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A method based on exact constraint for supporting space-based large mirror with a diameter of 2.8m

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ABSTRACT

The larger the aperture of the mirror, the more sensitive it was to the disturbance of temperature and gravity load. So that it was more difficult to guarantee the accuracy of the surface shape. Therefore, the large aperture mirror needs exact constraint and positioning especially. In this paper, the exact constraint theory was applied. The constraint line diagram of 2.8m diameter mirror was drawn by dual line rule. Parallel mechanical connections add constraints and series mechanical connections add the degrees of freedom. According to this principle, the tangential flexible support structure and the axial flexible support structure were designed synthetically by the constraint line diagram, by which the 6 degrees of freedom of the mirror was constrained. There is neither overconstraint nor underconstraint. By means of finite element analysis calculation, under temperature rise 3 working conditions, the surface shape accuracy RMS value of the mirror was 7.5nm. Under self-gravity load on the mirror with gravity compensation device, the surface shape accuracy RMS value of the mirror was 6.4nm. The results showed, this support method finally realized the exact constraint of this large aperture mirror, and sensitivity to temperature and gravity load disturbances was within acceptable range.

1. Introduction

The development of reflective mirrors with large aperture became a central as well as an urgent problem in the development of space camera with long focal length and large aperture. The surface shape precision of a reflective mirror directly influenced image quality of space camera [1]. A reflective mirror will experience environments such as transportation, reversal, launch, and flight on orbit etc. The accuracy and stability of the position and pointing of a reflective mirror were necessary condition for a remote sensor to function well. The repeatability of the position and pointing of a reflective mirror, which will make sure of the smooth process of manufacturing and will decrease machining period and cost, was also needed during the course of manufacturing.

Apparently, a reflective mirror can't work when underconstraint happens, because it needs to keep stable position and pointing in an optical system. But in practical application overconstraint were usually seen. However, the overconstraint will cause stress which will result in deformation of a reflective mirror. In addition, the stress will transfer when the change of exterior condition including temperature and vibration etc. happens, which makes it difficult to guarantee the shape precision. Consequently, the mirror (especially large one) was element of high precision which need Exact Constraint, that is, there is no one degree of freedom

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Fig. 1. Parallel constraint lines.

overconstrained and no one degree of freedom unconstrained [2].

The primary mirror of some space remote camera, a reflective mirror made of SiC, was studied in this paper. It had a diameter of 2.8 m in round shape. Discuss the principle of Exact Constraint, and draw constraint line diagram of the mirror. Then a combination of axial support structure and radial support structure, was devised based on the exact constraint theory. Finally, analyzed sensitivity to temperature and gravity load disturbances. Gravity compensation through hydraulic system will not be detailed in this paper.

2. Principle of exact constraint

An unconstrained rigid object has six degrees of freedom usually identified as three Translations(T) and three Rotations(R). A object was connected with another one, resulted in degree of freedom decrease in a direction, the line in this direction define the constraint line. The effect of constraint line was that all point lie in the constraint line of the object can only move in the perpendicular to constraint line, rather than move along the constraint line. Constraints at different position of constraint line were equivalent, so that one of two Constraints lie in a line was redundancy constraint.

In the same plane, any pair of constraints intersecting a given point were functionally equivalent to any other pairs intersected at this point. However, the angle between two constrain lines can't be close to zero. Two parallel constrain lines can be regard as intersectant constrain lines whose point of intersection was at infinity. Thus, Translations(T) was equivalent to Rotations(R) which axis was at infinity. Similarly, two parallel constrain lines can be substituted for any other two constrain lines that were parallel to them.

As Fig. 1 (I.C. instant center) indicates, we may consider parallel constraint lines to intersect at infinity such as railroad tracks appear at a distance. An object that rotates about a distant center appears to translate. With this background, we may conceptualize three translations and three rotations as being equivalent to six rotational degrees of freedom where three axes are at infinity.

Inferences can be made from the effect of the constraint line: The axis (R) of the rotational degree of freedom of an object must intersect with each constraint line (C) acting on the object. Here, parallelism can be equivalent to the intersection of intersection points at infinity. This is the basis of exact constraint design. Thus the dual line rule can be summed up as below: When the constraint line (C) between two objects is applied, it is always possible to find that the degree of freedom diagram (R) that is dual with the constraint line diagram exists between two objects, vice versa. Then, given any of these graphs containing *n* nonredundant lines, and their dual line diagram will contain *6-n* lines. And Each line in a line diagram must intersect with all the lines in its dual line diagram as well [3].

3. Statement of mirror

The main consideration about the material selection of mirror include physical performance, machinability, mechanical property, stability, security etc. The larger stiffness-to-density ratio was, the less deformation caused by unit load. Thermal distortion was coefficient of expansion-coefficient of heat conductivity ratio. The smaller thermal distortion was, the smaller thermal inertia, and the better thermal stability was. the property of several kinds of material in common use for mirror were shown in Table 1.

SiC which had low density, small thermal distortion, high stiffness-to-density ratio and fine thermal conductivity had obvious comprehensive superiority. And that it can be polished well because of maturing mirror surface Modification process. SiC was an extremely ideal selection of bulk material for mirror. It has been widely used in aerospace and ground equipment. We chose the material of invar steel whose coefficient of expansion was very close to Sic as the material of the part that directly connected with mirror.

Table 1

Several optics material attribute.

| | SiC | Al | Zerodur | ULE | fused quartz |
|--|------|------|---------|------|--------------|
| Density/ρ(kg/m ³) | 3050 | 2710 | 2530 | 2200 | 2230 |
| modulus of elasticity/ E(Gpa) | 407 | 69 | 90.6 | 67 | 64 |
| coefficient of heat conductivity/\(W/mK) | 180 | 167 | 1.64 | 1.3 | 1.13 |
| coefficient of expansion/ $\alpha(10^{-6}/K)$ | 2.2 | 23.9 | 0.05 | 0.03 | 3.25 |
| stiffness-to-density ratio/ $E/\rho(10^6 \text{ m})$ | 13.3 | 2.6 | 3.7 | 3.1 | 2.9 |
| thermal distortion $\alpha/\lambda(10^{-8} \text{ m/w})$ | 1.2 | 14.3 | 3.0 | 2.3 | 288 |



Fig. 2. Model of the mirror.

The mirror researched in this paper was circular mirror with diameter of 2.8 m, its central thickness was 200 mm. Lightweight structure selected open back structure. Lightweight hole adopted triangle because of its better stiffness stability [4]. The mirror model is shown in Fig. 2.

4. Exact constraint scheme of mirror

Obviously, Mirror requires exact constraints. That is, constrain 6 degrees of freedom, Neither overconstraint, nor underconstraint. According to Dual Line Diagram rule, six nonredundant constraints were required for exact constrained mirror. There are also many ways to draw constraint line diagram for mirror. Considering the symmetry of structure, three parallel axial constraints in the perpendicular to the back plane of the mirror and the tangential constraints of three symmetric distributions in a plane parallel to the back were adopted. Its constraint diagram is shown in Fig. 3.

The cross point of the three parallel constraints at the bottom of the mirror and the back plane of the mirror cannot be collinear. These three axially constrained lines constrain the mirror's three degrees of freedom: *Z* translation and *X*, *Y* rotation, respectively. Three constraint lines tangentially pairwise intersected that were not intersected at one point constrained the *X*, *Y* translation and *Z* rotation of the mirror.

5. Design of axial and tangential supporting structures

When designing flexible structures, flexible thin plate is the most commonly used flexible element. As shown in Fig. 4, the stiffness in the X direction and Z direction are respectively [3]:

$$K_X = \frac{wtE}{a}, \quad K_Z = \frac{wt^3E}{4a^3}$$

Its stiffness ratio is

$$\frac{K_X}{K_Z} = 4 \left(\frac{a}{t}\right)^2$$

It can be seen that as long as the parameters of thin plate are selected properly, the Z direction stiffness is much smaller than the x



Fig. 3. Constraint line diagram of the mirror.



Fig. 4. Flexible thin plate.



Fig. 5. Model of the tangential constraint structure.



Fig. 6. Model of the axial constraint structure.



Fig. 7. The finite element model of mirror assembly.



Fig. 8. Surface fringe under a 3-degree temperature rise load.



Fig. 9. Surface fringe under self-gravity with gravity compensation device.

direction stiffness. Therefore, it can be assumed that the thin plate releases the degree of freedom in *Z* translation [5,6]. By the same token, the plate also releases two rotational degrees of freedom *X*, *Y* rotation. Another commonly used flexible component, ideal flexible long rods provide rigid constraints only along axial direction, while releasing five other degrees of freedom [3].

In the tangential constraint structure design of mirror in this paper, the limit form of a very short flexible plate is used at both ends to release a rotational degree of freedom. a flexible thin plate was connected in the middle to form an integrated flexible structure. Each flexible structure was equal to a constraint in the constraint line diagram, that is, it was equivalent to an ideal flexible long rod with spherical hinge at both ends [7]. The tangential constraint structure is shown in Fig. 5.

Parallel mechanical connections add constraints; series mechanical connections add the degrees of freedom. The simulation of spherical hinge follows this principle. Each end of the structure uses a short flexible plate to release a rotational degree of freedom. Intermediate flexible thin plates was used to release of two other rotational degrees of freedom.

Each axial constrained structure should eventually be equivalent to an ideal flexible long rod with spherical hinges at both ends. Two rotational degrees of freedom are released in the limit form of two very short flexible thin plates that are perpendicular to each other at both ends, a cross-stiffened plate was connected in the middle to release another rotational degree of freedom to form an integrated flexible structure [8].

The axial constraint structure is shown in Fig. 6.

6. Sensitivity analysis of temperature and gravity

The finite element model of mirror assembly and its partial enlarged view were shown in Fig. 7.

The existence of overconstraint will cause the local stress of the mirror to be larger and surface shape precision decline. In order to fully demonstrate the necessity and effectiveness of exact constraints for mirrors, it needs to be verified by finite element analysis and calculation [9,10]. Fig. 8 is a surface fringe of a 3-degree temperature rise load. The shape accuracy RMS value of the mirror after temperature change was 7.5 nm. The results showed that the precision of the surface shape of the exact constrained mirror is as small as $1/84.4\lambda$ when it is disturbed by temperature fluctuation. It had less effect on imaging quality.

In addition, the mirror also needs exact constraint and positioning as the basic condition in the ground processing state. By using

finite element analysis software, the effect of self-gravity load on the mirror is calculated with gravity compensation device. Fig. 9 shows the mirror deformation fringe under this condition. After data processing, the surface shape precision of reflecting mirror can reach 6.4 nm, about $1/98.8\lambda$, which is higher than the requirement of optical index.

7. Conclusion

Any precision instrument required exact constraint and positioning. In this paper, the exact constraint theory is applied. The exact constraint diagram of 2.8 m diameter mirror is drawn by dual line rule. The tangential flexible support structure and the axial flexible supporting structure were designed synthetically by the constraint diagram. This positioning support structure provided exact position and gesture for the reflective mirror, and guaranteed its repeatability and reliability. It provided a good theoretical basis for the practical application. Schematic design and structure design based on exact constraint principle can be applied any other precision instrument and machinery. We used to neglect hidden troubles of the overconstraint, or to solve them by the cost of mass etc., which is very unacceptable to an optical cell with high precision. The overconstraint can be avoided when distribution and realization of the degrees of freedom are considered in the phase of design.

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