

$$\tilde{Y} = \begin{bmatrix} y(1) & y(2) & \dots & y(m_1) \\ y(2) & y(3) & \dots & y(m_1+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(m_2) & y(m_2+1) & \dots & y(N_{\tilde{Y}}) \end{bmatrix}, \tag{9}$$

where $m_1 \leq m_2 < N_{\tilde{Y}}$. Notice that the order of the system should be much smaller than the dimension of $\tilde{Y}$, which is a necessary but easy to satisfy assumption.

Next, what we need to do is to extract the matrix containing outliers from $\tilde{Y}$. This step is easy to achieve by solving the rank minimization problem, which can be described as follows:

$$\begin{aligned} &\min \|Y\|_* + \lambda \|E\|_1, \\ &\text{s.t.} Y + E = \tilde{Y}, \end{aligned} \tag{10}$$

where $\|\cdot\|_*$ represents the nuclear norm of the matrix, λ is a regulatory factor, and it can be chosen based on [25]. By solving the convex optimization problem shown in [29], the following matrix containing the outliers information can be obtained

$$E = \begin{bmatrix} e(1) & e(2) & \dots & e(m_1) \\ e(2) & e(3) & \dots & e(m_1+1) \\ \vdots & \vdots & \ddots & \vdots \\ e(m_2) & e(m_2+1) & \dots & e(N_{\tilde{Y}}) \end{bmatrix}. \tag{11}$$

The first row of matrix E can be treated as the outliers. However, due to the presence of noise and the relatively intense changes in output data, some extra outliers (can be called ripples) may be mistakenly introduced into E. It is clear that the presence of these ripples weakens the sparsity of outliers. Taking this into account, this study introduces a threshold to eliminate the effect of ripples, which can be summarized as follows:

$$e(k) = 0, \text{if } |e(k)| < \gamma (k = 1, 2, \dots, m_1), \tag{12}$$

where γ is a tuning threshold and usually a smaller value. Considering that the outliers are much larger than the real output, and the ripples are often small. Therefore, the choice of a appropriate γ also depends on the magnitude of the outliers. Besides, it should be pointed out that the choice of threshold cannot induce the subjectivity to the problem. Because some of the large noise or intense changes in output data, as mentioned earlier, will produce some smaller ripple, the threshold is introduced to eliminate the effects of these ripples.

It can be seen from the above discussion that the rank minimization problem is transformed into a nuclear norm minimization problem. This problem is very easy to solve with the help of convex optimization tools. In addition, unlike the method in [25], this work introduces a threshold, and it makes some misjudged outliers set to 0, which ensures the sparseness of outliers.

3.3 | FDSIM for FOSs

In this section, how a state-space realization can be estimated using frequency domain data will be discussed. Firstly, the FDSIM with known commensurate order will be given.

With the help of (2), a FOS is easily transformed to s-domain, and let $s = \mathrm{j}\omega$, one can get

$$\begin{bmatrix} G(\mathrm{j}\omega) \\ (\mathrm{j}\omega)^{\alpha}G(\mathrm{j}\omega) \\ \vdots \\ (\mathrm{j}\omega)^{(q-1)\alpha}G(\mathrm{j}\omega) \end{bmatrix} = \Gamma_q X(\mathrm{j}\omega) + \mathrm{O}_q \begin{bmatrix} I_m \\ (\mathrm{j}\omega)^{\alpha} I_m \\ \vdots \\ (\mathrm{j}\omega)^{(q-1)\alpha} I_m \end{bmatrix}, \quad (13)$$

where q is an auxiliary order controlling the number of block-row in Γ_q that is the extend observability matrix with $q \geq n$ block rows

$$\Gamma_q = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{q-1} \end{bmatrix}, \quad (14)$$

and O_q is a lower block-triangular Toeplitz matrix with the structure

$$\mathrm{O}_q = \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{q-2}B & CA^{q-3}B & \dots & D \end{bmatrix}. \quad (15)$$

The Equation (13) holds for all ω, and using the data sampled at different frequencies ω_k, $k = 1, 2, \dots, M$, the M vector relations can be coalesced into

$$\boldsymbol{G} = \Gamma_q \boldsymbol{X} + \mathrm{O}_q \boldsymbol{I_m}, \quad (16)$$

where

$$\boldsymbol{G} = \begin{bmatrix} G_1 & G_2 & \dots & G_M \end{bmatrix}, \quad (17)$$

and

$$G_k = \begin{bmatrix} G(\mathrm{j}\omega_k) \\ (\mathrm{j}\omega_k)^{\alpha}G(\mathrm{j}\omega_k) \\ \vdots \\ (\mathrm{j}\omega_k)^{(q-1)\alpha}G(\mathrm{j}\omega_k) \end{bmatrix}. \quad (18)$$

In order to facilitate calculation, the following notation is introduced

$$\boldsymbol{G}^{\mathrm{re}} = \begin{bmatrix} \mathrm{Re}(\boldsymbol{G}) & \mathrm{Im}(\boldsymbol{G}) \end{bmatrix}. \quad (19)$$

The definitions of $\boldsymbol{X}, \boldsymbol{I_m}, \boldsymbol{X}^{\mathrm{re}}, \boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}}$ are similar to $\boldsymbol{G}$ and $\boldsymbol{G}^{\mathrm{re}}$, respectively. Then the complex matrix (16) is equivalent to the following expression

$$\boldsymbol{G}^{\mathrm{re}} = \Gamma_q \boldsymbol{X}^{\mathrm{re}} + \mathrm{O}_q \boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}}. \quad (20)$$

This formula is widely used in many subspace identification methods. The next problem to be solved is to determine the system matrices A, B, C, D through the known matrix $\boldsymbol{G}^{\mathrm{re}}$ and $\boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}}$.

After getting the information matrices, the FDSIM for FOS can be summarized as the following procedures.

- Compute the orthogonal projection $\boldsymbol{G}^{\mathrm{re}}/(\boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}})^{\perp}$

$$\boldsymbol{G}^{\mathrm{re}}/(\boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}})^{\perp} = \boldsymbol{G}^{\mathrm{re}}\left\{I - (\boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}})^{\mathrm{T}}\left[\boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}}(\boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}})^{\mathrm{T}}\right]\boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}}\right\}^{-1}. \quad (21)$$

A more efficient and stable way to get orthogonal projection may be QR-factorization:

$$\begin{bmatrix} \boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}} \\ \boldsymbol{G}^{\mathrm{re}} \end{bmatrix} = \begin{bmatrix} R_{11}^{\mathrm{T}} & 0 \\ R_{12}^{\mathrm{T}} & R_{22}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} Q_1^{\mathrm{T}} \\ Q_2^{\mathrm{T}} \end{bmatrix}, \quad (22)$$

and

$$\boldsymbol{G}^{\mathrm{re}}/(\boldsymbol{I}^{\mathrm{re}}_{\boldsymbol{m}})^{\perp} = R_{22}^{\mathrm{T}}. \quad (23)$$

- Perform SVD on R_{22}^{T}

$$R_{22}^{\mathrm{T}} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad (24)$$

where the diagonal elements of Σ is the non-zero singular values of R_{22}^{T}, and the selection of Σ depends on the a priori information, dimension of system matrix A.

- Estimate the extend observability matrix $\widehat{\Gamma}_q$

$$\widehat{\Gamma}_q = U_1 \Sigma^{\frac{1}{2}}. \quad (25)$$

- Calculate A and C

$$\begin{aligned} C &= \widehat{\Gamma}_q(1 : r, :), \\ A &= \widehat{\Gamma}_q(1 : \mathrm{end} - r - 1, :)^{\dagger}\widehat{\Gamma}_q(r + 1 : \mathrm{end}, :), \end{aligned} \quad (26)$$

where the notation † indicates the pseudo inverse of matrix.

- Using the least squares method to determine B and D

$$\begin{bmatrix} \mathrm{vec}(L_R) \\ \mathrm{vec}(L_I) \end{bmatrix} = \begin{bmatrix} I_m \otimes M_R \\ I_m \otimes M_I \end{bmatrix} \mathrm{vec}(\begin{bmatrix} B \\ D \end{bmatrix}), \quad (27)$$

where ⊗ is Kronecker product, vec(·) arranges each column of the matrix into a column, and

$$L_R + \mathrm{j}L_I = \begin{bmatrix} G(\mathrm{j}\omega_1) \\ \vdots \\ G(\mathrm{j}\omega_M) \end{bmatrix} \in \mathbb{C}^{rM\times m}, \quad (28)$$

$$M_R + \mathrm{j}M_I = \begin{bmatrix} C\left[(\mathrm{j}\omega_1)^{\alpha}I_n - A\right]^{-1} & I_r \\ \vdots & \vdots \\ C\left[(\mathrm{j}\omega_M)^{\alpha}I_n - A\right]^{-1} & I_r \end{bmatrix} \in \mathbb{C}^{rM\times(n+r)}. \quad (29)$$

From the above discussion, the frequency domain identification algorithm does not involve the differential of the data, which greatly reduces the computational complexity. In addition, when a system has time delay characteristic, one can modify only (17) as follows:

$$\boldsymbol{G} = \begin{bmatrix} G_1 & G_2 & \dots & G_M \end{bmatrix} \begin{bmatrix} \mathrm{e}^{\mathrm{j}\omega_1\tau} & 0 & \dots & 0 \\ 0 & \mathrm{e}^{\mathrm{j}\omega_2\tau} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathrm{e}^{\mathrm{j}\omega_M\tau} \end{bmatrix}. \quad (30)$$

Then FDSIM algorithm will be still applicable to the system with time delay.

The previous discussion is based on the commensurate order α is given. When the commensurate order α is unknown, just define an objective function as follows:

$$J(\alpha) = \sum_{k=1}^{M} \left| \hat{G}(\mathrm{j}\omega_k, \alpha) - G(\mathrm{j}\omega_k) \right|^2. \tag{31}$$

Then the entire identification problem is completely transformed into a nonlinear optimization problem:

$$\min_{\alpha} \sum_{k=1}^{M} \left| \hat{G}(\mathrm{j}\omega_k, \alpha) - G(\mathrm{j}\omega_k) \right|^2, \quad \text{s.t.} 0 < \alpha < 2. \tag{32}$$

To solve the optimization problem [32], many optimization algorithms can be applied after setting a termination condition $J(\alpha) \le \varepsilon$, and in this paper, the *fmincon* function in MATLAB is selected.

In addition, the presence of noise will undoubtedly affect the identification performance. In the above FDSIM, the number of non-zero singular values in Σ will be increased due to the effect of noise. The noise information in the frequency domain data is mainly contained in the high frequency band. Under this case, the low frequency data can be intercepted to eliminate the impact of noise, whose validity will be illuminated more intuitively in numerical simulation. The entire steps for identifying the parameters of FOS can be summarized as following steps and flow chart, as shown in Figure 2.

Step1: Design periodic excitation signal and excite the FOS with it. Collect the time domain signals: input $u(t)$ and output $y(t)$.

Step2: Data preprocessing. Construct the data matrix in accordance with (9) and detect the outliers using (10).

Step3: Obtain the frequency domain data. Using the last few periods signals to calculate FRF, and only retain the data in low frequency band used for the next step.

Step4: Note the number of iterations $k = 0$, guess a initial value of α, and estimate $\hat{A}, \hat{B}, \hat{C}, \hat{D}$ by means of FDSIM.

Step5: Perform the iterative operation to obtain the next estimation using an optimization algorithm.

Step6: Let $k = k + 1$ and return to the previous step until the termination condition is satisfied.

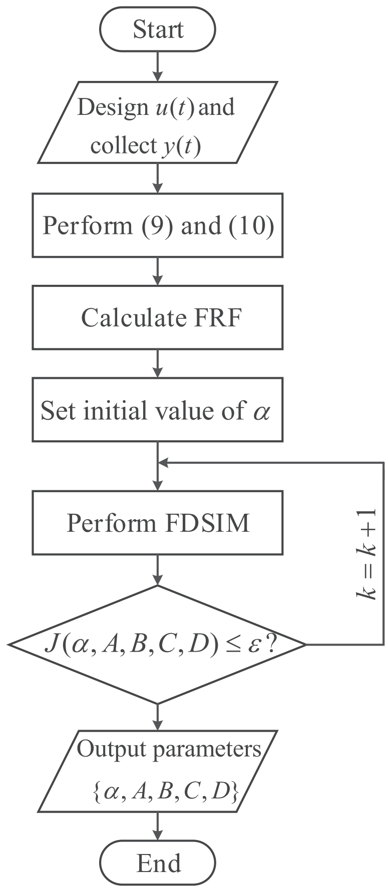


FIGURE 2 The flow chart of identifying the parameters of FOS

4 | NUMERICAL SIMULATION

In this section, the validity of the algorithm is verified by numerical simulation. Consider an FOS in [14] as follows:

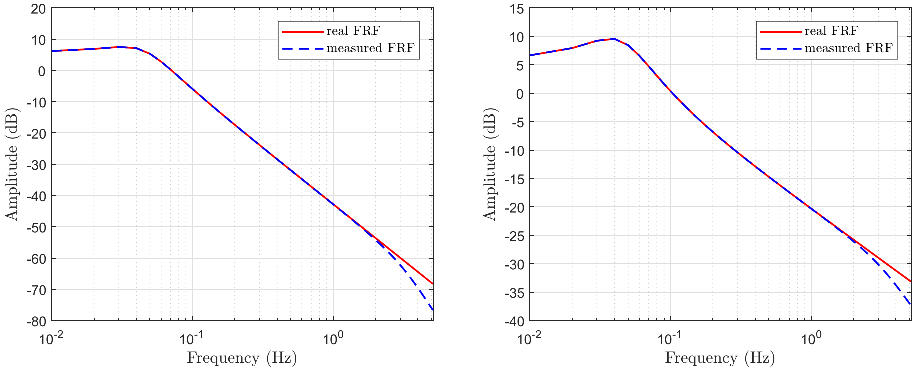
$$\begin{cases} \mathscr{D}^{0.9}x(t) = \begin{bmatrix} 0 & -0.1 \\ 1 & -0.2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t), \\ y(t) = \begin{bmatrix} 0 & 0.2 \\ 0.5 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u(t) + \xi(t), \end{cases} \tag{33}$$

where $\xi(t)$ is the sum of Gauss noise and outliers. The transfer function of system (33) could be obtainedas

$$\begin{cases} G_1 = \frac{b_{12}s^{1.8}+b_{11}s^{0.9}+b_{10}}{s^{1.8}+a_1 s^{0.9}+a_0} = \frac{0.2}{s^{1.8}+0.2s^{0.9}+0.1}, \\ G_2 = \frac{b_{22}s^{1.8}+b_{21}s^{0.9}+b_{20}}{s^{1.8}+a_1 s^{0.9}+a_0} = \frac{0.5s^{0.9}+0.2}{s^{1.8}+0.2s^{0.9}+0.1}. \end{cases} \tag{34}$$

The Caputo definition is chosen to numerically implement this FOS. In this example, the initial conditions of the system are set as $y(t) = y(0) = 0, -\infty \leqslant t < 0$. The input signal is a random phase multisine, and the system is excited by $i = 3$ periods of input signal. Select the sample frequency $f_s = 20.48$Hz, and measure $N = 2048$ points of the input/output signals corresponding to a period of excitation, which mean the spectral resolution $f_0 = 0.01$Hz.

- **FRF measurements.** When the measured points of the last period are considered as the stead-state response. One can get the measurement of FRF, and Figure 3 demonstrates the measured FRF without noise of system (33), which indicates that the measured FRF is almost consistent with the actual values, and confirms that the method of measuring FRF is very effective. It should be noted that a small part of measured FRF



(A) FRF of subsystem G_1. **(B)** FRF of subsystem G_2.

FIGURE 3 Measurement of FRF under periodic excitation [Color figure can be viewed at wileyonlinelibrary.com]

TABLE 1 The quality of estimated outliers

	mean(error)	std(error)	False Rate	Reliablity
outlier1	0.0115	0.1833	0	93.33%
outlier2	0.0268	0.4914	0.01	90.84%

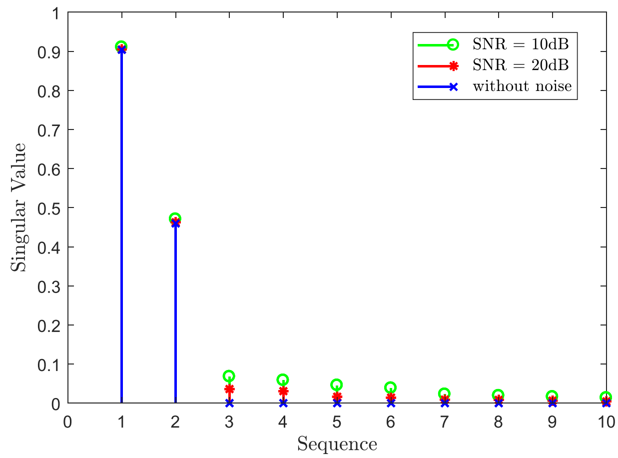


FIGURE 4 Singular in the presence of noise [Color figure can be viewed at wileyonlinelibrary.com]

should be discarded even if there is no noise, and this measurement error is unavoidable.

- **Examining the effects of noise.** As aforementioned, when the system output was corrupted by noise, the measured FRF will not be same as the consequence in Figure 3, and the data in the high frequency band will be diverged from the actual data, which means the influence of noise is mainly concentrated in the high frequency band. These data containing noise are disadvantageous for the acquisition of Σ, because the singular values except for Σ are not zero. However, by intercepting low frequency band data, the impact will be eliminated. To illustrate the effectiveness of intercepting low band data to suppress noise, consider the singular value of Σ in three cases: $10dB$, $20dB$ and without noise; besides, the auxiliary order $q = 5$, and the number of frequency domain sampling points $M = 12$.

  As can be seen from Figure 4, the number of non-zero singular values is two when there is no noise, which corresponds exactly to the order of (33) $n = 2$. With the increase of noise, the singular values from the third one are no longer zero. However, compared to the first two values, this part of singular values is obviously very small. Therefore, the method of truncating low frequency data is available to suppress the influence of noise. Also, it is needed to point out the number of frequency domain data points depends on the bandwidth of the system, which requires a rough estimation about the bandwidth of the system to be identified.
- **Outliers detection.** The interception of data in low and middle frequency bands contributes to restrain the effect of noise. However, when the output signals are simultaneously contaminated by the outliers and noise, this method may no longer be effective. Thus, the signal must be preprocessed to eliminate the effect of outliers before the FRF acquired. Assume that the data are corrupted by 20 dB noise and outliers accounting for 5% of total number of sampling points.

  Due to the high data volume in the matrix, the operation speed will be slow. A natural practice is to divide them into groups and then perform the optimization algorithm separately. Let $m_1 = 10$, $m_2 = 400$, $\lambda = 0.6$, $\gamma = 1.1$, and then the optimization problem (10) is performed by CVX-Toolbox of MATLAB and simulated on a 1.7 GHz processor with 8 GB memory.

  As can be seen from Figure 5, all of the outliers in signals are estimated by the proposed method, and most of the estimated outliers are quite close to the actual values. For these two sets of estimated outliers, one can evaluate the performance from two aspects: one is the error (mean and standard deviation); the second is false rate and reliability, and they can be defined as follows:

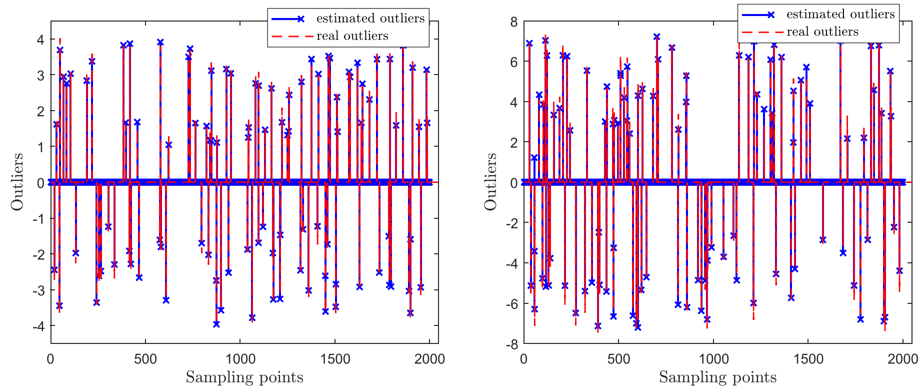


FIGURE 5 Comparison of real outliers with estimated outliers [Color figure can be viewed at wileyonlinelibrary.com]

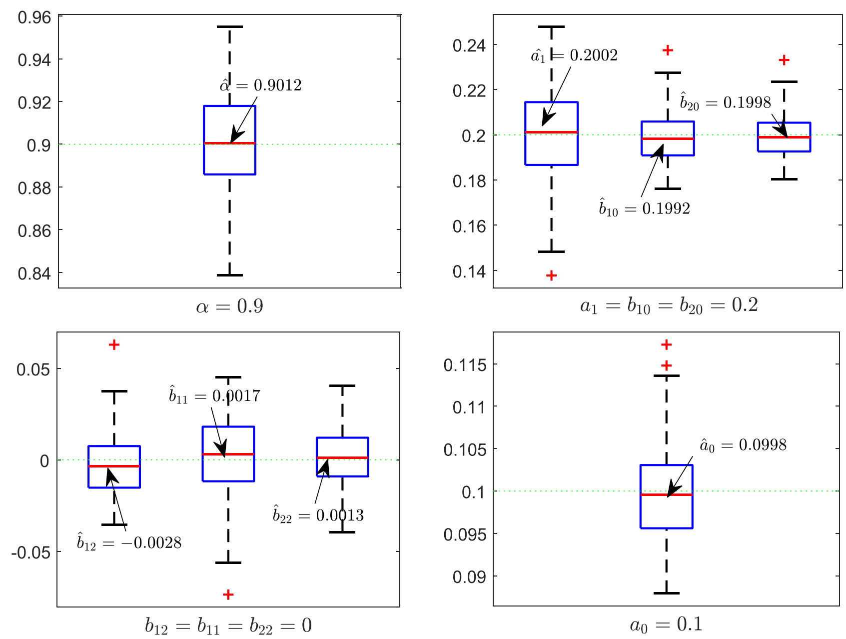


FIGURE 6 The distribution of estimated parameters [Color figure can be viewed at wileyonlinelibrary.com]

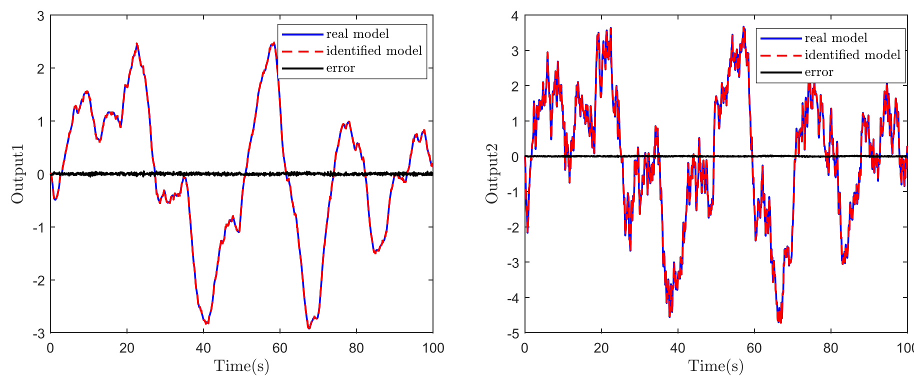


FIGURE 7 Comparison between the real model output and the identified one [Color figure can be viewed at wileyonlinelibrary.com]

False Rate: $\frac{|N_e - N_r|}{N_r}$, where N_e and N_r are the number of estimated outliers and the real one, respectively;
Reliablity: $|1 - \frac{\|e\|}{\|o\|}| \times 100\%$, where e is a vector consisting of the error of each estimated outlier, and o is a vactor of the real outliers.

The errors and the indexes defined above are shown in the following table. It can be seen from Table 1 that these estimated outliers are reliable enough.

- **Determination of parameters and fractional order.** After the signal preprocessing, the parameters of the system can be calculated using FDSIM. In this simulation, 200 Monte Carlo trials are performed, and the statistical estimated parameters are recorded in Figure 6.

From Figure 6, the mean of these estimated parameters can be obtained as follows:

$$\begin{cases} \hat{G}_1 = \frac{-0.0028s^{1.8024}+0.0017s^{0.9012}+0.1992}{s^{1.8024}+0.2002s^{0.9012}+0.0998}, \\ \hat{G}_2 = \frac{0.0013s^{1.8024}+0.4995s^{0.9012}+0.1998}{s^{1.8024}+0.2002s^{0.9012}+0.0998}. \end{cases}$$

The relative error criterion [26] as shown in (35) is used to evaluate the quality of estimated parameters

$$\delta = \frac{\|\eta - \hat{\eta}\|}{\|\eta\|} \times 100\%, \tag{35}$$

where $\eta = [a_1 a_0 b_{12} b_{11} b_{10} b_{22} b_{21} b_{20}]^{\mathrm{T}}$ is a vector of the parameters to be estimated, and $\hat{\eta}$ represents the vector with estimated parameters.

With (35), one can get $\delta = 0.62\%$, which means the system parameters are well estimated after the outliers processed. More intuitive result can be seen from Figure 7, which compares the output response of real model with estimated model's. It shows that the outputs of estimated model and real model are almost identical.

5 | CONCLUSIONS

In this paper, the identification of MIMO commensurate FOS is investigated. Firstly, a special periodic excitation signal is designed to obtain the accurate FRF, which is different from other frequency domain identification studies with the known data in frequency domain. In addition, the impact of noise is eliminated by truncating the data in the low frequency band. The outliers in the signal are also considered, and they are estimated by a matrix decomposition method with the help of nuclear norm and 1-norm. Meanwhile, a threshold value is introduced to make the estimated outliers more meeting the feature of sparseness. Finally, the system is identified by the FDSIM. The entire procedures are used for the identification of a MIMO FOS, and the effectiveness is confirmed with a typical numerical example. The future research work can be directed to eliminate the effect of noise on high-frequency band, and improve usage rate of the data in frequency domain.

ACKNOWLEDGMENTS

The work described in this paper was fully supported by the National Natural Science Foundation of China (No. 61601431, No. 61573332), the Anhui Provincial Natural Science Foundation (No. 1708085QF141), the Fundamental Research Funds for the Central Universities (No. WK2100100028) and the General Financial Grant from the China Postdoctoral Science Foundation (No. 2016M602032).

ORCID

Yiheng Wei https://orcid.org/0000-0002-0080-5365
Yong Wang https://orcid.org/0000-0002-6773-6544

REFERENCES

1. C. A Monje et al., *Fractional Order Systems and Controls: Fundamentals and Applications*, Springer, New York, 2010.
2. L. Liu et al., *Heat Conduction with Fractional Cattaneo–Christov Upper-Convective Derivative Flux Model*, Int. J. Thermal Sci. **112** (2017), 421–426.
3. Y. Z. Povstenko, *Fractional heat conduction equation and associated thermal stresses in an infinite solid with spherical cavity*, Quart. J. Mech. Appl. Math. **61** (2008), no. 4, 523–547.
4. K. Lazopoulos, D. Karaoulanis, and A. Lazopoulos, *On fractional modelling of viscoelastic mechanical systems*, Mech. Res. Commun. **78** (2016), 1–5.
5. W. Yu et al., *Frequency domain modelling and control of fractional-order system for permanent magnet synchronous motor velocity servo system*, IET Control Theory Appl. **10** (2016), no. 2, 136–143.
6. Y. G. Tang et al., *Identification of fractional-order systems with time delays using block pulse functions*, Mech. Syst. Signal Process. **91** (2017), 382–394.
7. Z. Gao, X. C. Lin, and Y. Zheng, *System identification with measurement noise compensation based on polynomial modulating function for fractional-order systems with a known time-delay*, ISA Trans. **79** (2018), 62–72.
8. K. Kothari, U. Mehta, and J. Vanualailai, *A novel approach of fractional-order time delay system modeling based on haar wavelet*, ISA Trans. **80** (2018), 371–380.
9. G. Liu et al., *Parameter identification of fractional order system using enhanced response sensitivity approach*, Commun. Nonlinear Sci. Numer. Simul. **67** (2019), 492–505.
10. J. C. Wang et al., *Fully parametric identification for continuous time fractional order hammerstein systems*, J. Franklin Inst. **357** (2020), no. 1, 651–666.
11. P. Van Overschee and B. De Moor, *Subspace Identification for Linear Systems: Theory-Implementation-Applications*, Springer, New York, 2012.
12. Y. Abe, M. Inoue, and S. Adachi, *Subspace identification method incorporated with a priori information characterized in frequency domain*, 15th European Control Conference (ECC), Aalborg, Denmark, 2016, pp. 1377–1382.
13. O. Cois et al., *Fractional state variable filter for system identification by fractional model*, European Control Conference (ECC), Porto, Portugal, 2001, pp. 2481–2486.
14. Y. S. Hu et al., *Subspace-based continuous-time identification of fractional order systems from non-uniformly sampled data*, Int. J. Syst. Sci. **47** (2016), no. 1, 122–134.
15. Y. Dai et al., *Modulating function-based identification for fractional order systems*, Neurocomputing **173** (2016), 1959–1966.
16. Y. G. Tang et al., *Parameter identification of fractional order systems using block pulse functions*, Signal Process. **107** (2015), 272–281.
17. J. Schoukens et al., *Frequency response function measurements in the presence of nonlinear distortions*, Automatica **37** (2001), no. 6, 939–946.

18. R. Pintelon and J. Schoukens, *System Identification: A Frequency Domain Approach*, John Wiley & Sons, New Jersey, 2012.
19. R. S. Smith, *Nuclear norm minimization methods for frequency domain subspace identification*, American Control Conference (ACC), Montreal, Canada, 2012, pp. 2689–2694.
20. R. S. Smith, *Frequency domain subspace identification using nuclear norm minimization and Hankel matrix realizations*, IEEE Trans. Autom. Control **59** (2014), no. 11, 2886–2896.
21. Y. S. Hu et al., *Subspace-based continuous-time model identification using nuclear norm minimization*, IFAC-PapersOnLine **48** (2015), no. 28, 338–343.
22. Y. H. Wei et al., *A note on short memory principle of fractional calculus*, Fract. Calc. Appl. Anal. **20** (2017), no. 6, 1382–1404.
23. G. A. Zou, B. Wang, and Y. Zhou, *Existence and regularity of mild solutions to fractional stochastic evolution equations*, Math. Model. Nat. Phenom. **13** (2018), no. 1, 1–15.
24. X. K. He et al., *Multisine input design for LTI systems by uniform design and its application to aerodynamic system*, 12th IEEE International Conference on Control and Automation (ICCA), Kathmandu, Nepal, 2016, pp. 541–546.
25. R. Z. Cui et al., *An innovative parameter estimation for fractional-order systems in the presence of outliers*, Nonlinear Dyn. **89** (2017), no. 1, 453–463.
26. R. Z. Cui et al., *An innovative parameter estimation for fractional order systems with impulse noise*, ISA Trans. **82** (2018), 120–129.

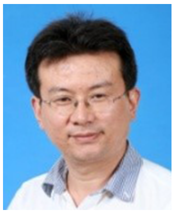
AUTHOR BIOGRAPHIES

Zongyang Li is a Ph.D. candidate in University of Science and Technology of China. He received his B.E. degree in Department of Electronic Science and Technology from the University of Science and Technology of China in 2015. His research interests include fractional order system identification, numerical computation, and controller design for ac motor drives and linear motor drives.

Yiheng Wei is currently an associate professor in Department of Automation of the University of Science and Technology of China. He received his B.E. and Ph.D. degrees from the Northeast University and the University of Science and Technology of China in 2010 and 2015, respectively. He was a postdoctoral fellow of the City University of Hong Kong and the University of Science and Technology of China from 2015 to 2017. He is now a visiting scholar of the University of California, Merced. His research interests include fractional calculus, numerical computation, and stochastic learning.

Jiachang Wang is a Ph.D. candidate in University of Science and Technology of China. He received his B.E. degree in Automation from the Zhengzhou University in 2016. He is currently participating in the Ph.D. joint training program between University of Science and Technology of China and INSA Centre Val de Loire, France. His research interests include fractional order system identification, numerical computation, and discrete modulating function method.

Yongting Deng was born in Shandong, China, in 1987. He received the B.E. degree from the China University of Petroleum, China, and the M.S. and Ph.D. degrees from the Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, China, in 2015. He is currently an Associate Professor with the Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences. His research interests include controller design for ac motor drives and linear motor drives, intelligent control, and digital control using the DSP and FPGA implementations.



Jianli Wang was born in Shandong, China, in 1971. He received the Ph.D. degree from the Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, China, in 2002. He is currently a professor with the Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences. He has authored/co-authored more than 100 publications in his main areas of research, which are optical-electric telescope, high-resolution imaging, and high-precision machine control techniques.

Yong Wang received the B.E. degree in automatic control from the University of Science and Technology of China in 1982, the M.E. and Ph.D. degrees in navigation, guidance, and control from the Nanjing Aeronautical Institute. He is currently a full professor in University of Science and Technology of China. He is leading several research groups that focus on vibration control, vehicle control and fractional order system control. Professor Wang is the recipient of the 2013, 2014 and 2015 Chinese Academy of Sciences Outstanding Instructor.

How to cite this article: Li Z, Wei Y, Wang J, Deng Y, Wang J, Wang Y. Frequency domain subspace identification of fractional order systems using time domain data with outliers. *Asian J Control*. 2020;1-11. https://doi.org/10.1002/asjc.2370