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# Design method for off-axis aspheric reflective optical system with extremely low aberration and large field of view 

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#### Abstract

This paper proposes a grouping design method based on a combination of spatial ray tracing and aberration correction to construct the initial structure for an off-axis multi-reflective aspheric optical system. This method establishes a mathematical parameter model of the optical system to satisfy the aberration balance and multiconstraint control requirements. The simulated annealing particle swarm algorithm is applied to calculate the initial optical system structure. Finally, an extreme ultraviolet (EUV) lithography projection objective with sixreflective aspheric mirrors is used as an example to verify the reliability and effectiveness of this method. A 0.33 numerical aperture EUV lithographic objective with wavefront error better than $1 / 70 \lambda(\lambda=13.5 \mathrm{~nm})$ root mean square (RMS) is obtained. © 2020 Optical Society of America


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## 1. INTRODUCTION

When compared with refractive optical systems, reflective optical systems offer several advantages, including an absence of chromatic aberrations, an optical path that can be folded that makes the structure compact, and low sensitivity to changes in both temperature and air pressure [1-3]. Development of off-axis reflective optical systems is an effective approach to increase the system's field of view and improve the image quality by eliminating obscurations. Therefore, the design of off-axis reflective optical systems, which are often used in space cameras, telescopes, and infrared/ultraviolet systems, has received extensive attention [3-7]. The aberration of the off-axis reflection system is not easy to be corrected due to the asymmetric structure compared with the coaxial system. Therefore, the off-axis reflection system often contains some advanced surfaces such as freeform surfaces to improve the imaging quality. The main reason why extreme ultraviolet (EUV) systems mostly use offaxis reflection systems is that there is no suitable transmission material that can maintain a high transmittance in the ultraviolet band. Recently, EUV lithography projection objective with large field of view off-axis six-reflective mirrors is a typical application of off-axis multi-reflective optical systems. The EUV lithography projection objective is a microscopic objective with extremely high imaging quality requirements. Imaging using an EUV lithography projection objective requires resolution near
the ultra-diffraction limit and a wavefront error of better than $1 / 50 \lambda$, where $\lambda$ is the working wavelength [8,9].

The design of ultra-diffraction-limited and extremely low aberration optical systems is strongly reliant on aberration balancing, which poses major challenges to optical designers. Optical design software mainly solves the problems of optical system optimization. The software uses a local optimization algorithm based on the damped least squares method to calculate the minimum value of the multi-dimensional variable space error function. In most cases, the final optimization result is a locally optimal solution that is close to the initial structure and thus has great limitations [10]. The use of optical design software for design optimization is strongly dependent on the selection of the initial structure [11]. The construction of this initial structure therefore plays the most important role in the optical system design. For extremely low aberration optical systems in particular, which are more sensitive to the aberration balance, it is essential to construct an initial structure that satisfies both the aberration balance and multi-constraint control requirements.

Over the past few years, several optical design methods for the initial structures of reflective optical systems have been proposed in the literature, including the paraxial search method, the $y-\bar{y}$ design method, and the grouping design method. Bal proposed the paraxial search method, in which a paraxial model is used to enumerate the first-order aberrations of the
optical system exhaustively [12,13]. However, the paraxial search method cannot control the constraints effectively, and the numbers that satisfied the constraints were too small, which affected the calculation efficiency strongly. Lerner et al. applied the $y-\bar{y}$ design method to provide a solution for the optical system structure $[14,15]$. This method uses the height of a marginal ray and the main ray incident on the surface of the optical element to solve for the radius of curvature and the distance required to construct the initial structure. The heights of the main ray and the marginal ray on the optical surface for each structure are not easy to determine, and the method is not universal. When the off-axis reflection optical system contains a large number of components, the number of calculations required will then increase greatly, which affects the method's design efficiency. Hudyma applied the grouping design method to the design of an off-axis six-reflective optical system by dividing the optical system into two groups, but did not provide a specific design method [16]. Li's group proposed a grouping design method with a real ray tracing model that was developed to acquire spherical initial configurations for reflective optical systems using more than six mirrors. They divided the off-axis reflective optical system into three groups, which allowed the structural parameters of each mirror group to be calculated through a real ray tracing approach based on constraint control [17,18]. Finally, they connected the structural parameters of the three mirror groups. This method introduced the real ray tracing approach to solve the constraint problem in the solution for the initial structure, but it did not consider the aberration balance and relied entirely on the optimization process of the optical design software to correct the aberration. This will have two effects. First, the optimization process may produce large disturbances, meaning that the structure will deviate too far from the initial structure and make the constraints difficult to control. Second, the aberration balance process during the optical design software optimization may increase the residual aberration values of both low-order aberrations and high-order aberrations and then increase the design's residual values. This method is thus not conducive to the realization of an extremely low aberration system.

This paper proposes a grouping design method based on spatial ray tracing and aberration correction to solve the aberration balance and multi-constraint control problems that occur during the initial structure construction process. This method can improve the influence of disturbances on the initial structure
during the optimization process, improve the imaging quality, and provide a potential starting point for the design of an off-axis multi-reflection optical system with extremely low aberration and a large field of view. In this paper, a grouping design method based on spatial ray tracing and aberration correction is applied to an EUV lithography projection objective optical system to design an off-axis six-reflective aspheric extremely low aberration optical system with a large field of view. The design process is detailed as follows. (1) The optical system is divided into two groups, which are the objective lens group and the image lens group. (2) Using a combination of aberration theory and spatial ray tracing theory, the configurational parameters of the optical system are parameterized, and a mathematical model is established to calculate the corresponding parameters for the objective lens group and the image lens group. (3) The objective lens group and the image lens group are combined, and the simulated annealing particle swarm algorithm is used to calculate the initial structure's parameters. (4) The initial structure is then optimized. We use the numerical aperture asymptotic method to optimize the design, based on the fact that a small aperture system has a small aberration property that effectively avoids trapping into a local minimum value. Finally, we complete the design of the off-axis six-reflective optical system that combines engineering feasibility with a full field of view, a chord length of 26 mm , and a width of 2 mm , which gives a root mean square (RMS) wavefront error of better than $1 / 70 \lambda$, where $\lambda$ is the working wavelength.

## 2. GROUP DESIGN BASED ON SPATIAL RAY TRACING AND ABERRATION CORRECTION

In this paper, as shown in Fig. 1, the off-axis six-reflective optical system is divided into two groups: the objective lens group and the image lens group. The main optical system parameters include the system demagnification $\beta$, the center object height $y$, the center image height $y_{\mathrm{im}}$, the object side numerical aperture NAO, and the image side numerical aperture NA, from which we can obtain $y_{\mathrm{im}}=y * \beta$ and $\mathrm{NAO}=\mathrm{NA} * \beta$. The objective lens group and the image side lens group are spliced together at the intermediate image position, which should obey the principles of object-image matching, pupil matching, and demagnification matching. Using the principles of grouping design and aberration theory, the structural parameters of both the objective lens group and the image lens group of the optical system are then parameterized. Spatial ray tracing is introduced


Fig. 1. Schematic of the off-axis six-reflective aspheric optical system.


Fig. 2. Schematic diagram of the Group 1 configuration.
to control the constraints, including obscurations, lens distances, image telecentricity, the lens apertures, and the angles of incidence; we then parameterize these constraints. Finally, we establish mathematical models to calculate the corresponding parameters for the optical system.

## A. Aberration Analysis of the Objective Lens Group <br> (Group 1)

As shown in Fig. 2, the center object height is $y$, the object paraxial marginal ray angle is $u_{1}$, and the stop is located on $M_{2}$. Rays are emitted from the object point via $M_{1}, M_{2}, M_{3}, M_{4}$ to the intermediate image points. $d_{1}, d_{2}, d_{3}, d_{4}$ represent the lens thicknesses from $M_{1}$ to these intermediate image points, respectively. $h_{1}, h_{2}, h_{3}, h_{4}$ represent the paraxial marginal ray heights on $M_{1}, M_{2}, M_{3}, M_{4}$, respectively. The parameters $r_{1}, r_{2}, r_{3}, r_{4}$ and $k_{1}, k_{2}, k_{3}, k_{4}$ represent the radii of curvature and the conics of $M_{1}, M_{2}, M_{3}, M_{4}$, respectively. $l_{1}, l_{2}, l_{3}, l_{4}$ and $l_{1}^{\prime}, l_{2}^{\prime}, l_{3}^{\prime}, l_{4}^{\prime}$ represent the object distances and image distances of $M_{1}, M_{2}, M_{3}, M_{4}$, respectively.

$$
\begin{cases}P=\left(\frac{\Delta u}{\Delta \frac{1}{n}}\right)^{2} \Delta \frac{u}{n} & W=\frac{\Delta u}{\Delta \frac{1}{n}} \Delta \frac{u}{n}  \tag{2}\\ \Pi=\frac{\Delta(n u)}{n n^{\prime}} & \phi=\frac{1}{b} \Delta \frac{u}{n}, \\ K=\frac{k}{r^{3} \Delta n} & J=\text { nuy }\end{cases}
$$

Next, we introduce the following parameters based on the paraxial approximation:

$$
\left\{\begin{array} { l } 
{ \alpha _ { 1 } = \frac { l _ { 2 } } { l _ { 1 } ^ { \prime } } \approx \frac { h _ { 2 } } { h _ { 1 } } }  \tag{3}\\
{ \alpha _ { 2 } = \frac { l _ { 3 } } { l _ { 3 } ^ { \prime } } \approx \frac { h _ { 3 } } { h _ { 2 } } , } \\
{ \alpha _ { 3 } = \frac { l _ { 4 } } { l _ { 3 } ^ { \prime } } \approx \frac { h _ { 4 } } { h _ { 3 } } }
\end{array} \quad \left\{\begin{array}{l}
\beta_{1}=\frac{l_{1}^{\prime}}{l_{1}} \\
\beta_{2}=\frac{l_{2}^{\prime}}{l_{2}} \\
\beta_{3}=\frac{l_{3}^{\prime}}{l_{3}} \\
\beta_{4}=\frac{l_{4}^{\prime}}{l_{4}}
\end{array}\right.\right.
$$

For a reflective optical system, $n_{1}=n_{2}^{\prime}=n_{3}=n_{4}^{\prime}=1$, $n_{1}^{\prime}=n_{2}=n_{3}^{\prime}=n_{4}=-1$, and $h_{1}=l_{1} u_{1}, h_{2}=\alpha_{1} l_{1} u_{1}, h_{3}=$ $\alpha_{1} \alpha_{2} l_{1} u_{1}, h_{4}=\alpha_{1} \alpha_{2} \alpha_{3} l_{1} u_{1}$.

According to paraxial optical theory, we can obtain the following expressions for the radii of curvature and the thicknesses:

$$
\left\{\begin{array}{l}
r_{1}=\frac{2 \beta_{1} l_{1}}{1+\beta_{1}}  \tag{4}\\
r_{2}=\frac{2 \alpha_{1} \beta_{1} \beta_{2} l_{1}}{1+\beta_{2}} \\
r_{3}=\frac{2 \alpha_{1} \alpha_{2} \beta_{1} \beta_{2} \beta_{3} l_{1}}{1+\beta_{3}} \\
r_{4}=\frac{2 \alpha_{1} \alpha_{2} \alpha_{3} \beta_{1} \beta_{2} \beta_{3} \beta_{4} l_{1}}{1+\beta_{4}}
\end{array}, \quad\left\{\begin{array}{l}
d_{1}=\beta_{1} l_{1}-\alpha_{1} \beta_{1} l_{1} \\
d_{2}=\alpha_{1} \beta_{1} \beta_{2} l_{1}-\alpha_{1} \alpha_{2} \beta_{1} \beta_{2} l_{1} \\
d_{3}=\alpha_{1} \alpha_{2} \beta_{1} \beta_{2} \beta_{3} l_{1}-\alpha_{1} \alpha_{2} \alpha_{3} \beta_{1} \beta_{2} \beta_{3} l_{1} \\
d_{4}=\alpha_{1} \alpha_{2} \alpha_{3} \beta_{1} \beta_{2} \beta_{3} l_{1}
\end{array}\right.\right.
$$

The third-order monochromatic aberrations mainly include spherical aberrations, coma, astigmatism, field curvature, and distortion, which are represented by $S_{\mathrm{I}}, S_{\mathrm{II}}, S_{\mathrm{III}}, S_{\mathrm{IV}}, S_{\mathrm{V}}$ and by the following expressions, respectively [3,11]:

$$
\begin{cases}S_{\mathrm{I}}=\sum h P+\sum b^{4} K & \text { parameters for G1 are }  \tag{1}\\ S_{\mathrm{II}}=\sum y P-J \sum W+\sum h^{3} y K & \\ S_{\mathrm{III}}=\sum \frac{y^{2}}{h} P-2 J \sum \frac{y}{h} W+J^{2} \sum \phi+\sum b^{2} y^{2} K \\ S_{\mathrm{IV}}=J^{2} \sum \frac{\Pi}{h} & \\ S_{V}=\sum \frac{y^{3}}{h^{2}} P-3 J \sum \frac{y^{2}}{h^{2}} W+J^{2} \sum \frac{y}{h}\left(3 \phi+\frac{\Pi}{b}\right)-J^{3} \sum \frac{1}{h^{2}} \triangle \frac{1}{n^{2}}+\sum h y^{3} K\end{cases}
$$

$$
\begin{align*}
& \int G_{1} S_{\mathrm{I}}=\frac{l_{1} u_{1}^{4}}{4 \beta_{1}^{3} \beta_{2}^{3} \beta_{3}^{3} \beta_{4}^{3}}\left[-\left(1+\beta_{1}\right) \beta_{2}^{3} \beta_{3}^{3} \beta_{4}^{3}\left(\left(-1+\beta_{1}\right)^{2}+\left(1+\beta_{1}\right)^{3} k_{1}\right)+\alpha_{1}\left(1+\beta_{2}\right) \beta_{3}^{3} \beta_{4}^{3}\left(\left(-1+\beta_{2}\right)^{2}+\left(1+\beta_{2}\right)^{3} k_{2}\right)\right. \\
& \left.-\alpha_{2}\left(1+\beta_{3}\right) \beta_{4}^{3}\left(\left(-1+\beta_{3}\right)^{2}+\left(1+\beta_{3}\right)^{3} k_{3}\right)+\alpha_{1} \alpha_{2} \alpha_{3}\left(1+\beta_{4}\right)\left(\left(-1+\beta_{4}\right)^{2}+\left(1+\beta_{4}\right)^{3} k_{4}\right)\right] \\
& G_{1} S_{\text {II }}=u_{1}^{3} y\left[-\frac{\left(-1+\beta_{1}\right)^{2}\left(1+\beta_{1}\right)}{\alpha_{1}}+\frac{\left(-1+\alpha_{2}\right) \beta_{4}^{3}\left(1-\beta_{3}^{2}+\beta_{3}^{3}\right)+\left(1-\beta_{1}+\beta_{1}^{2}+\beta_{1}^{3}\right) \beta_{2}^{3} \beta_{3}^{3} \beta_{4}^{3}}{4 \beta_{1}^{2} \beta^{2} \beta_{3}^{3} \beta_{4}^{3}}\right. \\
& \left.-\frac{\alpha_{3}\left(-1+\alpha_{2}-\beta_{3}\right)\left(-1+\beta_{4}\right)^{2}\left(1+\beta_{4}\right)-\beta_{3}\left(1+\beta_{4}-\beta_{4}^{2}+\alpha_{2} \beta_{4}^{3}\right)}{4 \beta_{1}^{2} \beta_{2}{ }^{2} \beta_{3}^{3} \beta_{4}^{3}}\right] \\
& +\frac{\left(-1+\alpha_{1}\right)\left(1+\beta_{1}\right)^{2} k_{1} u_{1}^{3} y}{4 \alpha_{1} \beta_{1}^{2}}+\frac{\left(-1+\alpha_{2}\right)\left(1+\beta_{3}\right)^{3} k_{3} u_{1}^{3} y}{4 \beta_{1}^{2} \beta_{2}^{2} \beta_{3}^{3}} \\
& -\frac{\left(\alpha_{3}\left(-1+\alpha_{2}-\beta_{3}\right)+\beta_{3}\right)\left(1+\beta_{4}\right)^{3} k_{4} u_{1}{ }^{3} y}{4 \beta_{1}^{2} \beta_{2} \beta_{3}^{3} \beta_{4}^{3}} \\
& G_{1} S_{\text {III }}=-\frac{u_{1}^{2} y^{2}}{4 \alpha_{1}^{2} \alpha_{2} \alpha_{3} \beta_{1} \beta_{2} \beta_{3}^{3} \beta_{4}^{3} l_{1}}\left(\alpha_{1}^{2} \alpha_{2} \alpha_{3}\left(1+\beta_{1}\right)^{3} \beta_{2} \beta_{3}^{3} \beta_{4}^{3}\left(1+k_{1}\right)-\alpha_{1}\left(\beta_{3}^{2}\left(1+\beta_{4}\right)^{3}\left(1+k_{4}\right)\right)\right. \\
& +\alpha_{2} \alpha_{3}\left(1+\beta_{1}\right) \beta_{2} \beta_{3}^{3} \beta_{4}^{3}\left(\left(-1+\beta_{1}\right)^{2}+\left(1+\beta_{1}\right)^{2} k_{1}\right)+\alpha_{3}^{2}\left(1-\alpha_{2}+\beta_{3}\right)^{2}\left(1+\beta_{4}\right)\left(\left(-1+\beta_{4}\right)^{2}\right. \\
& \left.+\left(1+\beta_{4}\right)^{2} k_{4}\right)+\alpha_{3}\left(-\left(-1+\alpha_{2}\right)^{2} \beta_{4}^{3}\left(1+k_{3}\right)+\beta_{3}^{3} \beta_{4}^{3}\left(-1-k_{3}+\alpha_{2}\left(-\alpha_{2}\left(1+k_{3}\right)\right.\right.\right. \\
& \left.+2\left(1+\beta_{2}\left(1-\beta_{1}+\beta_{1}^{2}+\beta_{1}^{3}+\left(1+\beta_{1}\right)^{2} k_{1}+k_{3}\right)\right)\right)+\left(-1+\alpha_{2}\right) \beta_{3}\left(2\left(1+k_{4}\right)\right. \\
& \left.+\beta_{4}\left(2+6 k_{4}+\beta_{4}\left(-2+6 k_{4}+\beta_{4}\left(1+\alpha_{2}+3 k_{3}-3 \alpha_{2} k_{3}+2 k_{4}\right)\right)\right)\right)+\beta_{3}^{3}\left(-2\left(1+k_{4}\right)\right. \\
& \left.\left.\left.\left.-\beta_{4}\left(2+6 k_{4}+\beta_{4}\left(-2+6 k_{4}+\beta_{4}\left(1+3 k_{3}+\left(-2+\alpha_{2}\right) \alpha_{2}\left(-1+k_{3}\right)+2 k_{4}\right)\right)\right)\right)\right)\right)\right) \\
& G_{1} S_{\mathrm{IV}}=\frac{u_{1}^{2} y^{2}}{4 \alpha_{1} \alpha_{2} \alpha_{3} \beta_{1} \beta_{2} \beta_{3} \beta_{4} l_{1}}\left(-1-\beta_{4}+\alpha_{3} \beta_{4}+\alpha_{3} \beta_{3} \beta_{4}-\alpha_{2} \alpha_{3} \beta_{3} \beta_{4}-\alpha_{2} \alpha_{3} \beta_{2} \beta_{3} \beta_{4}\right. \\
& \left.+\alpha_{1} \alpha_{2} \alpha_{3} \beta_{2} \beta_{3} \beta_{4}+\alpha_{1} \alpha_{2} \alpha_{3} \beta_{1} \beta_{2} \beta_{3} \beta_{4}\right) \\
& G_{1} S_{\mathrm{V}}=\frac{u_{1} y^{3}}{4 l_{1}{ }^{2}}\left(\left(1+\beta_{1}\right)^{2}\left(3+\beta_{1}+k_{1}+\beta_{1} k_{1}\right)-\frac{\left(1+\beta_{1}\right)\left(\left(-1+\beta_{1}\right)^{2}+\left(1+\beta_{1}\right)^{2} k_{1}\right)}{\alpha_{1}{ }^{3}}\right. \\
& +\frac{1-3 k_{1}-\beta_{1}\left(5+9 k_{1}+3 \beta_{1}\left(3+\beta_{1}\right)\left(1+k_{1}\right)\right)}{\alpha_{1}}-\frac{1}{\alpha_{1}^{2} \alpha_{2}^{2} \alpha_{3}^{2} \beta_{3}^{3} \beta_{4}^{3}}\left(\alpha_{3}^{3}\left(-1+\alpha_{2}-\beta_{3}\right)^{3}\left(1+\beta_{4}\right)\left(\left(-1+\beta_{4}\right)^{2}+\left(1+\beta_{4}\right)^{2} k_{4}\right)\right. \\
& +\alpha_{3}\left(-1+\alpha_{2}-\beta_{3}\right) \beta_{3}^{2}\left(1+\beta_{4}\right)\left(3+6 \beta_{4}-\beta_{4}^{2}+3\left(1+\beta_{4}\right)^{2} k_{4}\right)+\beta_{3}^{3}\left(1+\beta_{4}\right)^{2}\left(1+k_{4}+\beta_{4}\left(3+k_{4}\right)\right) \\
& +\alpha_{3}^{2}\left(-\left(-1+\alpha_{2}\right)^{3} \beta_{4}^{3}\left(1+k_{3}\right)+\beta_{3}^{3}\left(3\left(1+\beta_{4}\right)+\beta_{4}\left(3+9 k_{4}+\beta_{4}\left(-3+\alpha_{2} \beta_{4}\left(1-\alpha_{2}\left(\alpha_{2}+3 \beta_{1}\left(-1+\beta_{1}+\beta_{1}^{2}\right)\right)\right.\right.\right.\right.\right. \\
& \left.\left.\left.-3 \alpha_{2}\left(1+\beta_{1}\right)^{2} k_{1}-\left(-1+\alpha_{2}\right)^{3} \beta_{4} k_{3}+3\left(3+\beta_{4}\right) k_{4}\right)\right)\right)+\left(-1+\alpha_{2}\right)^{2} \beta_{3}\left(3\left(1+k_{4}\right)\right. \\
& \left.+\beta_{4}\left(3+9 k_{4}+\beta_{4}\left(-3+9 k_{4}+\beta_{4}\left(2+\alpha_{2}+3 k_{3}-3 \alpha_{2} k_{3}+3 k_{4}\right)\right)\right)\right)+\left(-1+\alpha_{2}\right) \beta_{3}^{3}\left(-6\left(1+k_{4}\right)+\beta_{4}\left(-6\left(1+3 k_{4}\right)\right.\right. \\
& \left.\left.\left.\left.\left.-k_{4}\left(-6+18 k_{4}+k_{4}\left(1+3 k_{3}+\left(-2+\alpha_{2}\right) \alpha_{2}\left(-1+3 k_{3}\right)+6 k_{4}\right)\right)\right)\right)\right)\right)\right) \tag{5}
\end{align*}
$$

## B. Aberration Analysis of the Image Lens Group

## (Group 2)

As shown in Fig. 3, the center object height of the image lens group is $y_{5}$, the center image height of the image lens group is $y_{\mathrm{im}}$, and the object paraxial marginal ray angle of the image lens group is $u_{5}$. Rays are emitted from the intermediate image point via $M_{5}$ and $M_{6}$ to the image point. $d_{5}$ and $d_{6}$ represent the lens thicknesses from $M_{5}$ to $M_{6}$ and $M_{6}$ to the image plane, respectively. $b_{5}$ and $h_{6}$ represent the paraxial marginal ray heights on $M_{5}$ and $M_{6}$, respectively. $r_{5}, r_{6}$ and $k_{5}, k_{6}$ represent the radii of curvature and the conics of $M_{5}$ and $M_{6}$, respectively. $l_{5}, l_{6}$ and $l_{5}^{\prime}, l_{6}^{\prime}$ represent the object distances and image distances of $M_{5}$ and $M_{6}$, respectively.

Based on the paraxial approximation, we now introduce the following parameters:

$$
\alpha_{5}=\frac{l_{6}}{l_{5}^{\prime}} \approx \frac{h_{6}}{h_{5}} \quad\left\{\begin{array}{l}
\beta_{5}=\frac{l_{5}^{\prime}}{l_{5}}  \tag{6}\\
\beta_{6}=\frac{l_{6}^{\prime}}{l_{6}}
\end{array} .\right.
$$

For a reflective optical system, $n_{5}=n_{6}^{\prime}=1, n_{5}^{\prime}=n_{6}=-1, h_{5}=l_{5} u_{5}, h_{6}=\alpha_{1} l_{5} u_{5}$.
According to paraxial optical theory, we can also obtain the following expressions for the radii of curvature and thicknesses:

$$
\left\{\begin{array}{l}
r_{5}=\frac{2 \beta_{5} l_{5}}{1+\beta_{5}}  \tag{7}\\
r_{6}=\frac{2 \alpha_{5} \beta_{5} l_{6}}{1+\beta_{6}}
\end{array}, \quad\left\{\begin{array}{l}
d_{5}=\beta_{5} l_{5}-\alpha_{5} \beta_{5} l_{5}=l_{5} \\
d_{6}=\alpha_{5} \beta_{5} \beta_{5} l_{5}
\end{array}\right.\right.
$$



Fig. 3. Schematic diagram of the Group 2 configuration.

The expression for the distance between the entrance pupil position and $M_{5}$ in Group 2 is

$$
l_{\mathrm{enpG} 2}=-\frac{\beta_{5} l_{5}\left(1+\beta_{6}+\alpha_{5} \beta_{5} \beta_{6}\right)}{1+\beta_{6}+\alpha_{5} \beta_{5} \beta_{6}+\alpha_{5} \beta_{5}^{2} \beta_{6}}
$$

The expression for the distance between the exit pupil position and $M_{5}$ in Group 1 is

$$
\begin{aligned}
l_{\operatorname{expG1}}^{\prime} & =l_{\operatorname{expG1}}-\left(d_{4}-l_{5}\right) \\
& =\frac{\alpha_{1} \alpha_{2} \alpha_{3} \beta_{1} \beta_{2} \beta_{3}^{2} \beta_{4}^{2} l_{1}}{\alpha_{3}\left(-1+\alpha_{2}-\beta_{3}\right)+\beta_{3}+\beta_{3} \beta_{4}}+l_{5}
\end{aligned}
$$

The expressions for the third-order aberration coefficient parameters for G2 are:

The third-order aberration coefficients for the off-axis six-reflective-surface optical system can be expressed as

$$
\left\{\begin{array}{l}
S_{1}=G_{1} S_{\mathrm{II}}+G_{2} S_{\mathrm{I}}  \tag{10}\\
S_{\mathrm{II}}=G_{1} S_{\mathrm{II}}+G_{2} S_{\mathrm{II}} \\
S_{\mathrm{III}}=G_{1} S_{\mathrm{III}}+G_{2} S_{\mathrm{III}} \\
S_{\mathrm{IV}}=G_{1} S_{\mathrm{IV}}+G_{2} S_{\mathrm{IV}} \\
S_{\mathrm{V}}=G_{1} S_{\mathrm{V}}+G_{2} S_{\mathrm{V}}
\end{array}\right.
$$

## 3. ERROR FUNCTION CALCULATION OF INITIAL STRUCTURE USING PARTICLE SWARM SIMULATED ANNEALING ALGORITHM

Using the aberration theory presented above, we obtain the structural parameters and the third-order aberration coefficients of the optical system. We then introduce real ray tracing and calculate the optical system constraints, including obscurations, lens thicknesses, image telecentricity, the lens apertures, and the angles of incidence. To balance the third-order aberrations and control constraints, we established a mathematical model of the above parameters. The model's error function can be expressed as

$$
\begin{aligned}
F= & f\left(\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{5}, \beta_{1}, \beta_{2}, \beta_{3}, \beta_{4}, \beta_{5}, \beta_{6}, k_{1}, k_{2}, k_{3},\right. \\
& \left.k_{4}, k_{5}, k_{6}\right) \\
= & \left|S_{\mathrm{I}}\right|+\left|S_{\mathrm{II}}\right|+\left|S_{\mathrm{III}}\right|+\left|S_{\mathrm{IV}}\right|+\left|S_{\mathrm{V}}\right|+\mid \text { constraints } \mid \\
= & \left|S_{\mathrm{I}}\right|+\left|S_{\mathrm{II}}\right|+\left|S_{\mathrm{III}}\right|+\left|S_{\mathrm{IV}}\right|+\left|S_{\mathrm{V}}\right|+\text { Sum(|Obscuration } \mid \\
& +\mid \text { BWD }|+| \text { TEL }|+| \text { RED }|+| \text { APE }|+| \text { AOI } \mid)
\end{aligned},
$$

where the constraints in the equation represent the sum of constraints listed above, respectively. Obscuration, BWD, TEL,

$$
\begin{align*}
G_{2} S_{\mathrm{I}}= & \frac{l_{5} u_{5}^{4}}{4 \beta_{5}^{3} \beta_{6}^{3}}\left[\left(1+\beta_{5}\right) \beta_{6}^{3}\left(-\left(-1+\beta_{5}\right)^{2}-\left(1+\beta_{5}\right)^{3} k_{5}\right)+\alpha_{5}\left(1+\beta_{6}\right)\left(\left(-1+\beta_{6}\right)^{2}+\left(1+\beta_{6}\right)^{3} k_{6}\right)\right] \\
G_{2} S_{\mathrm{II}}= & \frac{y_{2} u_{5}^{3}}{4 \alpha_{5} \beta_{5}^{2} \beta_{6}^{2}}\left(-\left(1+\beta_{5}\right) \beta_{6}^{3}\left(1+\beta_{6}\right)\left(\left(-1+\beta_{5}\right)^{2}+\left(1+\beta_{5}\right)^{3} k_{5}\right)+\alpha_{5}\left(-1+k_{6}+\beta_{6}\left(-1+k_{6}\right.\right.\right. \\
& \left.\left.+\beta_{6}\left(-1+3 k_{6}+\beta_{6}\left(\beta_{6}+k_{5}+\beta_{5}\left(-1+\beta_{5}+\beta_{5}^{2}+\left(3+\beta_{5}\left(3+\beta_{5}\right)\right) k_{5}\right)+\left(3+\beta_{6}\right) k_{6}\right)\right)\right)\right) \\
G_{2} S_{\mathrm{III}}= & -\frac{u_{5}^{2} y_{2}^{2}}{4 \alpha_{5}^{2} \beta_{5} \beta_{6} l_{5}}\left(\alpha_{5}^{2}\left(1+\beta_{5}\right)^{3} \beta_{6}\left(1+k_{5}\right)+\left(1+\beta_{5}\right) \beta_{6}\left(1+\beta_{6}\right)^{3}\left(\left(-1+\beta_{5}\right)^{2}+\left(1+\beta_{5}\right)^{3} k_{5}\right)\right. \\
& \left.-\alpha_{5}\left(1+\beta_{6}\right)\left(1+k_{6}+\beta_{6}\left(\beta_{6}+2 k_{5}+2 \beta_{5}\left(-1+\beta_{5}+\beta_{5}^{2}\left(3+\beta_{5}\left(3+\beta_{5}\right)\right) k_{5}\right)+\left(2+\beta_{6}\right) k_{6}\right)\right)\right)  \tag{8}\\
G_{2} S_{\mathrm{IV}}= & \frac{u_{5}^{2} y_{2}{ }^{2}}{\alpha_{5} \beta_{5} \beta_{6} l_{5}}\left(-1-\beta_{6}+\alpha_{5} \beta_{6}+\alpha_{5} \beta_{5} \beta_{6}\right) \\
G_{2} S_{\mathrm{V}}= & \frac{u_{5} y_{2}^{3}}{4 \alpha_{5}^{2} l_{1}^{2}}\left(\alpha_{5}^{3}\left(1+\beta_{5}\right)^{2}\left(3+k_{5}+\beta_{5}+k_{5} \beta_{5}\right)-\left(1+\beta_{5}\right)\left(1+\beta_{6}\right)^{3}\left(\left(-1+\beta_{5}\right)^{2}+\left(1+\beta_{5}\right)^{3} k_{5}\right)\right. \\
& -\alpha_{5}^{3}\left(1+\beta_{5}\right)\left(1+\beta_{6}\right)\left(-1+3 k_{5}+3 \beta_{5}\left(2+\beta_{5}\right)\left(1+k_{5}\right)+\alpha_{5}\left(1+\beta_{6}\right)^{3}\left(\beta_{6}+3 k_{5}\right.\right. \\
& \left.\left.+3 \beta_{5}\left(-1+\beta_{5}+\beta_{5}^{2}+\left(3+\beta_{5}\left(3+\beta_{5}\right)\right) k_{5}\right)+k_{6}+\beta_{6} k_{6}\right)\right)
\end{align*}
$$

## C. Matching Grouping

From the principles of object-image matching, pupil matching, and demagnification matching, we can obtain the following expressions:

RED, AOI, and APE represent the obscurations, back working distance, image telecentricity, reduction in G2, max angle of incidence, and max aperture of optical system, respectively. The error function $F$ reflects the value of the third-order

$$
\left\{\begin{array}{l}
\beta=\beta_{1} \beta_{2} \beta_{3} \beta_{4} \beta_{5} \beta_{6}  \tag{9}\\
y_{2}=\beta_{1} \beta_{2} \beta_{3} \beta_{4} y=\frac{y_{\mathrm{im}}}{\beta_{5} \beta_{6}} \\
u_{5}=\frac{u_{1}}{\beta_{1} \beta_{2} \beta_{3} \beta_{4}} \\
l_{\text {enp }}=-\frac{\beta_{5} l_{5}\left(1+\beta_{6}+\alpha_{5} \beta_{5} \beta_{6}\right)}{1+\beta_{6}+\alpha_{5} \beta_{5} \beta_{6}+\alpha_{5} \beta_{5}^{2} \beta_{6}}=l_{\exp G 1}^{\prime}=\frac{\alpha_{1} \alpha_{2} \alpha_{3} \beta_{1} \beta_{2} \beta_{3}^{2} \beta_{4}^{2} l_{1}}{\alpha_{3}\left(-1+\alpha_{2}-\beta_{3}\right)+\beta_{3}+\beta_{3} \beta_{4}}+l_{5}
\end{array} .\right.
$$

aberrations of the optical system and the system's ability to control these constraints. The error function $F$ can be controlled by setting weights of third-order aberrations and constraints. In this model, the weights of aberration are higher than the weights of constraints. A smaller value of $F$ indicates smaller third-order aberrations in the initial structure, better control of the constraints, and greater potential to achieve high imaging quality. Therefore, the essential approach to solve for the initial structure is dependent on the method used to solve for the minimum value of the error function, and the physical process of solving for the structural parameters of the optical system is thus transformed into a mathematical process to solve for the minimum value of the parameter error function. The main goal is to determine how to solve for the minimum values of the high-dimensional nonlinear parameter equations. In this paper, the particle swarm simulated annealing algorithm is applied to solve for the initial structure of the off-axis six-reflective optical system [19-21]. This algorithm offers the following advantages: fast convergence, an ability to jump out of local minima effectively, and a global optimization algorithm that does not rely on the initial values and is thus suitable for high-dimensional nonlinear optimization problems. As shown in Fig. 4, the main algorithm design process is as follows:

Step 1: Calculate the parameters of the off-axis six-mirror optical system and initialize the positions and velocities of the particles in the population randomly.

Step 2: Set the initial temperature. A larger initial temperature will increase the probability of obtaining a high-quality solution, but the time required will also increase. Therefore, the process for determination of the initial temperature should consider both the calculation efficiency and the optimization quality.

Step 3: Evaluate the fitness of each particle and calculate the individual best value and the global best value, where the individual best value represents the best solution found for each particle, and then find a global value from these best solutions, which is called the global best value solution.

Step 4: Using the simulated annealing algorithm and the particle swarm algorithm, update the position and the velocity of each particle.

Step 5: Based on the fitness value, update the individual best value and the global best value solution for each particle.

Step 6: The cooling down stage. Here, as T decreases, the algorithm becomes stable, and the probability of selection of a poor solution thus decreases. Finally, T drops to the condition required to terminate the iteration.

Step 7: The termination condition. If the termination condition is met (i.e., the error is good enough or the maximum number of cycles has been reached), then exit; otherwise, return to step 3.

Step 8: From the calculation results based on the algorithm, we obtain the initial structural parameters for the off-axis aspheric system with extremely low aberration and a large field of view.

The initial structure of the off-axis six-reflective optical system is solved using the particle swarm simulated annealing algorithm. The main parameters for this algorithm are the number of particles $N=1000$, the learning factor $c_{1}=2.05$, the


Fig. 4. Flow chart for the particle swarm simulated annealing algorithm.


Fig. 5. Error function convergence curve.
learning factor $c_{2}=2.05$, the annealing constant 0.42 , and the maximum number of iterations $M=300$.

Figure 5 shows the variation in the error function with the number of iterations. Each point in the curve is the global best error function value at the specified iteration. The error function


Fig. 6. Schematic diagrams of different initial structures for the off-axis six-reflective aspheric optical system.
decreased after 190 iterations and converged to a low value close to zero. We used the grouping design method based on the combination of spatial ray tracing and aberration correction to obtain several different initial structures. The diagrams of these initial structures are shown in Fig. 6.

## 4. OPTIMIZATION AND PERFORMANCE

To optimize the designs of the initial structures shown above, we first added high-order aspheric coefficients to the conic surfaces

166.67 MM

Fig. 7. Schematic diagram of the optimized structure for the off-axis six-reflective aspheric optical system with extremely low aberration and a large field of view.

Table 1. Specifications of the Off-Axis Six-Reflective Aspheric Optical System with Extremely Low Aberration and a Large Field of View

| Item | Specification |
| :--- | :---: |
| Numerical Aperture | 0.33 |
| Field | $26 \mathrm{~mm} \times 2 \mathrm{~mm}$ arc |
| Demagnification | 4 |
| RMS wavefront error | $0.014 \lambda$ |
| (Z5-Z37) |  |
| Max distortion | 1 nm |
| Max telecentricity error | $0.1^{\circ}$ |
| Max asphere departure | $55 \mu \mathrm{~m}$ |
| Total track | 1426 mm |
| Back working distance | 38 mm |
| Max angle of incident | $24.05^{\circ}$ |

to obtain high imaging quality and also maintained the constraints, causing the optimized result to show a small deviation from the initial structure. Then, we controlled the steps for the disturbances for each variable quantity during the optimization


Fig. 8. Distortion on full image field.


Fig. 9. Wavefront error RMS on full image field.


Fig. 10. Modulation transfer function (MTF).
process (where a step that is too large may skip an extreme value, and a step that is too small may fall into a local minimum).

Using the optimization principles above, we have obtained an off-axis six-reflective optical system design with high imaging quality. Figure 7 shows the best design layout for this off-axis six-mirror optical system, and the specific system parameters are given in Table 1. Figure 8 shows the distribution of distortions on a full image field. Fig. 9 shows the distribution of wavefront error RMS on a full image field. Figure 10 shows the modulation transfer function (MTF) curve, which is close to the diffraction limit.

## 5. CONCLUSION

This paper has proposed a grouping design method based on a combination of spatial ray tracing and aberration correction to enable quick and effective construction of the initial structure of an off-axis six-reflective aspheric optical system. In the construction of the initial structure, both an aberration balance and multi-constraint control were realized. We completed the design of the off-axis six-reflective optical system with suitable engineering feasibility to give an RMS wavefront error of better than $1 / 70 \lambda$ RMS.

The method proposed in this article can be extended to offaxis multi-reflection optical systems. For structures containing more reflective mirrors, this method can still be used to design the initial structures of off-axis multi-reflective optical systems to achieve both an aberration balance and control of multiple constraints. The method can provide a good starting point for design of off-axis multi-reflective optical systems with the potential to optimize these systems to have both extremely low aberration and large fields of view.

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