# Analytical design method of three-mirror anastigmatic telescope with mirror spacings as free design parameters

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**Abstract.** We present an analytical design method for the three-mirror anastigmatic (TMA) telescope with mirror spacings as the free design parameter. After the optical designer determines, the system focal length and the mirror spacings according to the design requirements, the design solutions of all TMA telescopes that meet the conditions can be obtained directly according to the formulas for the mirror radius and conic constants derived. The method here can predetermine the mirror position and system envelope size before design, and can quickly give all design solutions. We give a design example using the method and compare and discuss all design solutions of TMA. © 2020 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.JATIS.6.4.044007]

**Keywords:** optical design; telescope; three-mirror anastigmat; mirror spacings; analytical design.

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# 1 Introduction

The three-mirror anastigmat (TMA) can simultaneously correct the four primary aberrations of spherical aberration, coma, astigmatism, and field curvature, so it can achieve diffraction-limited imaging quality in a larger field of view and a compact design. Since the design concept of the TMA was proposed in the 1970s,<sup>1,2</sup> its application in the field of astronomical exploration and remote sensing has become more and more extensive. The Vera C. Rubin Observatory's telescope (formerly known as Large Synoptic Survey Telescope) is a modified TMA of Paul–Baker design.<sup>3,4</sup> The next major space-borne observatory, the James Webb Space Telescope, is a 6.6-m field-biased, obscured, TMA telescope.<sup>5</sup> The SuperNova/Acceleration Probe (SNAP) telescope is also a TMA design, which is designed to precisely measure the expansion history of the universe.<sup>6</sup>

For the optical design of the TMA, Korsch<sup>2</sup> used the transverse magnification of the secondary mirror (SM) and tertiary mirror (TM), and the pupil magnification of the SM as the free design parameters and developed a general set of solutions in 1972. Robb<sup>7</sup> deduced the formulas for the radius of curvature, the spacings, and the conic constants using the *f*-number of the primary mirror (PM), the combined focal length of the PM and SM, and the distance between the focal point of the PM and SM systems and the vertices of the PM and TM as free design parameters, and the numerical solution method is used to solve the structural parameters. Lee and Yu<sup>8</sup> proposed the solution of the design parameters of the TMA system with the height of the marginal rays on the SM, the PM and SM spacing, and the SM and TM spacing as free design parameters. In addition, some people use the blocking ratio of the PM and SM and the transverse magnification of the PM and SM as free design parameters.<sup>9</sup>

The above-mentioned design method of the TMA optical system has the problem that the free design parameters are not intuitive structural parameters (such as mirror spacings). This will

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cause the optical designer to be unable to directly define the position of the mirrors and the envelope size of the optical system before optical design. Therefore, for the convenience of design, Terebizh<sup>10</sup> proposed an analytical algorithm with mirror diameters and the distance between the SM and TM as free design parameters.

The existing methods can only solve the design parameters of the positive-negative-positive power combined TMA system. In fact, there are additional forms of optical power combination (such as positive-positive-negative optical power combination) TMA system. Korsch<sup>11</sup> proposed a more general TMA system design method that can solve multiple combinations of optical powers, but this method uses the paraxial ray-height ratios of mirrors as the design input.

In view of the above problems, we propose an analytical design method for the TMA telescope with the mirror spacings as the free design parameters. This method can predetermine the position of each mirror and image surface before design, which facilitates the design of TMA systems with mirror position requirements (for example, the PM and TM are on the same substrate<sup>12</sup>). It is convenient to define the envelope size of the optical system in advance. The optical parameter calculation formula in this paper is completely analytical, and the free design parameters can be directly substituted into the formula to calculate the complete set of optical design parameters. The entire design process is very simple, and it is convenient for quickly giving all the design solutions of the TMA telescopes that meet the requirements, which is helpful for the rapid design of the TMA systems.

### 2 Analytical Design Process of TMA Telescopes

The TMA telescopes have a total of nine design parameters, which are the radius of curvature, the mirror spacings, and the conic constants of the PM, SM, and TM. In this method, the three mirror spacings are used as the free design parameters, and the radius of curvature and the conic constants of the three mirrors must be solved.

According to the first-order parameter conditions and aberration elimination conditions of the TMA optical system, the solution of the optical design parameters must meet the following six conditions:

- a. Focal length condition: the effective focal length is a certain value.
- b. Imaging conditions: the height of the paraxial marginal ray on the image surface should be zero.
- c. Spherical aberration condition: the third-order spherical aberration of the system is zero.
- d. Coma aberration condition: the third-order coma of the system is zero.
- e. Astigmatism condition: the third-order astigmatism of the system is zero.
- f. Field curvature condition: the third-order field curvature of the system is zero.

According to Seidel's aberration theory, among the above six conditions, the conditions a, b, and f are only related to the surface radius of curvature and the mirror spacings. In addition to the radius of curvature, the conditions c, d, and e are additionally related to the conic constants. Therefore, the conditions a, b, and f can be used to first solve the radius of curvature of the mirrors, and then the conditions c, d, and e can be used to solve the conic constants.

#### 2.1 Solution of the Radius of Curvature

Using paraxial marginal ray tracing Eqs. (1) and (2), condition a and b can be expressed as two equations about optical structure parameters. The sign convention in this paper is the same as that in Ref. 13.

$$u_{j}' = (n_{j}u_{j} - y_{j}\phi_{j})/n_{j}', \tag{1}$$

$$y_{i+1} = y_i + u_i' d_i,$$
 (2)

where  $u_j/u_j'$  is the slope angle of the marginal ray in the local object/image space of the surface j,  $n_j/n_j'$  is the refractive index of the local object/image space of the surface j,  $d_j$  is the spacing

between surface j and surface j + 1,  $y_j$  is the height of the marginal ray on surface j, and  $\phi_j$  is the optical power of surface j, which can be calculated by Eq. (3).

$$\phi_j = (n_j' - n_j)/r_j, \tag{3}$$

where  $r_j$  is the radius of curvature of surface *j*. The subscript *j* represents the PM, SM, and TM for the TMA telescope, respectively, when it equals to 1, 2, and 3.

According to the equation for calculating the focal length of the TMA system as shown as

$$f = -y_1/u_3',$$
 (4)

the condition a can be expressed in the form shown as

$$f_{\rm eff} = \frac{r_1 r_2 r_3}{(8d_1 d_2 - 4d_1 r_2 - 4d_2 r_1 + 4d_1 r_3 + 4d_2 r_2 + 2r_1 r_2 - 2r_1 r_3 + 2r_2 r_3)}.$$
 (5)

At the same time, the condition b can be expressed as

$$8d_{1}d_{2}d_{3} - 4d_{1}d_{2}r_{3} - 4d_{1}d_{3}r_{2} - 4d_{2}d_{3}r_{1} + 4d_{1}d_{3}r_{3} + 4d_{2}d_{3}r_{2} + 2d_{1}r_{2}r_{3} + 2d_{2}r_{1}r_{3} + 2d_{3}r_{1}r_{2} - 2d_{2}r_{2}r_{3} - 2d_{3}r_{1}r_{3} + 2d_{3}r_{2}r_{3} - r_{1}r_{2}r_{3} = 0.$$
(6)

When the third-order field curvature of the TMA system is zero, it needs to meet

$$\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_3} = 0. \tag{7}$$

Equation (7) is the mathematical expression of the condition f.

Simultaneously Eqs. (5), (6), and (7), solving equations, we can get two sets of analytical solutions of radius of curvature for the three mirrors, as shown in Eqs. (8) and (9).

The first set of solutions is

$$r_{1} = \frac{d_{1}f(d_{2} - 2d_{3} + 2f) - \sqrt{d_{1}d_{2}f(4d_{3}^{3} + f(d_{1}(d_{2} - 4d_{3}) + 4(d_{2} - 2d_{3})d_{3} + 4d_{3}f))}}{d_{3}^{2} + f(d_{2} - 2d_{3} + f)},$$

$$r_{2} = \frac{-d_{1}d_{2}f - \sqrt{d_{1}d_{2}f(d_{1}d_{2}f + 4d_{3}((d_{3} - f)^{2} + (-d_{1} + d_{2})f))}}{(d_{3} - f)^{2} + (-d_{1} + d_{2})f)},$$

$$r_{3} = \frac{d_{1}d_{2}f - 2d_{2}d_{3}(d_{2} - d_{3} + f) + \sqrt{d_{1}d_{2}f(d_{1}d_{2}f + 4d_{3}((d_{3} - f)^{2} + (-d_{1} + d_{2})f))}}{d_{1}f - (d_{2} - d_{3} + f)^{2}}.$$
(8)

The second set of solutions is

$$r_{1} = \frac{d_{1}f(d_{2} - 2d_{3} + 2f) + \sqrt{d_{1}d_{2}f(4d_{3}^{3} + f(d_{1}(d_{2} - 4d_{3}) + 4(d_{2} - 2d_{3})d_{3} + 4d_{3}f))}}{d_{3}^{2} + f(d_{2} - 2d_{3} + f)},$$

$$r_{2} = \frac{-d_{1}d_{2}f + \sqrt{d_{1}d_{2}f(d_{1}d_{2}f + 4d_{3}((d_{3} - f)^{2} + (-d_{1} + d_{2})f))}}{(d_{3} - f)^{2} + (-d_{1} + d_{2})f)},$$

$$r_{3} = \frac{d_{1}d_{2}f - 2d_{2}d_{3}(d_{2} - d_{3} + f) - \sqrt{d_{1}d_{2}f(d_{1}d_{2}f + 4d_{3}((d_{3} - f)^{2} + (-d_{1} + d_{2})f))}}{d_{1}f - (d_{2} - d_{3} + f)^{2}}.$$
(9)

#### 2.2 Solution of Conic Constants

The third-order spherical aberration, third-order coma, and third-order astigmatism of the TMA system can be calculated using the Seidel formula and expressed as a function of the structural parameters.

Therefore, the conditions c, d, and e can be expressed as

Spherical aberration: 
$$S_I(k_1, k_2, k_3, d_1, d_2, r_1, r_2, r_3, y_1) = 0,$$
 (10)

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Coma: 
$$S_{II}(k_1, k_2, k_3, d_1, d_2, r_1, r_2, r_3, s_{pr1}, u_{pr1}, y_1) = 0,$$
 (11)

Astigmatism: 
$$S_{III}(k_1, k_2, k_3, d_1, d_2, r_1, r_2, r_3, s_{pr1}, u_{pr1}, y_1) = 0,$$
 (12)

where  $k_j$  is the conic constant of surface j,  $s_{pr1}$  is the distance from the entrance pupil to the PM,  $u_{pr1}$  is the slope angle of the chief ray, that is, the half angle of view, and  $y_1$  is the height of the marginal ray on the PM, which is the radius of the entrance pupil. Equations (10), (11), and (12) are all linear equations about conic constants, and their analytical solutions are

$$k_{1} = -\frac{\begin{pmatrix} r_{3}^{2}(-r_{2}(2d_{1}^{2}(d_{2}+2r_{1})-4d_{1}r_{1}(d_{2}+r_{1})+r_{1}^{2}(2d_{2}+r_{1}))\\ +r_{2}^{2}(d_{1}-r_{1})(d_{1}-d_{2}-r_{1})+d_{2}r_{1}(r_{1}-2d_{1})^{2})\\ +r_{1}r_{3}(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})^{2}\\ +d_{2}r_{1}(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})^{2}\\ +d_{2}r_{1}(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})^{2}\\ +d_{2}r_{1}(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})^{2}\\ +d_{2}r_{1}(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})^{2}\\ +d_{2}r_{1}(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})^{2}\\ +r_{3}(2d_{1}-d_{2})(r_{1}-2d_{1})^{2}+d_{1}d_{2}(r_{1}-2d_{1})^{2}\\ +r_{3}(2d_{1}+r_{2})(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})^{2}\\ +(2d_{1}d_{2}-d_{1}r_{2}+d_{2}r_{2})(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})^{2}\\ +(2d_{1}d_{2}-d_{1}r_{2}+2d_{2}r_{2})(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})) \cdot\\ \\ k_{3} = \frac{\begin{pmatrix} -r_{3}(r_{2}r_{3}(-2d_{1}^{2}+d_{1}(r_{1}-r_{2})+r_{1}r_{2})\\ +(4d_{1}d_{2}-d_{1}r_{2}+2d_{2}r_{2})(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})) \cdot\\ \\ (2d_{1}(2d_{2}-r_{2}+r_{3})+2d_{2}(r_{2}-r_{1})+r_{1}r_{2}-r_{1}r_{3}+r_{2}r_{3})\\ d_{2}(2d_{1}d_{2}-d_{1}r_{2}+d_{2}r_{2})(4d_{1}d_{2}-2d_{1}r_{2}-2d_{2}r_{1}+2d_{2}r_{2}+r_{1}r_{2})^{2} -1. \quad (13)$$

It can be seen from Eq. (13) that the position of the stop has nothing to do with the calculation result of the conic constants of the TMA system.

### 2.3 Design Process of TMA Telescopes

The design process of the TMA telescope is as follows:

- Determine the focal length of the system according to the application requirements. If one wants to design a TMA system with an intermediate image plane, the focal length should be set to a positive value; if one wants to design a TMA system without an intermediate image plane, the focal length should be set to a negative value.
- 2. According to the requirements of the envelope size of the system or the requirements of the mirror position, the values of the mirror spacings  $d_1$ ,  $d_2$ , and  $d_3$  are determined.
- 3. Calculate the radius of curvature of the mirrors according to Eqs. (8) and (9), and two sets of solution are obtained, each with a specific combination of optical powers.
- 4. According to the calculation result of the radius of curvature, the conic constants of the mirrors are calculated according to Eq. (13). The solution of each set of radii of curvatures obtained in step 3 corresponds to a solution of a set of conic constants. Therefore, when the focal lengths of the system are positive and negative, respectively, the optical design parameters of four sets of TMA telescopes with different power combinations are obtained.

# **3 Design Examples**

A TMA telescope design example is given below. The specific design constraints are:

Entrance pupil diameter: 125 mm, focal length:  $\pm 500$  mm,

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*d*<sub>1</sub>: -200 mm, *d*<sub>2</sub>: 200 mm, *d*<sub>3</sub>: -200 mm.

According to the design method in this paper, the design parameters of four groups of optical systems as shown in Tables 1–4 can be calculated. The third-order aberration coefficients of each surface are also given in the tables, which are calculated at a field angle of paraxial image height of 1 mm, at a wavelength of 587.5618 nm. These four groups of optical systems simultaneously satisfy the elimination of third-order spherical aberration, coma, astigmatism, and field curvature. Table 1 shows the telescope parameters of +500-mm focal length, positive-negative-positive-positive-negative power combination form; Table 2 gives the telescope parameters of +500-mm focal length, positive-negative parameters of -500-mm focal length, negative-positive-negative power combination form; Table 3 shows the telescope parameters of -500-mm focal length, negative-positive-negative power combination form; Table 4

 Table 1
 Telescope parameters of +500-mm focal length, positive-negative-positive power combination.

Surface name	Surface type	Radius of curvature (mm)	Conic constant	Thickness (mm)	W <sub>040</sub> (waves)	W <sub>131</sub> (waves)	W <sub>220</sub> (waves)	W <sub>222</sub> (waves)
PM	Conic	-452.471	-0.859	-200	4.060	2.078	0.000	0.038
SM	Conic	-126.026	-6.896	200	-1.819	-0.868	0.077	0.019
ТМ	Conic	-174.679	-0.191	-200	-2.241	-1.210	-0.077	-0.057
Image <sup>a</sup>	Flat	Infinity	_	_	0.000	0.000	0.000	0.000

<sup>a</sup>The aberration coefficients of the image plane are the sum of the aberration coefficients of all surfaces in the system, similarly hereinafter.

Table 2Telescope parameters of +500-mm focal length, positive-positive-negative powercombination.

Surface name	Surface type	Radius of curvature (mm)	Conic constant	Thickness (mm)	W <sub>040</sub> (waves)	W <sub>131</sub> (waves)	W <sub>220</sub> (waves)	W <sub>222</sub> (waves)
PM	Conic	-89.902	-0.661	-200	3031.906	102.821	0.000	0.296
SM	Conic	183.997	-0.047	200	-1863.932	-68.360	-0.114	-0.084
ТМ	Conic	60.393	-0.147	-200	-1167.974	-34.461	0.114	-0.212
Image	Flat	Infinity		_	0.000	0.000	0.000	0.000

 Table 3
 Telescope parameters of -500 mm focal length, negative-positive-negative power combination.

Surface name	Surface type	Radius of curvature (mm)	Conic constant	Thickness (mm)	W <sub>040</sub> (waves)	W <sub>131</sub> (waves)	W <sub>220</sub> (waves)	W <sub>222</sub> (waves)
PM	Conic	8647.580	10223.338	-200	-102.650	-0.011	0.000	-0.003
SM	Conic	604.325	2.941	200	130.124	1.026	-0.004	0.037
ТМ	Conic	649.731	27.289	-200	-27.474	-1.015	0.004	-0.034
Image	Flat	Infinity	_	_	0.000	0.000	0.000	0.000

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**Table 4** Telescope parameters of -500 mm focal length, positive-negative-positive power combination.

Surface name	Surface type	Radius of curvature (mm)	Conic constant	Thickness (mm)	W <sub>040</sub> (waves)	W <sub>131</sub> (waves)	W <sub>220</sub> (waves)	W <sub>222</sub> (waves)
PM	Conic	-647.580	-1.338	-200	-3.310	-1.015	0.000	0.026
SM	Conic	-240.689	-1.530	200	2.601	1.231	0.024	-0.024
ТМ	Conic	-383.064	-0.252	-200	0.709	-0.217	-0.024	-0.003
Image	Flat	Infinity	_	_	0.000	0.000	0.000	0.000



**Fig. 1** The four optical systems obtained by the design method in this paper. (a), (b), (c), and (d) correspond to the optical design parameters in Tables 1, 2, 3, and 4, respectively. The numbers 1, 2, and 3 in the figure represent the PM, SM, and TM.

shows the telescope parameters of -500-mm focal length, positive-negative-positive power combination form. Figure 1 shows a structural diagram of four groups of optical systems.

The configuration (a) in Fig. 1 is one of the two types of TMA proposed by Korsch in 1972.<sup>2</sup> The James Webb Space Telescope and The SNAP telescope belong to the configuration (a) in Table 1. This type of TMA often has the stop set on or in front of the PM. The key feature and main advantage of this type of TMA are that it is a reimaging design with an accessible internal image plane and an accessible exit pupil, which allows the placement of field stops, Lyot stops, and cold stops (for infrared systems) for improved stray light suppression. It can be designed as an unobstructed TMA system using off axis in both aperture and field angle, as shown in Fig. 2. The image quality can be further improved after changing all mirrors to tenth-order asphere surfaces, which can reach an image quality level close to the diffraction limit in a 3° to 4° circular field of view, as shown in Fig. 3. The size of the circle is used to indicate the RMS geometric spot diameter. The circle in green indicates that the rms geometric spot diameter at the field of view point is <0.01 mm.

The configuration (b) in Fig. 1 has serious defects. The third-order aberration coefficients contributed by each mirror is very large. For example, the third-order spherical aberration coefficient of the PM reaches more than 3000 waves. Therefore, although the sum of the third-order aberration coefficients of the system is zero (excluding distortion), the remaining high-order

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Fig. 2 The unobstructed TMA system obtained from the configuration (a) using off axis in both aperture and field angle.



Fig. 3 The rms spot diameter versus field angle diagram for the unobstructed TMA system obtained from the configuration (a).

aberrations are very large, especially the fifth-order spherical aberration coefficient  $W_{060}$  is 165 waves, so the system cannot be used in imaging. However, we found that if the mirror spacings are set appropriately, the above-mentioned defects can be improved. We increased the mirror spacings and set  $d_1$ ,  $d_2$ , and  $d_3$  to -1700, 4400, and -2380 mm, respectively, and then recalculated all optical parameters. Similar to the processing method of configuration (a), we can get the optical system shown in Fig. 4 by off-axis and optimization.

The rms spot diameter vs. field angle diagram is shown in Fig. 5.

It can be seen from Fig. 5 that the system can reach an image quality level close to the diffraction limit in a wide arc-shaped banded field of view after increasing the mirror spacings. The full field of view in the x direction can reach about  $30^\circ$ , and the full field of view in the y direction can reach about  $5^\circ$ . The system has an intermediate image plane and a real exit pupil, which can achieve good stray light suppression. Its disadvantage is that the system length is longer, about 8 to 9 times the focal length.

The configuration (c) in Fig. 1 also has the problem that the third-order aberration coefficients of the mirrors are too large, and the high-order aberrations remain in the system, although it is not as serious as configuration (b). We also solved this problem by setting the mirror spacings appropriately.  $d_1$ ,  $d_2$ , and  $d_3$  are set to -750, 1500, and -750 mm, and we recalculated all optical parameters. The optical system shown in Fig. 6 can be obtained using the off-axis field of view and optimization. The rms spot diameter versus field angle diagram of this system is shown in Fig. 7.



Fig. 4 The unobstructed TMA system obtained from the configuration (b) using off axis in both aperture and field angle.



Fig. 5 The rms spot diameter versus field angle diagram for the unobstructed TMA system obtained from the configuration (b).



Fig. 6 The unobstructed TMA system obtained from the configuration (c) using off-axis field of view.

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Fig. 7 The rms spot diameter versus field angle diagram for the unobstructed TMA system obtained from the configuration (c).

This system is also known as the WALRUS TMA system. The system can achieve good image quality close to the diffraction limit in a wide arc-shaped banded field of view. The x direction field of view can reach about 50° or even 60°, and the y direction field of view can reach about 5°, which is very suitable for wide-field imaging applications. The disadvantage is that the distance from the SM to TM is large, typically three times the effective focal length.

The configuration (d) in Fig. 1 is also one of the two types of TMA proposed by Korsch.<sup>2</sup> The Paul–Baker design adopted by the Vera C. Rubin Observatory's telescope is a special case of this type of TMA telescope. This configuration is also called the non-reimaging TMA or reflective triplet, which is the reflective equivalent of the refractive Cooke triplet. The stop is always at or near a negative SM with a positively powered mirror on each side of the stop. The off-axis field of view and the off-axis aperture is used to avoid the obstruction of the SM. The optical system shown in Fig. 8 can be obtained after optimization.

The rms spot diameter versus field angle diagram of this system is shown in Fig. 9.

Because there is no intermediate image, this design form can support a larger field than the reimaging TMA design form, which can achieve good image quality close to the diffraction limit within a wide field of view of (8° to 10°)  $\times$ (4° to 3°). It has the advantages of good telecentricity and compact envelope size. The axial length of the system is typically 1/3 times the effective focal length. However, it is not appropriate for thermally cooled systems, as the exit pupil is virtual and therefore inaccessible for the placement of a cold stop.



Fig. 8 The unobstructed TMA system obtained from the configuration (d) using off axis in both aperture and field angle.

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Fig. 9 The rms spot diameter versus field angle diagram for the unobstructed TMA system obtained from the configuration (d).

### 4 Conclusion

In this paper, an analytical design method of TMA telescope with mirror spacings as the free design parameters is proposed. Using paraxial ray tracing and Seidel's aberration theory, according to the first-order parameter conditions and third-order field curvature elimination conditions of the TMA telescope, the calculation formula for the radius of curvature of the TMA telescope is derived. According to the elimination conditions of three-order spherical aberration, coma, and astigmatism, the calculation formula for the mirror conic constants is derived. Finally, we explain the design process through a design example and discussed the advantages and disadvantages of four types of TMA systems

The advantage of the method in this paper is that the mirrors and image position and system envelope size can be predetermined before optical design, and at the same time it is convenient to quickly give all the design solutions of the TMA telescope that meet the conditions.

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