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Method for designing error-resistant phase-shifting algorithm

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ABSTRACT

We present a method for designing error-resistant phase-shifting algorithms to suppress error sources in phase-shifting interferometry. Firstly, the partial-differential processing is applied to the weighted least squares algorithm to obtain error sensitivity equations. Sequentially, bound equations are obtained to minimize error sensitivity. Finally, the bound equations are solved to determine the weights, and the error-resistant phase-shifting algorithms are developed. Aiming at a self-developed interferometer, the proposed method is used to design phase-shifting algorithms which are resistant to given error sources. Theoretical analysis and numerical simulations of the self-designed algorithms compared with a commercial algorithm are completed. Theoretical analysis indicates that the self-designed algorithms meet the desired requirements. Numerical simulations verify the correctness of theoretical analysis. And the comparisons show that the self-designed algorithm is more resistant to error sources that needs to be suppressed. These results verify the proposed method and demonstrate its effectiveness.

1. Introduction

In phase-shifting interferometry, the measured object is the phase difference of reference and test wave fields. Phase-shifter introduces additional OPD (optical path difference), which yields sequential interferograms. These interferograms are captured by detector, and then are processed using a given phase-shifting algorithm to calculate the phase difference [1]. This technique reduces the influence of contrast, and yields good results even if the interferograms have poor contrast. Furthermore, it reduces the influence of background and nonuniformity of the light source, and provides high accuracy [2].

In phase-shifting interferometers, PZT (piezoelectric transducers) is commonly used as phase shifter, CCD (charge-coupled device) or CMOS (complementary metal oxide semiconductor) is used as detector, and laser is typically used as light source. The nonideal performance of these devices usually causes measurement errors. Therefore, the phase-shifting algorithm should be designed to suppress these error sources.

To date, many methods have been reported on designing phase-shifting algorithms to minimize particular errors. Freischlad [3] proposed a method to evaluate the performance of phase-shifting algorithms through Fourier theory and Zhang [4] used this theory to derive a new error-resistant algorithm. De Groot [5] designed algorithms via window functions such as the Hanning window. Surrel [6] presented a characteristic polynomial theory for designing phase-shifting algorithms, and Zhu [7] designed a new algorithm by overlapping averaged results of the old algorithm, making the new algorithm more insensitive to phase-shift error. Phillion [8] proposed a method based on recursion rules to design a new algorithm from the old one. Shi [9] presented an effective approach to derive phase-shifting algorithms based on the self-convolution of a rectangle window, and designed an algorithm to suppress the phase-shift error and detector-response error simultaneously. However, to the best of our knowledge, these methods do not discuss the error sensitivity of algorithms, but only focus on suppressing phase-shift error, detector-response error, or both, while ignoring the other error sources.

This paper thus proposes an effective method for designing error-resistant algorithms. By applying the partial-differential method, we get sensitivity relationship between weighted least-square algorithm and error sources, which named error sensitivity equations. When the error sensitivity vanishes, the weighted least-square algorithm is insensitive to corresponding error sources. And then, we get bound equations. Sequentially, the bound equations are solved to determine the weights, and finally, phase-shifting algorithms are obtained that are resistant to

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corresponding error sources. The designed error-resistant phase-shifting algorithms effectively reduce the performance requirements for phase shifter, detector, and light source.

This paper is organized as follows. Section 2 introduces the weighted least-square phase-shifting algorithm. Section 3 introduces the main error sources (phase-shift error, detector-response error, and light-source-instability), analyses sensitivity relationship between weighted least-square algorithm and these error sources, and then derives the bound equations. Section 4 takes a self-designed interferometer as an example to design algorithms which are capable of suppressing the first-, second-, and third-order phase-shift error, the first- and second-order detector-response error, and the first- and second-order light-intensity–instability. Section 5 uses Fourier transform theory and numerical simulations to evaluate the performance of self-designed algorithms. The self-designed algorithms are compared with Zygo 13-frames algorithm [10,11], and the results demonstrate the effectiveness of the proposed method.

2. Weighted least-square algorithm

In phase-shifting interferometry, the nth irradiance $I_n(x,y)$ at a point (x,y) could be expressed as follows [12],

$$I_n(x,y) = A(x,y) + B(x,y) \times \cos\left[\phi(x,y) + \delta_n(x,y)\right] \tag{1}$$

where $A(x,y)$ is the background intensity, $B(x,y)$ is the amplitude of modulation, the quantity to be measured $\phi(x,y)$ is the phase difference of the wave fields that interfere, and $\delta_n(x,y)$ is the phase shift of the nth irradiance. Eq. (1) can be rewritten as,

$$I_n(x,y) = A(x,y) + B_{cos}(x,y)\cos\left[\delta_n(x,y)\right] + B_{sin}(x,y)\sin\left[\delta_n(x,y)\right] \tag{2}$$

where,

$$\begin{aligned} B_{cos}(x,y) &= B(x,y)\cos\left[\phi(x,y)\right] \\ B_{sin}(x,y) &= -B(x,y)\sin\left[\phi(x,y)\right] \end{aligned} \tag{3}$$

Then, the phase difference can be calculated from $B_{sin}(x,y)$ and $B_{cos}(x,y)$ by,

$$\phi(x,y) = \arctan\left[-\frac{B_{sin}(x,y)}{B_{cos}(x,y)}\right] \tag{4}$$

The phase at point (x,y) is only determined by the intensity and phase shift at this point, so we omit the explicit dependence (x,y) on position. Phase-shifting algorithm can be developed from the principle of weighted least-square estimation [13]. With the weight function, the error function ϵ could be defined as,

$$\epsilon = \sum_{n=1}^{N} w_n\left(I_n - \widetilde{I}_n\right)^2 = \sum_{n=1}^{N} w_n\left[A + B_{cos}\cos(\delta_n) + B_{sin}\sin(\delta_n) - \widetilde{I}_n\right]^2 \tag{5}$$

where $\widetilde{I}_n$ represents the nth actual irradiance and w_n is the nth weight. The ϵ is minimized when the derivatives of ϵ with respect to A, B_{sin} and B_{cos} vanish. This condition yields the following matrix equation,

$$\begin{bmatrix} \sum_{n=1}^{N} w_n & \sum_{n=1}^{N} w_n\cos(\delta_n) & \sum_{n=1}^{N} w_n\sin(\delta_n) \\ \sum_{n=1}^{N} w_n\cos(\delta_n) & \sum_{n=1}^{N} w_n\cos^2(\delta_n) & \sum_{n=1}^{N} w_n\sin(\delta_n)\cos(\delta_n) \\ \sum_{n=1}^{N} w_n\sin(\delta_n) & \sum_{n=1}^{N} w_n\sin(\delta_n)\cos(\delta_n) & \sum_{n=1}^{N} w_n\sin^2(\delta_n) \end{bmatrix} \times \begin{bmatrix} A \\ B_{cos} \\ B_{sin} \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N} w_n\widetilde{I}_n \\ \sum_{n=1}^{N} w_n\widetilde{I}_n\cos(\delta_n) \\ \sum_{n=1}^{N} w_n\widetilde{I}_n\sin(\delta_n) \end{bmatrix} \tag{6}$$

If the weights are selected to satisfy the following conditions,

$$\begin{cases} \sum_{n=1}^{N} w_n = 1 \\ \sum_{n=1}^{N} w_n\cos(\delta_n) = \sum_{n=1}^{N} w_n\sin(\delta_n) = \sum_{n=1}^{N} w_n\sin\left(2\delta_n\right) = 0 \\ \sum_{n=1}^{N} w_n\cos^2(\delta_n) = \sum_{n=1}^{N} w_n\sin^2(\delta_n) = Q \end{cases} \tag{7}$$

where Q is a non-zero constant, the phase difference ϕ can be calculated by,

$$\phi = \arctan\left[-\frac{\sum_{n=1}^{N} w_n\widetilde{I}_n\sin(\delta_n)}{\sum_{n=1}^{N} w_n\widetilde{I}_n\cos(\delta_n)}\right] \tag{8}$$

Eq. (8) is the formula of weighted phase-shifting algorithm. When appropriate weights are selected, weighted phase-shifting algorithm could be resistant to given error sources.

3. Sensitivity analysis for error sources

In this section, we analyze sensitivity of weighted phase-shifting algorithm to phase-shift error, detector-response error, and light-source-instability, which are main error sources in phase-shift interferometry. And finally, we obtain the bound equations to minimize the influence of these error sources.

3.1. Phase-shift error

In the case of a linear or nonlinear miscalibration, the actual nth phase shift δ'_n may be expressed as a polynomial of ideal phase shift δ_n [14]. Note that in the first irradiance, phase shift is zero, so $\delta'_1 = \delta_1 = 0$. For equal-interval phase shift, $\delta_n = (n-1)\delta$, where δ is single phase shift interval. When k order phase-shift errors exist, δ'_n is a k order polynomial of frames number n, as,

$$\delta'_n = \left[\left(1+\zeta_1\right)(n-1) + \zeta_2(n-1)^2 + \cdots + \zeta_k(n-1)^k\right]\delta \tag{9}$$

where, ζ_k is the coefficient of the kth order phase-shift error. The phase-shift error defined as the difference between δ'_n and δ_n is,

$$\Delta\delta_n = \left[\zeta_1(n-1) + \zeta_2(n-1)^2 + \cdots + \zeta_k(n-1)^k\right]\delta \tag{10}$$

From the partial derivative of Eq. (8) with respect to δ_n, we get phase extraction error caused by phase-shift error (see Box I). Using Eq. (7), the denominator and numerator of Eq. (11) can be simplified respectively as,

$$\begin{aligned} denominator =& \left[B\cos(\phi)\sum_{n=1}^{N} w_n I_n\cos^2\left(\delta_n\right)\right]^2 \\ &+ \left[-B\sin(\phi)\sum_{n=1}^{N} w_n I_n\sin^2\left(\delta_n\right)\right]^2 \\ =& B^2Q^2 \end{aligned} \tag{12}$$

and,

$$\begin{aligned} numerator = B^2Q\Bigg\{&\cos(\phi)\left[\frac{\sin(\phi)}{2}\sum_{n=1}^{N} w_n\sin\left(2\delta_n\right)\Delta\delta_n \right. \\ &\left. + \cos(\phi)\sum_{n=1}^{N} w_n\sin^2\left(\delta_n\right)\Delta\delta_n\right] \\ &\sin(\phi)\left[\frac{\cos(\phi)}{2}\sum_{n=1}^{N} w_n\sin\left(2\delta_n\right)\Delta\delta_n \right. \\ &\left. + \sin(\phi)\sum_{n=1}^{N} w_n\cos^2\left(\delta_n\right)\Delta\delta_n\right]\Bigg\} \end{aligned} \tag{13}$$

$$\Delta\phi = \frac{B\left[\sum_{n=1}^{N} w_n I_n \cos\left(\delta_n\right) \sum_{n=1}^{N} w_n \sin\left(\delta_n\right) \sin\left(\phi+\delta_n\right) \Delta\delta_n - \sum_{n=1}^{N} w_n I_n \sin\left(\delta_n\right) \sum_{n=1}^{N} w_n \cos\left(\delta_n\right) \sin\left(\phi+\delta_n\right) \Delta\delta_n\right]}{\left[\sum_{n=1}^{N} w_n I_n \cos\left(\delta_n\right)\right]^2 + \left[\sum_{n=1}^{N} w_n I_n \sin\left(\delta_n\right)\right]^2} \tag{11}$$

Box I.

$$\Delta\phi = \frac{\sin^2(\phi) \sum_{n=1}^{N} w_n \cos^2\left(\delta_n\right) \Delta\delta_n + \cos^2(\phi) \sum_{n=1}^{N} w_n \sin^2\left(\delta_n\right) \Delta\delta_n + \frac{1}{2}\sin(2\phi) \sum_{n=1}^{N} w_n \sin\left(2\delta_n\right) \Delta\delta_n}{Q} \tag{14}$$

Box II.

Hence, we obtain the error sensitivity equation with respect to phase-shift error $\Delta\delta_n$ as Eq. (14) given in Box II . If the following conditions are satisfied,

$$\begin{cases} \sum_{n=1}^{N} w_n \cos^2\left(\delta_n\right) \Delta\delta_n = \sum_{n=1}^{N} w_n \sin^2\left(\delta_n\right) \Delta\delta_n \\ \sum_{n=1}^{N} w_n \sin\left(2\delta_n\right) \Delta\delta_n = 0 \end{cases} \tag{15}$$

the phase extraction error is a point-independent constant, which does not influence the measurement. Thus, we get the bound equations for suppressing k order phase-shift errors as,

$$\begin{cases} \sum_{n=1}^{N} (n-1)\, w_n \cos\left(2\delta_n\right) = \sum_{n=1}^{N} (n-1)^2\, w_n \cos\left(2\delta_n\right) = \cdots \\ \quad = \sum_{n=1}^{N} (n-1)^k\, w_n \cos\left(2\delta_n\right) = 0 \\ \sum_{n=1}^{N} (n-1)\, w_n \sin\left(2\delta_n\right) = \sum_{n=1}^{N} (n-1)^2\, w_n \sin\left(2\delta_n\right) = \cdots \\ \quad = \sum_{n=1}^{N} (n-1)^k\, w_n \sin\left(2\delta_n\right) = 0 \end{cases} \tag{16}$$

3.2. Detector-response error

The detector-response error is caused by the linear and nonlinear relationship between the irradiance incident upon a detector and the voltage it outputs. When k order detector-response errors exist, the output signal I_n' may be expressed as a k order polynomial of the incident irradiance I_n [15],

$$I_n' = \left(1+\eta_1\right) I_n + \eta_2 I_n^2 + \cdots + \eta_k I_n^k \tag{17}$$

The detector-response error is,

$$\Delta I_n = \eta_1 I_n + \eta_2 I_n^2 + \cdots + \eta_k I_n^k \tag{18}$$

From the partial derivative of Eq. (8) with respect to I_n, phase extraction error caused by detector-response error is expressed as,

$$\Delta\phi = \frac{\sum_{n=1}^{N} w_n I_n \sin\left(\delta_n\right) \sum_{n=1}^{N} w_n \Delta I_n \cos\left(\delta_n\right) - \sum_{n=1}^{N} w_n I_n \cos\left(\delta_n\right) \sum_{n=1}^{N} w_n \Delta I_n \sin\left(\delta_n\right)}{\left[\sum_{n=1}^{N} w_n I_n \cos\left(\delta_n\right)\right]^2 + \left[\sum_{n=1}^{N} w_n I_n \sin\left(\delta_n\right)\right]^2} \tag{19}$$

From Eq. (12), the denominator of Eq. (19) equals to constant B^2Q^2. Using Eq. (7), the numerator of Eq. (19) can be simplified as,

$$numerator = -BQ\left[\sin(\phi) \sum_{n=1}^{N} w_n \Delta I_n \cos\left(\delta_n\right) + \cos(\phi) \sum_{n=1}^{N} w_n \Delta I_n \sin\left(\delta_n\right)\right] \tag{20}$$

Thus, we get the error sensitivity equation with respect to detector-response error ΔI_n as,

$$\Delta\phi = -\frac{\sin(\phi) \sum_{n=1}^{N} w_n \Delta I_n \cos\left(\delta_n\right) + \cos(\phi) \sum_{n=1}^{N} w_n \Delta I_n \sin\left(\delta_n\right)}{BQ} \tag{21}$$

For linear detector-response error ($k = 1$), phase extraction error $\Delta\phi$ vanishes. For higher order harmonics ($k \geq 2$), phase extraction error $\Delta\phi$ can be expressed as,

$$\Delta\phi = -\frac{\sin(\phi)\eta_k \sum_{n=1}^{N} w_n I_n^k \cos\left(\delta_n\right) + \cos(\phi)\eta_k \sum_{n=1}^{N} w_n I_n^k \sin\left(\delta_n\right)}{BQ} \tag{22}$$

where I_n^k is the kth order detector response, and it can be expressed as,

$$I_n^k = \left[A + B\cos\left(\phi+\delta_n\right)\right]^k = \sum_{p+q=k} \frac{k!}{p!q!} A^p B^q \cos^q\left(\phi+\delta_n\right) \tag{23}$$

where q and p are integers from 0 to k, and the sum of q and p equals to k. If q is odd,

$$\cos^q\left(\phi+\delta_n\right) = \frac{1}{2^{q-1}} \sum_{m=0}^{\frac{q-1}{2}} C_q^m \cos\left[(q-2m)\left(\phi+\delta_n\right)\right] \tag{24}$$

If q is even,

$$\cos^q\left(\phi+\delta_n\right) = \frac{1}{2^{q-1}} \left\{ \sum_{m=0}^{\frac{q}{2}-1} C_q^m \cos\left[(q-2m)\left(\phi+\delta_n\right)\right] + \frac{1}{2} C_q^{\frac{q}{2}} \right\} \tag{25}$$

Considering the last term in square brackets in Eq. (25) is a point-independent constant, which does not influence the phase extraction error. Bring Eq. (23), Eq. (24) and Eq. (25) into Eq. (22), we get,

$$\begin{aligned} \Delta\phi = \frac{-1}{BQ} \Bigg\{ & \sin(\phi)\eta_k \sum_{n=1}^{N} \left[w_n \cos\left(\delta_n\right) \sum_{q=0}^{k} \xi_q \cos\left[q\left(\phi+\delta_n\right)\right] \right] \\ & + \cos(\phi)\eta_k \sum_{n=1}^{N} \left[w_n \sin\left(\delta_n\right) \sum_{q=0}^{k} \xi_q \cos\left[q\left(\phi+\delta_n\right)\right] \right] \Bigg\} \end{aligned} \tag{26}$$

where ξ_q is constant determined by A, B, q and k, and is independent of n. Thereby, Eq. (26) can be rewritten as,

$$\Delta\phi = \frac{-\eta_n}{BQ}\left\{ \sin(\phi)\sum_{q=0}^{k}\xi_q\left[\cos(q\phi)\sum_{n=1}^{N}w_n\cos(q\delta_n)\cos(\delta_n) - \sin(q\phi)\sum_{n=1}^{N}w_n\sin(q\delta_n)\cos(\delta_n)\right] + \cos(\phi)\sum_{q=0}^{k}\xi_q\left[\cos(q\phi)\sum_{n=1}^{N}w_n\cos(q\delta_n)\sin(\delta_n) - \sin(q\phi)\sum_{n=1}^{N}w_n\sin(q\delta_n)\sin(\delta_n)\right]\right\} \tag{27}$$

When $\Delta\phi$ vanishes, the algorithm is insensitive to the kth order detector-response error. It is obvious that if $q = 0{,}1$, $\Delta\phi = 0$. We get the following bound equations for suppressing the kth order detector-response error,

$$\begin{cases} \sum_{n=1}^{N}w_n\cos(2\delta_n)\cos(\delta_n) = \sum_{n=1}^{N}w_n\cos(3\delta_n)\cos(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N}w_n\cos(k\delta_n)\cos(\delta_n) = 0 \\ \sum_{n=1}^{N}w_n\sin(2\delta_n)\cos(\delta_n) = \sum_{n=1}^{N}w_n\sin(3\delta_n)\cos(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N}w_n\sin(k\delta_n)\cos(\delta_n) = 0 \\ \sum_{n=1}^{N}w_n\cos(2\delta_n)\sin(\delta_n) = \sum_{n=1}^{N}w_n\cos(3\delta_n)\sin(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N}w_n\cos(k\delta_n)\sin(\delta_n) = 0 \\ \sum_{n=1}^{N}w_n\sin(2\delta_n)\sin(\delta_n) = \sum_{n=1}^{N}w_n\sin(3\delta_n)\sin(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N}w_n\sin(k\delta_n)\sin(\delta_n) = 0 \end{cases} \tag{28}$$

Through above derivation, it is easy to get a conclusion that, if the algorithm is insensitive to the kth-order detector-response error, it is insensitive to the mth-order ($m < k$) detector-response error too. So, if Eq. (28) is satisfied, the algorithm is insensitive to k order detector-response errors.

3.3. Light-source-instability

The light-source-instability includes intensity instability and frequency instability. The former changes the background intensity and amplitude of modulation of the interferogram; and the latter introduces additional phase shift.

3.3.1. Intensity instability

The intensity $I_{lightsource}$ of the light source fluctuates over time t. When k order intensity instabilities exist, $I_{lightsource}$ can be expressed as k order polynomial of intensity I at $t = 0$, as,

$$I_{lightsource} = \left[1 + \gamma_1 t + \gamma_2 t^2 + \cdots + \gamma_k t^k\right] I \tag{29}$$

so, the nth irradiance I'_n may be written as a polynomial of frames number n in discrete form,

$$I'_n = \left[1 + \gamma_1(n-1) + \gamma_2(n-1)^2 + \cdots + \gamma_k(n-1)^k\right] I_n \tag{30}$$

The difference between the actual fringe pattern and the ideal fringe pattern is then expressed as,

$$\Delta I_n = \left[\gamma_1(n-1) + \gamma_2(n-1)^2 + \cdots + \gamma_k(n-1)^k\right] I_n \tag{31}$$

From the partial derivative of Eq. (8) with respect to I_n, we get the phase extraction error $\Delta\phi$ caused by the light-source-intensity instability ΔI_n, and it has the same formula with Eq. (21). Bring irradiance deviation expressed in Eq. (31) into Eq. (21), the phase extraction error can be rewritten as,

$$\Delta\phi = \frac{-1}{BQ}\left\{ A\sin(\phi)\gamma_1\sum_{n=1}^{N}w_n(n-1)\cos(\delta_n) + B\sin(\phi)\gamma_1\sum_{n=1}^{N}w_n(n-1)\cos(\phi+\delta_n)\cos(\delta_n) + \cdots + A\sin(\phi)\gamma_k\sum_{n=1}^{N}w_n(n-1)^k\cos(\delta_n) + B\sin(\phi)\gamma_k\sum_{n=1}^{N}w_n(n-1)^k\cos(\phi+\delta_n)\cos(\delta_n) + A\cos(\phi)\gamma_1\sum_{n=1}^{N}w_n(n-1)\sin(\delta_n) + B\cos(\phi)\gamma_1\sum_{n=1}^{N}w_n(n-1)\cos(\phi+\delta_n)\sin(\delta_n) + \cdots + A\cos(\phi)\gamma_k\sum_{n=1}^{N}w_n(n-1)^k\sin(\delta_n) + B\cos(\phi)\gamma_k\sum_{n=1}^{N}w_n(n-1)^k\cos(\phi+\delta_n)\sin(\delta_n)\right\} \tag{32}$$

when $\Delta\phi$ vanishes, the algorithm is insensitive to the k order light-source-intensity instabilities, and we get the bound equations for suppressing k order light-source-intensity instabilities as,

$$\begin{cases} \sum_{n=1}^{N}(n-1)w_n\sin(\delta_n) = \sum_{n=1}^{N}(n-1)^2w_n\sin(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N}(n-1)^k w_n\sin(\delta_n) = 0 \\ \sum_{n=1}^{N}(n-1)w_n\cos(\delta_n) = \sum_{n=1}^{N}(n-1)^2w_n\cos(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N}(n-1)^k w_n\cos(\delta_n) = 0 \\ \sum_{n=1}^{N}(n-1)w_n\sin(2\delta_n) = \sum_{n=1}^{N}(n-1)^2w_n\sin(2\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N}(n-1)^k w_n\sin(2\delta_n) = 0 \\ \sum_{n=1}^{N}(n-1)w_n\cos(2\delta_n) = \sum_{n=1}^{N}(n-1)^2w_n\cos(2\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N}(n-1)^k w_n\cos(2\delta_n) = 0 \end{cases} \tag{33}$$

3.3.2. Frequency instability

In interferometry, the phase difference is directly proportional to the frequency of the light source. Thus, the frequency instability Δv introduces the following additional phase difference,

$$\Delta\delta = \frac{2\pi \times OPD \times \Delta v}{C} \tag{34}$$

Here, OPD is the optical path difference and C is a constant. It is clear that additional phase difference is proportional to light-source-frequency instability. When the k order frequency instabilities exist, the frequency of nth irradiance may be expressed as k order polynomial of n. Thus, the additional phase difference caused by frequency instability is the same as Eq. (10). So, the bound equations for frequency instability can be expressed as Eq. (16) too.

3.4. Combination of bound equations

To design an algorithm that is insensitive to k order phase-shift errors, k order detector-response errors and k order light-source-instabilities, the following bound equations must be satisfied,

$$\begin{cases} \sum_{n=1}^{N} w_n = 1 \\ \sum_{n=1}^{N} w_n \cos(\delta_n) = \sum_{n=1}^{N} w_n \sin(\delta_n) = \sum_{n=1}^{N} w_n \cos(2\delta_n) \\ \quad = \sum_{n=1}^{N} w_n \sin(2\delta_n) = 0 \\ \sum_{n=1}^{N} (n-1) w_n \cos(\delta_n) = \sum_{n=1}^{N} (n-1)^2 w_n \cos(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N} (n-1)^k w_n \cos(\delta_n) = 0 \\ \sum_{n=1}^{N} (n-1) w_n \cos(2\delta_n) = \sum_{n=1}^{N} (n-1)^2 w_n \cos(2\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N} (n-1)^k w_n \cos(2\delta_n) = 0 \\ \sum_{n=1}^{N} (n-1) w_n \sin(\delta_n) = \sum_{n=1}^{N} (n-1)^2 w_n \sin(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N} (n-1)^k w_n \sin(\delta_n) = 0 \\ \sum_{n=1}^{N} (n-1) w_n \sin(2\delta_n) = \sum_{n=1}^{N} (n-1)^2 w_n \sin(2\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N} (n-1)^k w_n \sin(2\delta_n) = 0 \\ \sum_{n=1}^{N} w_n \cos(2\delta_n)\cos(\delta_n) = \sum_{n=1}^{N} w_n \cos(3\delta_n)\cos(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N} w_n \cos(k\delta_n)\cos(\delta_n) = 0 \\ \sum_{n=1}^{N} w_n \cos(2\delta_n)\sin(\delta_n) = \sum_{n=1}^{N} w_n \cos(3\delta_n)\sin(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N} w_n \cos(k\delta_n)\sin(\delta_n) = 0 \\ \sum_{n=1}^{N} w_n \sin(2\delta_n)\cos(\delta_n) = \sum_{n=1}^{N} w_n \sin(3\delta_n)\cos(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N} w_n \sin(k\delta_n)\cos(\delta_n) = 0 \\ \sum_{n=1}^{N} w_n \sin(2\delta_n)\sin(\delta_n) = \sum_{n=1}^{N} w_n \sin(3\delta_n)\sin(\delta_n) = \cdots \\ \quad = \sum_{n=1}^{N} w_n \sin(k\delta_n)\sin(\delta_n) = 0 \end{cases} \tag{35}$$

4. Error-resistant algorithms to obtain desired properties

To obtain the desired performance of an interferometer, we customize the error-resistant algorithm to suppress the given error sources. For example, the main error sources of the self-developed interferometer are the first-, second-, and third-order phase-shift error, the first- and second-order detector-response error, and the first- and second-order light-source-intensity instability. We use the proposed method to design phase-shifting algorithm to suppress these error sources. Single phase shift interval is $\pi/2$. Assuming the weights are symmetrical, the bound equations Eq. (35) are solved simultaneously to obtain the following weights as,

$$\begin{aligned} &w_1 = w_{13} = \frac{0.25(\beta-40)}{64}; \quad w_2 = w_{12} = \frac{0.25(\beta-39)}{32}; \\ &w_3 = w_{11} = \frac{0.25(\beta-36)}{32} \\ &w_4 = w_{10} = \frac{0.25(\beta-31)}{32}; \quad w_5 = w_9 = \frac{0.25(72-\beta)}{64}; \\ &w_6 = w_8 = \frac{0.25(51-\beta)}{16} \\ &w_7 = \frac{0.25(52-\beta)}{16}; \qquad 40 < \beta < 51 \end{aligned} \tag{36}$$

where β is an integer. In this case, we get 10 self-designed algorithms which have the same formula as follows,

$$\phi = \arctan\left[-\frac{w_2(I_2 - I_{12}) - w_4(I_4 - I_{10}) + w_6(I_6 - I_8)}{w_1(I_1 + I_{13}) - w_3(I_3 + I_{11}) + w_5(I_5 + I_9) - w_7 I_7}\right] \tag{37}$$

5. Theoretical analysis and numerical simulations

In this section, we use Freischlad's [3] spectral analysis method to analyze the self-designed algorithms. In this theory, phase-shifting is explained as a filtering process in the frequency domain, which allows us to analyze the performance of phase-shifting algorithms based on their frequency response. By using Parseval's identity, Eq. (8) can be rewritten in continuous format as,

$$\begin{aligned} \phi &= \arctan\left[-\frac{\int w(t)\widetilde{I}(t)\sin(\nu_0 t)dt}{\int w(t)\widetilde{I}(t)\cos(\nu_0 t)dt}\right] = \arg\left[\int W(t)\widetilde{I}(t)dt\right] \\ &= \arg\left[\int \widetilde{W}(\nu)\widetilde{I}^*(\nu)d\nu\right] \end{aligned} \tag{38}$$

where $\widetilde{W}(\nu)$ and $\widetilde{I}(\nu)$ are the Fourier transform of the window function $W(t)$ and the signal $\widetilde{I}(t)$, respectively. Considering the p order phase-shifting errors and q order detector-response errors, Shi [9] obtained the following approximate expression for phase-shifting algorithm,

$$\phi' = \arg\left\{\sum_{k=-q}^{q} \frac{\eta_k}{2}\exp(i\phi_{-k})\left[\widetilde{W}(k\nu_0) + k\sum_{j=1}^{p}(-i)^{j+1}\zeta_j \widetilde{W}^j(k\nu_0)\right]\right\} \tag{39}$$

Shi obtained the necessary and sufficient condition for $\phi' = \phi_1$ as follows: The Fourier transform of the window function must have $2q$ equidistant multiple roots of order $p+1$ except at $k=-1$, which gives,

$$\widetilde{W}^j(k\nu_0) = 0, \quad j = 0,1,\ldots,p, k = -q,\ldots,-2,0,1,\ldots,q \tag{40}$$

The window function for Algorithm A is,

$$\begin{aligned} W = [&w_1 \quad iw_2 \quad -w_3 \quad -iw_4 \quad w_5 \quad iw_6 \quad -w_7 \quad -iw_8 \quad w_9 \\ &iw_{10} \quad -w_{11} \quad -iw_{12} \quad w_{13}] \end{aligned} \tag{41}$$

Fig. 1 shows the normalized amplitude spectrum of Algorithm A and of Zygo 13-buckets algorithm. The Fourier transforms of the window function of Algorithm A clearly satisfy,

$$\widetilde{W}^j(k\nu_0) = 0, \quad j = 0,1,2,3, k = -q,\ldots,-2,0,1,2 \tag{42}$$

which means self-designed algorithms are simultaneously resistant to a phase-shift nonlinearity up to third-order and a signal nonlinearity up to second-order.

From Fig. 1, it is clear that, with the increase of β, the bandwidth of window function reduces, side-lobe level increases, and order of zeros at the integer harmonics of the window function remains unchanged. It means that, with the increase of β, the suppression capability of algorithm to random noise becomes stronger, the order of phase-shifting error which can be suppressed is unchanged, and the suppression capability of algorithm to higher order harmonics is weaker [8,16]. So, if random noise is large, an algorithm with larger β should be chosen, conversely, an algorithm with smaller β should be chosen.

From theoretical analysis, we get a conclusion that self-designed algorithms meet the desired requirements listed in Section 4. Next, we present some numerical simulations, with example of $\beta = 42$ named as

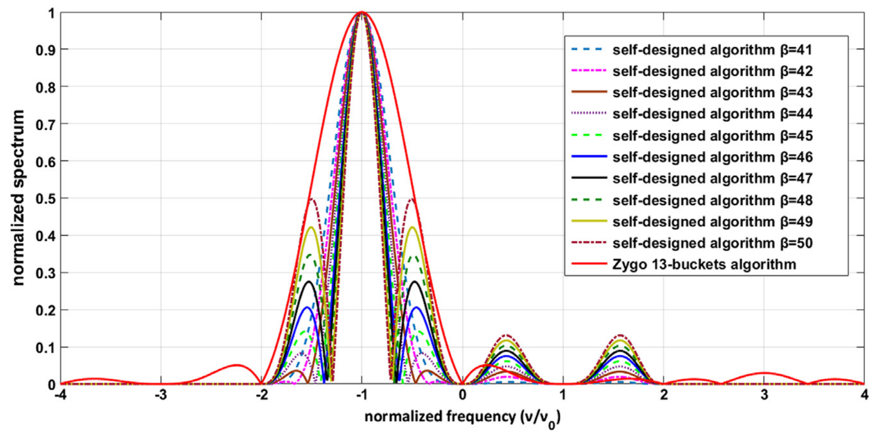

Fig. 1. Normalized amplitude spectrum of Algorithm A and of Zygo 13-buckets algorithm.

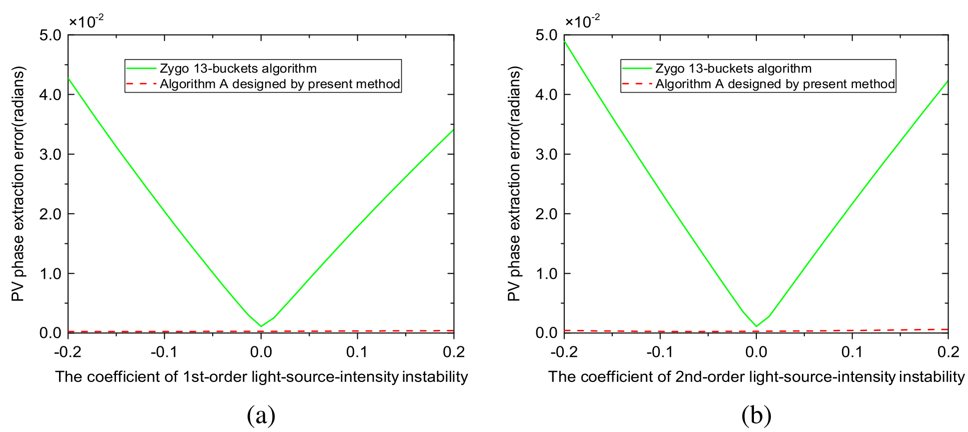

Fig. 2. (a) PV phase extraction error versus 1st-order light-source-intensity instability. (b) PV phase extraction error versus 2nd-order light-source-intensity instability.

Table 1
Settings of error sources for simulation of light-source-intensity instability.

Error sources	Value
1st-order phase-shift error	5%(*with phase unit radians*)
2nd-order phase-shift error	−0.5%(*with phase unit radians*)
3rd-order phase-shift error	0.025%(*with phase unit radians*)
1st-order detector-response error	10%(*at normalized gray level*)
2nd-order detector-response error	10%(*at normalized gray level*)

Algorithm A, to evaluate the performance of self-designed algorithms and the Zygo 13-frames algorithm.

PV (peak-to-valley) of phase difference $\phi(x, y)$ influences the phase extraction error. If the PV of $\phi(x, y)$ is less than a phase period 2π, the PV of phase extraction error is nearly proportional to the PV of $\phi(x, y)$, otherwise it is irrelevant. Therefore, we set the PV of $\phi(x, y)$ to be 10 radians (greater than 2π) in following numerical simulations.

Fig. 2(a) and (b) show the phase extraction error induced by the first- and second-order light-source-intensity instability, respectively. Table 1 lists the settings of the other error sources for this simulation. The abscissa gives the coefficient of light-source-intensity instability expressed in Eq. (29). The curves in Fig. 2 show clearly that the PV phase extraction error of Algorithm A is very small (from 0.0002 to 0.0006 radians) and changes less with the increase of the coefficient. However, the PV phase extraction error of Zygo 13-frames algorithm is quite different: it is nearly proportional to light-source-intensity instability, and it is much larger, even reaches 0.049 radians. So, Algorithm A is more insensitive to first- and second-order light-source-intensity instability.

Fig. 3(a)–(c) show the phase extraction errors induced by the first-, second-, and third-order phase-shifting errors, respectively. The Zygo 13-frames algorithm is sensitive to light-source-intensity instability, so this simulation is done without it, but with 10% first-order and 10% second-order detector response error (at normalized gray level). The red curves, which represent Algorithm A, shown in Fig. 3(a)–(c) are always below the green curves, which represent Zygo 13-frames algorithm. It means PV phase extraction error of Algorithm A is always smaller than that of the Zygo 13-frames algorithm. Even if the coefficients reach the maximum, the PV phase extraction error of Algorithm A is small than one third of the PV phase extraction error of Zygo 13-frames algorithm. This simulation proves that Algorithm A is more resistant to first-, second-, and third-order phase-shifting errors.

Fig. 4(a) and (b) show the phase extraction errors induced by the first- and second-order detector-response error, respectively. This simulation is done without light-source-intensity instability for the same reason as above, but with 5% first-order, −0.5% second-order, and 0.025% third-order phase-shift error (with the phase in units of radians). As shown in Fig. 4(a), both curves do not change with the increase of coefficient, it means both algorithms are resistant to first-order detector-response error. The curves in Fig. 4(b) are quite different. The red curve, which represents Algorithm A, is always below the green one, which represents Zygo 13-frames algorithm, and it changes much slower. And the green curve is 3 – 5 times larger than the red one. Obviously, Algorithm A is more resistant to first- and second-order detector-response errors.

In summary, Algorithm A, which is designed by the proposed method, is resistant to first-, second-, and third-order phase-shifting

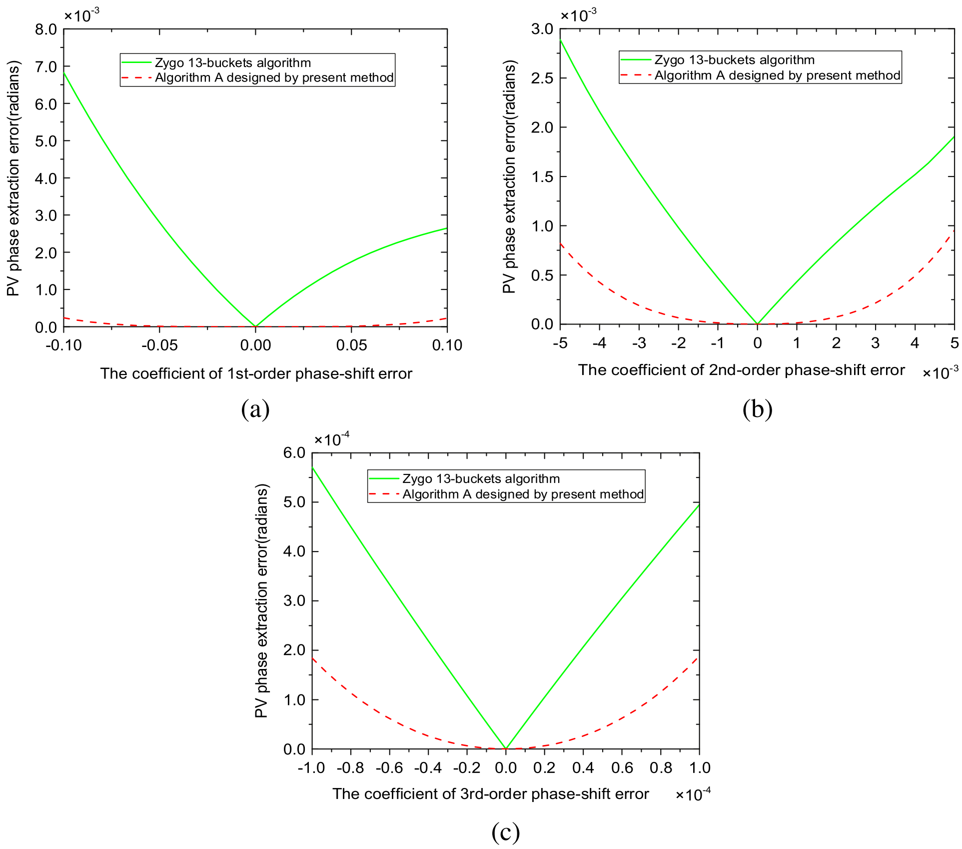

Fig. 3. (a) PV phase extraction error versus first-order phase-shift error. (b) PV phase extraction error versus second-order phase-shift error. (c) PV phase extraction error versus third-order phase-shift error.

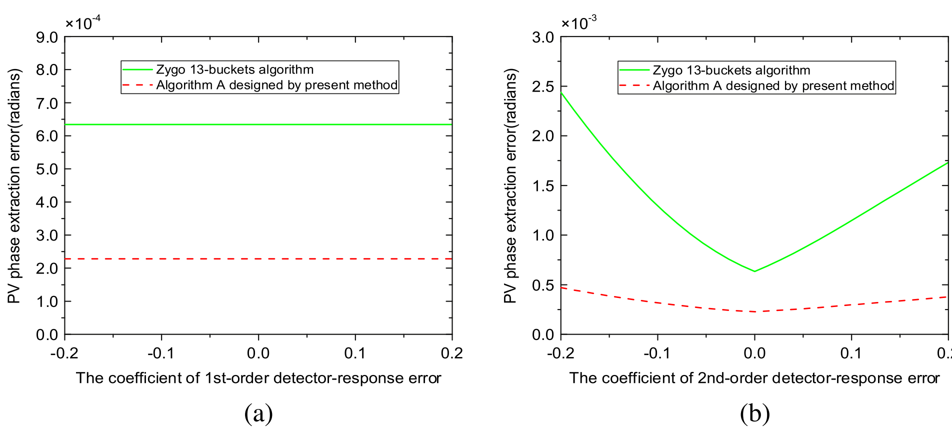

Fig. 4. (a) PV phase extraction error versus first-order detector-response error. (b) PV phase extraction error versus second-order detector-response error.

error, first- and second-order detector-response error, and first- and second-order light-source-intensity instability. These results verify the proposed method and demonstrate its effectiveness.

6. Conclusions

This paper proposes a comprehensive method to design effective phase-shifting algorithms which are resistant to various error sources, such as the multi-order phase-shift error, detector-response error, and light-source-instability. Aiming at a self-designed interferometer, we consider the error source induced by nonideal performance of devices, and use the proposed method to design phase-shifting algorithms which are capable of suppressing interested error sources. The self-designed algorithms are evaluated through Fourier transform spectral analysis and numerical simulations. The results indicate that the self-designed algorithms are resistant to the given error sources, meet the desired requirements, and its PV phase extraction error is much smaller than that of the Zygo 13-frames algorithm. Therefore, the proposed method is verified and its effectiveness is demonstrated.

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