# **Comparative analysis on phase shifting schemes in planar lightwave circuit devices**

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A comparative analysis on characteristics of the two phase shifting schemes, length difference scheme (LDS) and refractive index difference scheme (RIDS), is carried out on silica based phase shifters designed respectively by the two schemes. Results show that over the wavelength range of 1500-1600 nm, phase shifter designed by LDS possesses higher sensitivity to wavelength, and also higher immunity to waveguide fabrication imperfections, in terms of waveguide geometry, and waveguide refractive index as well; by contrast, phase shifter designed by RIDS has a wider working wavelength range, but it suffers from much higher sensitivity to waveguide fabrication imperfections.

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## 1. Introduction

Since the 1960s, when concept of integrated optics was initially proposed, integrated optical technologies have been developing rapidly, motivated by their potentially extensive application in optical information networks. Up to the latest years, integrated optical device fabrication technologies have been brought out from laboratory and into the realm of practical application, with coupler, modulator, wavelength division multiplexer, among other integrated optical elements, being extensively applied in optical communication and optical sensing networks.

Phase shifter, an integrated optical structure to generate phase difference between optical waveguide branches, is frequently applied in integrated optical devices based on interference principle, e.g. Mach-Zehnder interferometer [1], optical isolator [2,3], arrayed waveguide grating (AWG) [4,5], and optical mixer [6,7] as well. There exist two basic phase shifting schemes between optical waveguide branches: one is length difference scheme (hereafter named LDS), implemented by introducing waveguide length difference between waveguide branches of the same effective refractive index, as that widely applied in arrayed waveguides in AWG; the other is refractive index difference scheme (hereafter named RIDS), implemented by introducing effective refractive index difference between waveguide branches of the same geometrical length. The two schemes produce optical phase shifters with different characteristics, which could have a considerable impact on integrated optical devices that involves phase shifter in their performance, selection of phase shifting schemes thus plays a key role in optimizing these integrated optical devices. Difference between the two phase shifting schemes in their

characteristics has been noticed by researchers in the field of integrated photonics. Pierre Labeye and his team deem that phase shifter designed by RIDS possesses a wider operating wavelength range [8]; Loridat and his colleague have attempt to design achromatic optical device by optimizing waveguide structure [9]. On the other hand, fabrication imperfection is usually inevitable in the process of optical device production, and thus phase shifter of different layout may possess different fabrication imperfection sensitivity. However, there is few article that gives a quantitative analysis on differences between the two phase shifting schemes, and that, no investigation on fabrication imperfection sensitivity of phase shifting schemes has been reported.

In this paper, a comparative analysis is conducted on silica based phase shifters designed respectively by LDS and RIDS, in aspects of wavelength dependence, and immunity to fabrication imperfections as well.

## 2. Theory and method

The two phase shifting schemes are schematically presented in Fig.1, where Fig.1 (a) represents LDS, phase difference between the two waveguide branches (WG1-1 and WG1-2) being introduced by increasing length of WG1-2; while Fig.1 (b) represents RIDS, phase difference between WG2-1 and WG2-2 being introduced by locally increasing width of WG2-2.

For a phase shifter designed by RIDS, phase difference between the two waveguide branches, WG1-1 and WG1-2 shown in Fig. 1(a), can be expressed as

$$\Phi_1(W_0,\lambda) = \frac{2\pi}{\lambda} \cdot N_{eff}(W_0,\lambda) \cdot \Delta L$$
(1)

where  $\lambda$  is wavelength;  $W_0$  is waveguide width;  $N_{eff}$  is waveguide effective refractive index;  $\Delta L$  is length of LDS phase shifter.

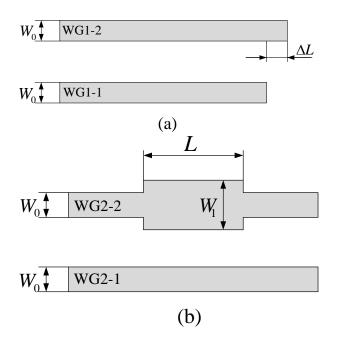


Fig. 1. Schematic diagram of the two phase shifter schemes, (a) and (b) representing LDS and RIDS respectively

Table 1. Main parameters of silica based integrated optical phase shifters

Parameters	Value
Refractive index contrast	0.45%
Waveguide core height	6.5 μm
Waveguide core width $W_0$	6.0~6.9 μm
Waveguide core width $W_1$	6.1~7.0 μm

For a phase shifter designed by LDS with phase difference  $\Phi_1(W_0, \lambda_0)$  at central wavelength  $\lambda_0$ , its phase difference dependence on operating wavelength  $\lambda$  can be written as follow.

$$\Phi_1(W_0,\lambda) = \Phi_1(W_0,\lambda_0) \cdot \frac{\lambda_0}{\lambda} \cdot \frac{N_{eff}(W_0,\lambda)}{N_{eff}(W_0,\lambda_0)}$$
(2)

By contrast, for a phase shifter designed by RIDS, phase difference between the two waveguide branches, can be expressed as:

$$\Phi_{2}(W_{0}, W_{1}, \lambda) = \frac{2\pi}{\lambda} \cdot \left[ N_{eff}(W_{1}, \lambda) - N_{eff}(W_{0}, \lambda) \right] \cdot L$$
(3)

where  $W_1$  is width of the phase shifter waveguide; *L* is length of the phase shifter.

For a phase shifter designed by RIDS with phase difference  $\Phi_2(W_0, W_1, \lambda_0)$  at central wavelength  $\lambda_0$ , its phase difference dependence on operating wavelength  $\lambda$  can be written as

$$\Phi_{2}(W_{0}, W_{1}, \lambda) = \Phi_{2}(W_{0}, W_{1}, \lambda_{0}) \cdot \frac{\lambda_{0}}{\lambda} \cdot \frac{N_{eff}(W_{1}, \lambda) - N_{eff}(W_{0}, \lambda)}{N_{eff}(W_{1}, \lambda_{0}) - N_{eff}(W_{0}, \lambda_{0})}$$
(4)

Silica based optical channel waveguide, with its core doped with GeO<sub>2</sub>, is selected for design of phase shifters, main parameters of waveguide being listed in Table 1. These parameters ensure waveguide working in singlemode regime over wavelength range of 1500-1600nm. Refractive index of waveguide materials in the wavelength range is calculated by the well-proved Sellmeier dispersion equation (11).

$$n^{2}(\lambda) - 1 = \sum_{i=1}^{3} \frac{[SA_{i} + X(GA_{i} - SA_{i})]\lambda^{2}}{\lambda^{2} - [SI_{i} + X(GI_{i} - SI_{i})]}$$
(5)

where  $SA_i$ ,  $SI_i$ ,  $GA_i$  and  $GI_i$  are Sellmeier coefficients for the SiO<sub>2</sub> and GeO<sub>2</sub>, respectively; *x* is GeO<sub>2</sub> concentration in mol%.

Effective refractive index of optical channel waveguides, with waveguide width varies in the range of 6.0~7.0  $\mu m$ , are obtained by numerically solving the Maxwell equations using a homemade program based on semi-vectorial method. Dependence of waveguide effective refractive index (TE mode) on waveguide width at different wavelength is shown in Fig. 2.

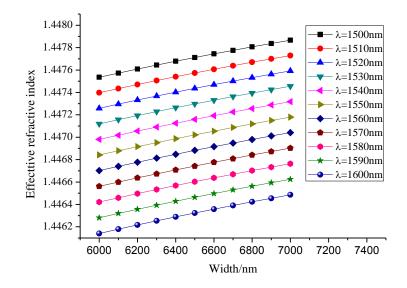


Fig. 2. Dependence of waveguide effective refractive index (TE mode) on waveguide width (color online)

## 3. Results and discussion

#### 3.1. Wavelength dependence

For convenience of comparison, a factor of  $\xi$  is defined, which represents normalized deviation of phase difference at operating wavelength  $\lambda$  from its nominal value (at central wavelength  $\lambda_0$ ).

$$\xi_{LDS} \equiv \frac{\Phi_1(W_0, \lambda) - \Phi_1(W_0, \lambda_0)}{\Phi_1(W_0, \lambda_0)} \tag{6}$$

$$\xi_{RIDS} \equiv \frac{\Phi_2(W_0, W_1, \lambda) - \Phi_2(W_0, W_1, \lambda_0)}{\Phi_2(W_0, W_1, \lambda_0)}$$
(7)

Dependence of the  $\xi$  factor on wavelength and for phase shifters designed respectively by LDS and RIDS are calculated and shown in Fig. 3. Fig. 3 (a) gives the  $\xi$ factor of a phase shifter designed by LDS with  $W_0 = 6.0 \mu m$ , in contrast with five phase shifters designed by RIDS with  $W_0 = 6.0 \mu m$ , and  $W_1$  increases from  $6.2 \mu m$  to  $7.0 \mu m$  with  $0.2 \mu m$  intervals. It can be seen that  $\xi$  factor of phase shifters designed by both LDS and RIDS decreases linearly with wavelength in the range of 1500-1600nm; compared with RIDS devices, LDS phase shifter is more sensitive to wavelength: slope of  $\xi$  with respect to wavelength of LDS phase shifter is  $-6.55 \times 10^{-4}/nm$ ; while for those RIDS phase shifters, their slope varies between  $-2.90 \times 10^{-4} \sim -2.42 \times 10^{-4}/nm$ .

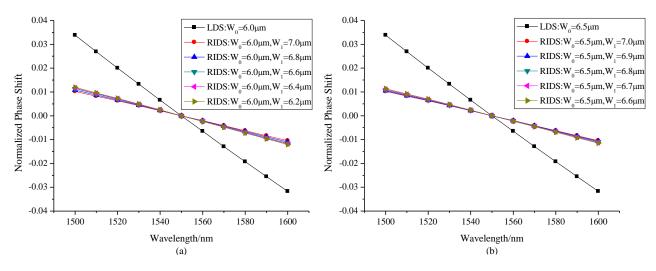


Fig. 3. Wavelength dependence comparison of the two phase shifters (color online)

Fig. 3(b) shows comparison of  $\xi$  factor between the two phase shifting schemes with alternative parameters, in which LDS phase shifter is of  $W_0 = 6.5 \mu m$ , five RIDS phase shifters are of  $W_0 = 6.5 \mu m$ , and  $W_1$  increasing from  $6.6 \mu m$  to  $7.0 \mu m$  with  $0.1 \mu m$  intervals. It can be seen from the similarities between Fig. 3(b) and Fig. 3(a) that while  $W_0$  takes different values, characteristics of phase shift deviation behavior maintain. As  $W_0$  and  $W_1$  varies

over broader scope, slopes of  $\xi$  factor with respect to wavelength are listed in Table 2, from which it can be seen that the difference between the two phase shifting schemes in their wavelength dependence behavior is distinguishable.

Difference between phase shifters designed respectively by RIDS and LDS in their wavelength dependence behavior can be illustrated on basis of the waveguide dispersion characteristics.

$W_1$	LDS	RIDS									
W <sub>0</sub>		6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
6.0	-6.55	-2.97	-2.90	-2.83	-2.77	-2.71	-2.65	-2.59	-2.53	-2.47	-2.42
6.1	-6.55		-2.83	-2.76	-2.7	-2.64	-2.58	-2.52	-2.46	-2.4	-2.35
6.2	-6.55			-2.69	-2.63	-2.57	-2.51	-2.45	-2.39	-2.33	-2.28
6.3	-6.55				-2.57	-2.50	-2.45	-2.38	-2.33	-2.27	-2.22
6.4	-6.55					-2.44	-2.38	-2.32	-2.26	-2.21	-2.15
6.5	-6.55						-2.32	-2.26	-2.2	-2.14	-2.09
6.6	-6.55							-2.19	-2.14	-2.08	-2.03
6.7	-6.55								-2.08	-2.02	-1.98
6.8	-6.55									-1.96	-1.92
6.9	-6.55										-1.88
7.0	-6.55										

*Table 2. Slope of*  $\xi$  *with respect to wavelength* (×10<sup>-4</sup>*nm*<sup>-1</sup>)

For the case of LDS, by substituting Eq. (2) into Eq. (6) and differentiate  $\xi$  with respect to  $\lambda$ , one can obtain slope of normalized phase shift deviation with respect to  $\lambda$ . Its value at  $\lambda_0$  can be given as:

$$\frac{\partial \xi_{LDS}}{\partial \lambda}\Big|_{\lambda=\lambda_0} ; \quad -\frac{1}{\lambda_0} + \frac{1}{N_{eff}(W_0,\lambda_0)} \frac{\partial N_{eff}(W_0,\lambda)}{\partial \lambda}\Big|_{\lambda=\lambda_0} (8)$$

where the first term on the right side of Eq. (8), with its value  $-6.45 \times 10^{-4} nm^{-1}$ , represents contribution of operating

wavelength to the phase difference error. The second term on the right side is normalized effective refractive index wavelength dispersion. According to data shown in Fig. 2, this term varies between  $-1.38 \times 10^{-5} nm^{-1}$  and  $-1.39 \times 10^{-5} nm^{-1}$ , while  $W_0$  increases from  $6.0 \mu m$  to  $7.0 \mu m$ . Therefore,  $\partial [\xi_{LDS}] / \partial \lambda$  at  $\lambda_0$  is about  $-6.59 \times 10^{-4} nm^{-1}$ , in close agreement with the corresponding values shown in Tab. 3.

While for the RIDS,  $\partial [\xi_{RIDS}] / \partial \lambda$  at  $\lambda_0$  can be derived from Eq. (4) and Eq (7):

$$\frac{\partial \xi_{RIDS}}{\partial \lambda}\Big|_{\lambda=\lambda_0} = -\frac{1}{\lambda_0} + \frac{1}{N_{eff}(W_1,\lambda_0) - N_{eff}(W_0,\lambda_0)} \frac{\partial \Big[N_{eff}(W_1,\lambda) - N_{eff}(W_0,\lambda)\Big]}{\partial \lambda}\Big|_{\lambda=\lambda_0}$$
(9)

With the value of  $W_1 - W_0$  sufficiently small, Eq. (8) is further approximated as:

$$\frac{\partial \xi_{RIDS}}{\partial \lambda} \bigg|_{\substack{\lambda = \lambda_0 \\ W = W_0}}; \quad -\frac{1}{\lambda_0} + \left[ \frac{\partial N_{eff}(W, \lambda)}{\partial W} \bigg|_{\substack{\lambda = \lambda_0 \\ W = W_0}} \right]^{-1} \left[ \frac{\partial^2 N_{eff}(W, \lambda)}{\partial W \partial \lambda} \bigg|_{\substack{\lambda = \lambda_0 \\ W = W_0}} \right]$$
(10)

where *W* is the width of the waveguide. The first term on the right side of Eq. (9), with its value  $-6.45 \times 10^{-4} nm^{-1}$ , represents contribution of operating wavelength to the phase difference error.  $\partial N_{eff}(W, \lambda) / \partial W$  in the second term

is slope of the curves in Fig. 2, as shown in Fig. 4, and  $\partial^2 N_{\text{eff}}(W,\lambda)/\partial W\partial\lambda$  is the slope of the curve in this figure. Using data shown in Fig. 2, the second term in Eq. (10) can be estimated to be  $4.06 \times 10^{-4} nm^{-1}$ . As a result,  $\partial [\xi_{RIDS}]/\partial\lambda$ 

From the above comparison, it can be seen that there are two factors that influence phase difference wavelength dependence behavior, for phase shifters designed by both LDS and RIDS. Operating wavelength deviation is one of primary importance, as can be seen from the relatively large value of the first terms on the right side of Eq. (8) and Eq. (10), compared with the second terms in the equations. The other factor is optical waveguide dispersion characteristics. For phase shifter designed by LDS, since waveguide refractive index decreases with increasing wavelength, the waveguide dispersion slightly enhanced dependence of phase difference on wavelength, as can be seen that second term on the right side of Eq. (8) have the same minus sign as the first term. While for phase shifter designed by the RIDS, with increasing of the waveguide width, increment of the waveguide effective refractive index is larger at longer wavelength, as shown in Fig. 4. Therefore the second term on the right side of Eq. (10) has opposite sign with respect to the first term, in other words, waveguide dispersion compensate part of contribution from operating wavelength. As a result, wavelength dependence of phase shifters designed by the RIDS is significantly improved.

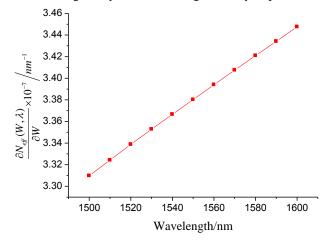


Fig. 4. Dependence of  $\partial N_{eff}(W, \lambda) / \partial W$  on wavelength (color online)

#### 3.2. Immunity to fabrication imperfections

Considering ubiquity of integrated optical waveguide device fabrication imperfections, in aspects of waveguide geometry (width and height of waveguide cross section) and waveguide refractive index profile, their influence on phase shifters are simulated for phase shifters designed respectively by the two schemes. Fig. 5 depicts simulated dependence of normalized phase deviation on fabrication imperfections. For the devices designed respectively by the two phase shifting schemes, the LDS phase shifter is of width  $W_0 = 6.5 \mu m$ , and the RIDS phase shifter is of

 $W_0 = 6.5 \mu m$  and  $W_1 = 7.0 \mu m$ . From this figure it can be seen that the phase shifter designed by LDS possess much higher immunity to waveguide fabrication imperfections compared with that designed by RIDS, in aspects of both waveguide core geometry and refractive index.

Difference between phase shifters designed respectively by the two schemes in their sensitivity to waveguide fabrication imperfections can be qualitatively illustrated, by taking waveguide width error as an example. For the sake of convenience, a factor of  $\eta$  is defined, which denotes the normalized deviation of phase shift from its nominal value, due to presence of waveguide width deviation  $\Delta W$ .

$$\eta_{LDS} = \frac{\Phi_1(W_0 + \Delta W, \lambda) - \Phi_1(W_0, \lambda)}{\Phi_1(W_0, \lambda)} \tag{11}$$

$$\eta_{RDS} \equiv \frac{\Phi_2(W_0 + \Delta W, W_1 + \Delta W, \lambda) - \Phi_2(W_0, W_1, \lambda)}{\Phi_2(W_0, W_1, \lambda)} \quad (12)$$

For a LDS phase shifter with a nominal shift value  $\Phi_1(W_0, \lambda_0)$  at  $\lambda_0$ , according Eq. (1), its length can be given as

$$\Delta L = \frac{\lambda}{2\pi} \cdot \frac{\Phi_1(W_0, \lambda_0)}{N_{eff}(W_0, \lambda_0)}$$
(13)

With presence of waveguide width deviation  $\Delta W$ , its phase shift can be written as

$$\Phi_1(W_0 + \Delta W, \lambda) = \frac{N_{eff}(W_0 + \Delta W, \lambda)}{N_{eff}(W_0, \lambda_0)} \Phi_1(W_0, \lambda_0)$$
(14)

As a result, slope of  $\eta_{LDS}$  with respect to waveguide width can be given as:

$$\frac{\partial \eta_{LDS}}{\partial W_0} = \frac{1}{N_{eff}(W_0, \lambda)} \frac{\partial N_{eff}(W_0, \lambda)}{\partial W_0}$$
(15)

By contrast, for a phase shifter designed by RIDS at  $\lambda_0$  with phase difference  $\Phi_2(W_0, W_1, \lambda_0)$ , its length is

$$L = \frac{\lambda}{2\pi} \cdot \frac{\Phi_2(W_0, W_1, \lambda)}{\left[N_{eff}(W_1, \lambda) - N_{eff}(W_0, \lambda)\right]}$$
(16)

With presence of waveguide width deviation  $\Delta W$  for waveguide with width of both  $W_0$  and  $W_1$ , phase difference can be given as

$$\Phi_2(W_0 + \Delta W, W + \Delta W, \lambda) = \frac{N_{eff}(W_1 + \Delta W, \lambda) - N_{eff}(W_0 + \Delta W, \lambda)}{N_{eff}(W_1, \lambda) - N_{eff}(W_0, \lambda)} \cdot \Phi_2(W_0, W_1, \lambda)$$
(17)

Slope of  $\eta_{RIDS}$  with respect with waveguide width is

$$\frac{\partial \eta_{RIDS}}{\partial W_0} = \frac{1}{N_{eff}(W_1,\lambda) - N_{eff}(W_0,\lambda)} \cdot \left[ \frac{\partial N_{eff}(W,\lambda)}{\partial W} \Big|_{W=W_1} - \frac{\partial N_{eff}(W,\lambda)}{\partial W} \Big|_{W=W_0} \right]$$
(18)

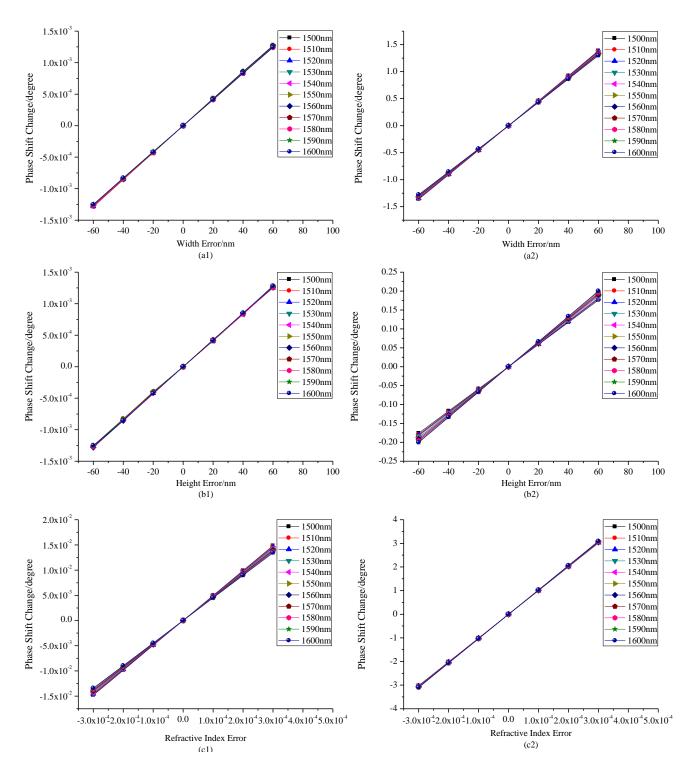


Fig. 5. Effect of fabrication imperfections on phase shifter designed respectively by LDS and RIDS. Where (a1), (b1), (c1) show LDS phase shifter with presence of fabrication imperfection in waveguide core width, height and refractive index respectively; (a2), (b2), (c2) show the corresponding case for RIDS phase shifter (color online)

According to Eq. (15) and Eq. (18), slope of  $\partial \eta_{LDS}$  and  $\partial \eta_{RDS}$  are given in Table 3, with  $W_0$  and  $W_1$  varies in the scope of between  $6.0 \mu m$  and  $7.0 \mu m$ . It can be seen clearly

that over the scope of waveguide width variation, phase shifters designed by LDS possess much higher immunity to waveguide fabrication imperfections than that designed by RIDS.

$W_1$	LDS	RIDS									
$W_0$		6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
6.0	3.81	-2264	-2390	-2428	-2445	-2455	-2459	-2461	-2461	-2460	-2437
6.1	3.73		-2519	-2513	-2508	-2505	-2501	-2496	-2492	-2487	-2459
6.2	3.64			-2507	-2503	-2500	-2496	-2491	-2488	-2482	-2450
6.3	3.55				-2499	-2497	-2493	-2487	-2484	-2478	-2442
6.4	3.46					-2496	-2490	-2483	-2480	-2474	-2431
6.5	3.37						-2484	-2477	-2474	-2468	-2417
6.6	3.29							-2469	-2469	-2462	-2400
6.7	3.21								-2468	-2458	-2375
6.8	3.13									-2449	-2327
6.9	3.06										-2203
7.0	2.99										

Table 3. The slope of normalized phase shift with waveguide width at wavelength 1550nm ( $\times 10^{-7}$  nm<sup>-1</sup>)

### 4. Conclusion

Characteristics of phase shifters designed respectively by LDS and RIDS are comparatively analyzed, in aspects of wavelength dependence as well as fabrication imperfections immunity. Results show that in wavelength range of 1500-1600nm, phase shifter designed by LDS is more sensitive to wavelength, and it possess much higher immunity to waveguide fabrication imperfections; as a contrast, phase shifter designed by RIDS have a higher operating wavelength range, but it is more sensitive to waveguide fabrication imperfections.

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