



Ultra-low-distortion optical system design based on tolerance sensitivity optimization

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ABSTRACT

This study theoretically analyzed the relationship between the primary aberrations and tolerances to meet the distortion requirements of an ultra-low-distortion optical system. Theoretical analysis shows that the distortion sensitivities of the optical system are related to the primary aberrations of the optical surfaces. Consequently, a method of optimizing the primary aberrations on the surface of the optical system to reduce the distortion sensitivities of sensitive tolerances is proposed. A distortion sensitivity analysis of manufacturing and assembly tolerance was carried out for the designed ultra-low-distortion and telecentric optical system. The distortion sensitivities of the sensitive tolerances were reduced by optimizing the primary aberrations on the surface of the optical system. A Monte Carlo analysis of the optimized ultra-low-distortion optical system showed that the proposed method has a remarkable effect on the distortion sensitivity optimization of the tolerances.

1. Introduction

As a basic artifact of all imaging optical systems, distortion exerts a strong influence in many applications, especially for photogrammetry cameras, standard lenses, star sensors, and star simulators, which require an image's coordinate to be positioned precisely. An optical system with ultra-low distortion is a basic way to guarantee such precise positioning [1–3]. Unfortunately, it is very difficult to acquire ultra-low distortion below the 0.1% level on actual wide-field systems, even with an excellent design result. For most of the design phase, relative distortion values are sometimes set as merit functions, but its sensitivity to tolerances is rarely considered. Therefore, the unavoidable errors brought by manufacturing and assembly [4–6], will cause the distortion to become unexpected. To meet the index requirements of an ultra-low-distortion optical system, it is of great importance that the distortion sensitivities of the manufacturing and assembly tolerances are analyzed and then specifically reduced.

Currently, there are few methods to reduce tolerance sensitivities [7–11]. Tolerance analysis analyzes image quality sensitivity, but distortion is the only primary aberration that does not affect image quality. Thus, previous studies have generally analyzed the influence of tolerances on the sensitivity of the modulation transfer function (MTF) of given optical system. For some ultra-low-distortion optical systems, the influence of tolerance distortion is neglected, which significantly increases the distortions of the optical systems.

Therefore, this paper proposes a method to reduce the distortion sensitivity of the optical system tolerances when optimizing the optical system. In the optimization phase of the optical system, the distortion sensitivities of the manufacturing and assembly tolerances of the optical system are analyzed, and parameters with high distortion sensitivities are found. Then, the primary aberrations of the sensitive surface are controlled to reduce the distortion sensitivity. The example analysis showed that this method had a remarkable effect on the distortion sensitivity optimization of the tolerances and could relax the tolerances without increasing the costs.

2. Relationship between primary aberrations and tolerances

High-order aberrations are small, much smaller than the primary aberrations, and the variation with the slight structural change of the optical system can often be ignored; therefore, the tolerances can be determined according to the size of the primary aberrations. However, for a high-quality optical system with large aberration compensation, the aberration produced by each optical surface is larger than the residual aberration of the optical system. At this time, a small change in some parameters is enough to destroy the performance of the optical system, so the tolerances are often very harsh.

The change of aberrations in the optical system with structural parameters consists of two parts: The first is the direct effect, which is called intrinsic change, and the second is the indirect effect, which is

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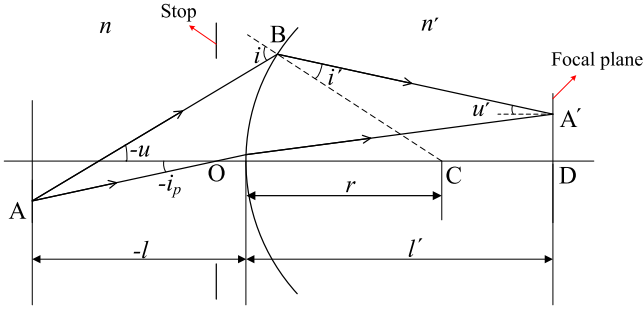


Fig. 1. Light path diagram.

called derived change, due to the intrinsic change, the Gaussian optical properties of the received beams in the latter group of the optical system are changed. The sum of the intrinsic change and derived change is the total aberration change [12].

2.1. Intrinsic change

In an optical system, when the thickness of a lens increases Δd , the l of the latter group changes to $l - \Delta d$, and l_p changes to $l_p - \Delta d$. That is, the object and the stop have moved, which indirectly affects the aberrations, and the intrinsic change is zero.

Taking axial light as an example, the intrinsic change of each aberration caused by the radius change can be obtained by the following:

$$\Delta S_I = S_I \left(2 \frac{\Delta i}{i} + \frac{\Delta i'}{i' - u} \right) \quad (1)$$

$$\Delta S_{II} = S_{II} \left(\frac{\Delta i}{i} + \frac{\Delta i'}{i' - u} + \frac{\Delta i_p}{i_p} \right) \quad (2)$$

$$\Delta S_{III} = S_{III} \left(\frac{\Delta i'}{i' - u} + \frac{2\Delta i_p}{i_p} \right) \quad (3)$$

$$\Delta S_{IV} = S_{IV} \times \Delta c \quad (4)$$

$$\Delta S_V = S_V \left(\frac{\Delta S_{III} + \Delta S_{IV}}{S_{III} + S_{IV}} + \frac{\Delta i_p}{i_p} \times \frac{\Delta i}{i} \right) \quad (5)$$

where S_n ($n = I, II, III, IV, V$) represents the primary spherical aberration coefficient, primary coma coefficient, primary astigmatism coefficient, primary field curvature coefficient, and primary distortion coefficient, respectively. ΔS_n ($n = I, II, III, IV, V$) is the change of the primary aberration coefficient after the change in radius. i, i', u, u' , and i_p represent the angle of the incident light, the angle of the exit ray, the angle between the incident light and the optical axis, the angle between the exit ray and the optical axis, and the incident angle of the principal light, respectively. $\Delta i, \Delta i'$, and Δi_p are the changes of i, i' , and i_p after the change in radius, and Δc is the change in curvature. Fig. 1 shows the light path of a single refractive surface.

2.2. Derived change

In an optical system, when one structural parameter has been changed, the altered beam accepted by the latter group of the optical system can always be seen as the result of the following four variations: object height change, object movement, stop movement, and stop diameter change. The total derived aberrations of the optical system back group (from the i th surface to the k th surface) are:

$$\Delta S_I = B [4S_{II} + J (u'_k{}^2 - u_i^2)] + 4S_I \left(\frac{\Delta u - Bu_p}{u} \right) \quad (6)$$

$$\Delta S_{II} = B [3S_{III} + S_{IV} + J (u'_k u'_{pk} - u_i u_{pi})] + 2S_{II} \left(\frac{\Delta u - Bu_p}{u} \right) + \Delta S_I \quad (7)$$

$$\Delta S_{III} = B [S_V + J (u'_{pk}{}^2 - u_{pi}^2)] + 2\Delta S_{II} \quad (8)$$

$$\Delta S_{IV} = 0 \quad (9)$$

$$\Delta S_V = BS_{Ip} - 2 \left(\frac{\Delta u - Bu_p}{u} \right) S_V + A (3S_{III} + S_{IV}) \quad (10)$$

$$A = -\frac{nu_p^2 \Delta l_p}{J}, \quad B = \frac{nu^2 \Delta l}{J} \quad (11)$$

where J is the Lagrange invariant, Δl_p is the movement change of the stop, and Δl is the movement change of the object. S_n ($n = I, II, III, IV, V, I_p$) represents the sum coefficients from the i th surface to the k th surface about the primary spherical aberration, the primary coma, the primary astigmatism, the primary field curvature, the primary distortion, and the stop spherical aberration, respectively.

Bringing Eq. (12) into the derived aberration expressions, the derived variations of the corresponding aberrations when the lens thickness is changed can be obtained.

$$\begin{cases} nu^2 \Delta l = -nu^2 \Delta d \\ nu_p^2 \Delta l_p = -nu_p^2 \Delta d \\ \Delta u = 0 \end{cases} \quad (12)$$

Bringing Eq. (13) into the derived aberration expressions, the derived variations of the corresponding aberrations when the curvature radius of the lens changes can be obtained.

$$\begin{cases} nu^2 \Delta l = (n - n') h^2 \Delta c \\ nu_p^2 \Delta l_p = (n - n') h_p^2 \Delta c \\ \Delta u = \left(1 - \frac{n}{n'} \right) h \Delta c \end{cases} \quad (13)$$

where h is the incident height of the incident light, and h_p is the incident height of the principal light.

2.3. Method of reducing tolerance distortion sensitivity

In an optical system, when the tolerances are $\Delta x_1, \Delta x_2, \dots, \Delta x_n$, and all tolerances are assumed to be independent of each other, the tolerance sensitivity M of the optical system is defined as [13,14]:

$$M = \sqrt{\sum_i \left(\frac{\partial \Phi}{\partial x_i} \Delta x_i \right)^2} \quad (14)$$

where x_i is the structural parameter of the optical system, e.g., lens thickness, and curvature radius. Φ is the merit function of the optical system, e.g., optical system modulation transfer function (MTF), root mean square (RMS) spot radius and distortion. Parameter sensitivity $\partial \Phi / \partial x_i$ indicates the rate of change of the objective function.

Theoretical analysis of the lens thickness and the curvature ratio showed that the influence of lens thickness change on optical system distortion could be obtained by Eq. (15), and that the radius change on the optical system distortion could be resolved by Eq. (16). It can be seen from Eqs. (15) and (16) that primary astigmatism, primary field curvature and primary distortion directly affect distortion sensitivity. Further analysis found that the changes of other processing and assembly parameters are also related to the primary aberrations. The smaller the primary aberration coefficient of the optical system, the smaller the influence of the parameter change on the system distortion, and the lower the distortion sensitivity of the optical system tolerances [15,16].

$$\Delta S_V = BS_{Ip} - 2S_V \left(\frac{\Delta u - Bu_p}{u} \right) + A (3S_{III} + S_{IV}) \quad (15)$$

$$\begin{aligned} \Delta S_V = S_V \left(\frac{\Delta S_{III} + \Delta S_{IV}}{S_{III} + S_{IV}} + \frac{\Delta i_p}{i_p} \times \frac{\Delta i}{i} \right) + BS_{Ip} \\ - 2S_V \left(\frac{\Delta u - Bu_p}{u} \right) + A (3S_{III} + S_{IV}) \end{aligned} \quad (16)$$

Optimizing the primary aberrations on the lens surface of the optical system can reduce the tolerance distortion sensitivity. In the design of the optical system and the optimization process of the distortion sensitivity, the primary aberrations of sensitive surfaces are constrained

Table 1

Optical design parameters.

Wavelength	632.8 nm
Entrance pupil	25.4 mm
Full field of view	16°
Focal length	90 mm
Distortion	$\leq 7.3 \times 10^{-3}\%$
Centroid shift	$\leq 1.0 \mu\text{m}$

by primary aberration operands, e.g., ASTI, FCUR, and DIST. When optimizing the primary aberrations of a sensitive surface, blindly pursuing the low aberrations of one surface is not allowed as that will cause the aberrations of the other surfaces to become larger. Therefore, comprehensive consideration should be taken into the optimization process. When the operands have little effect on the primary aberrations of each surface, the incident and refracted angles of the incident rays at a surface can be optimized. The absolute values of the incident and refracted angles on each surface are small, and thus, the sensitivity of the system is small [17].

3. Ultra-low-distortion optical system design and tolerance analysis

In the optical system calibration process based on grating, the machining error of grating period is very harsh, and the current level of industrial machining is far from the requirement [18,19]. Therefore, the designed ultra-low-distortion lens was used to rigorously calibrate the grating period. The performance index of the designed ultra-low-distortion optical system needs to match the parameters of the optical system.

3.1. Design parameters

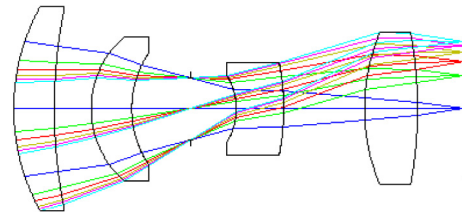
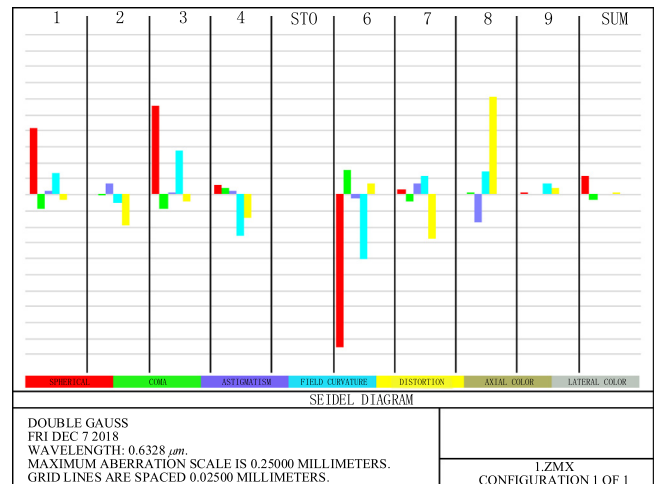
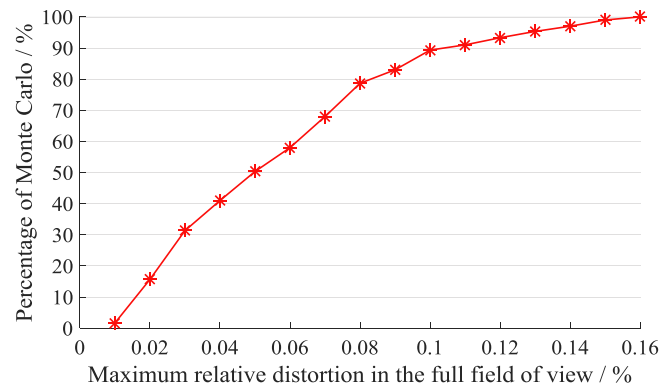
The ultra-low-distortion lens designed is a standard optical system used to calibrate the grating period, it belongs to the telecentric optical system and requires extremely low distortion in the full field of view. During grating calibration, the centroid of each imaging point on the image plane of the ultra-low-distortion lens is extracted. Therefore, in the design of the optical system, it is necessary to strictly control the centroid shift in the full field of view and the circular symmetry of the imaging points in the full field of view. The index requirements of the ultra-low-distortion optical system are shown in Table 1.

3.2. Optical system design

According to the design index requirements of the optical system, the optical system designed is a small aperture, large F number, monochromatic light incidence and telecentric optical system, so it is more reasonable to select a refractive optical system.

In the fully symmetric optical system, it is known from analyzing the structural characteristics that: the spherical aberration, astigmatism, field curvature, and axial chromatic aberration of the left are equal to the rights', so the aberrations of the whole optical system are twice as large as those half of the whole optical system. Coma, distortion, and magnification chromatic aberration of the left are equal in number to the rights', and the signs are opposite, so the aberrations of the whole optical system are zero [20]. Therefore, the ultra-low-distortion optical system presented in this paper uses double Gaussian as the initial structure to optimize, and minimize the number of lenses. At the same time, the machining rationality of the lenses should be taken into consideration.

Since the designed ultra-low-distortion lens is a standard lens, a large asymmetric aberration will cause an asymmetric degeneration of imaging points in the full field of view, thus affecting the centroid shift. Therefore, asymmetrical aberrations, such as coma and astigmatism, should be strictly controlled in the design phase. The relative distortion of the ultra-low optical system optimized by the optical design software Zemax is less than $4.3 \times 10^{-3}\%$, which meets the design index requirement. The structure is shown in Fig. 2. Fig. 3 shows the Seidel diagram with the largest aberration scale of 0.25 mm.

**Fig. 2.** Structure of the optical system.**Fig. 3.** Seidel diagram.**Fig. 4.** Distortion distribution curve.

3.3. Tolerance analysis

When the designed optical system has been manufactured and assembled, the imaging points in the full field of view require the center symmetry to be maintained to meet the centroid extraction accuracy. Therefore, in the tolerance analysis, the RMS spot radius is used as the evaluation criterion, and the back length of the optical system is used as the compensation.

After inverse sensitivity analysis, the tolerances of the optical system are distributed: the radius is 2 fringes, the lens thickness is 0.02 mm, the lens spacing is 0.02 mm, the surface eccentricity is 0.01 mm, the surface tilt is 0.02° , the surface irregularity is 0.25 fringes, the lens eccentricity is 0.01 mm, the lens tilt is 0.02° , the refractive index is 0.001, and the Abbe number is 0.1%.

After 500 Monte Carlo analysis, the distribution law of the maximum relative distortion in the full field of view was simulated. The statistical results are shown in Fig. 4. As shown in Fig. 4, in the 500 simulated

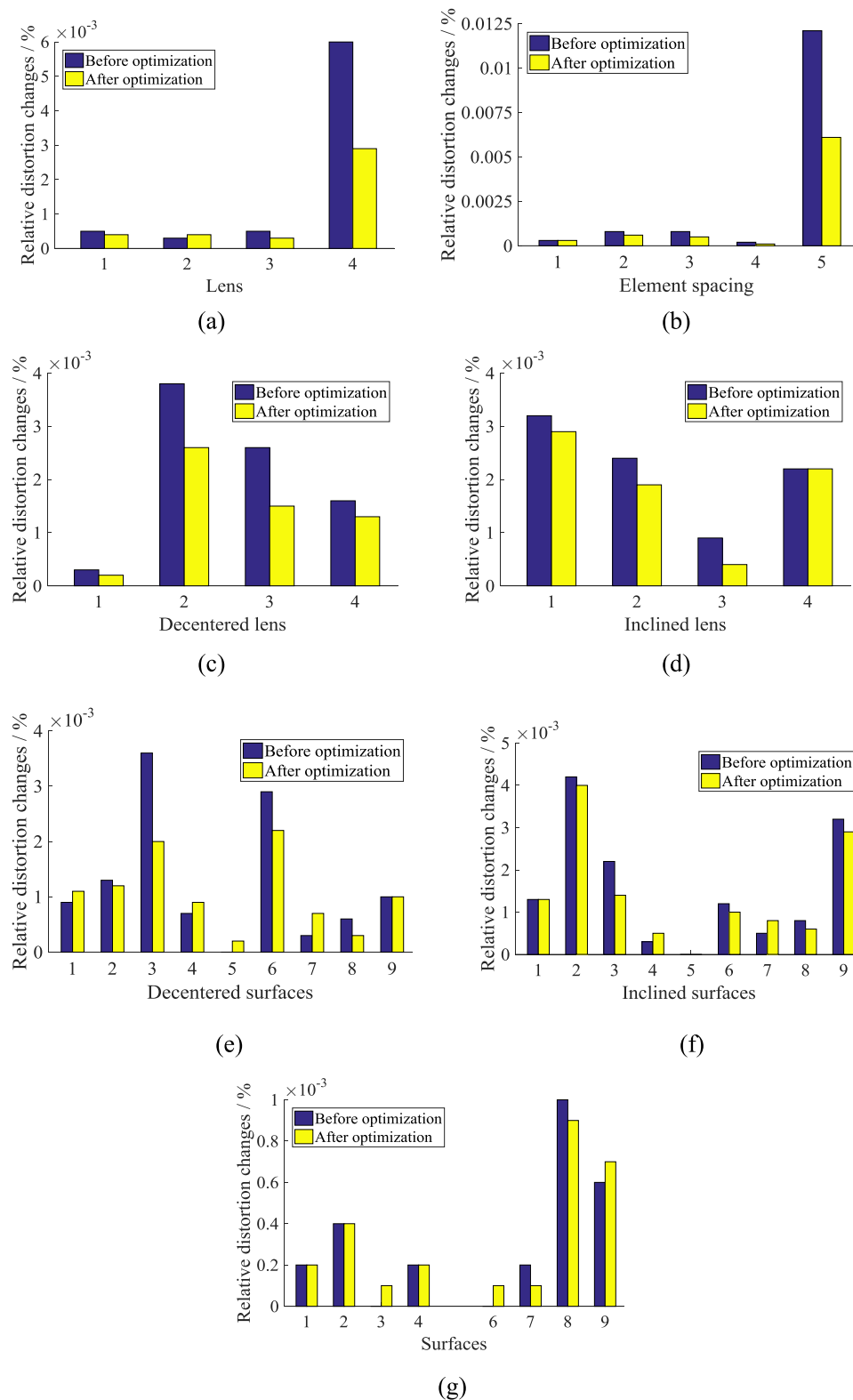


Fig. 5. Distortion sensitivities of the tolerances before and after optimization. (a) distortion sensitivity of the lens thickness, (b) distortion sensitivity of the element spacing, (c) distortion sensitivity of the lens decenter, (d) distortion sensitivity of the lens tilt, (e) distortion sensitivity of the surface decenter, (f) distortion sensitivity of the surface tilt, and (g) distortion sensitivity of the surface radius.

optical systems, to the 80% optical systems the maximum relative distortion is less than 0.085%, and in the worst lens the distortion is as high as 0.16%, which shows that the tolerances greatly influence the distortion.

Monte Carlo analysis shows that, in order to meet the requirements of the ultra-low-distortion lens after manufacturing and assembly, it is necessary to set very strict tolerances, and strict tolerances greatly improve the difficulty of machining and assembly, which are even higher than the existing machining and assembly accuracy. Therefore,

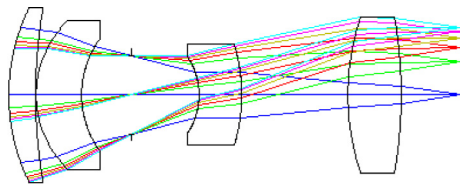


Fig. 6. Structure of the optimized optical system.

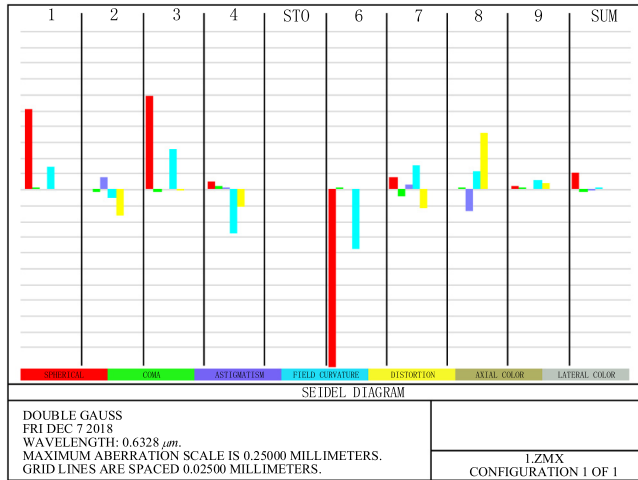


Fig. 7. Seidel diagram.

reducing the distortion sensitivities of the optical system tolerances is of great significance.

4. Example of tolerance distortion sensitivity optimization

4.1. Distortion sensitivity analysis of tolerances

The ultra-low distortion of the optical system designed in this paper has extremely low distortion, which will be seriously affected by the slight change of the machining and assembly errors. Therefore, the distortion sensitivity of the optical system tolerances should be analyzed. Machining tolerances include the following: the surface decenter, surface tilt, lens thickness, surface radius, surface irregularity, refractive index, and Abbe number. Assembly tolerances include the following: the lens spacing, lens decenter, and lens tilt.

The distortion sensitivity of the tolerances in the ultra-low-distortion optical system are shown in Fig. 5. Compared and analyzed the influence of various tolerances on the distortion sensitivity, from which we know that the tolerances have a large influence on the distortion sensitivity included: the thickness of the fourth lens, the back length, and the tilt tolerance of the second surface are generally higher than those of other parameters.

4.2. Distortion sensitivity optimization of tolerances

As can be seen from the Seidel diagram in Fig. 3, the astigmatism of the eighth surface, the field curvature of the sixth surface, and the distortion of the eighth surface are larger. Therefore, the primary aberrations of the sensitive surface of the original ultra-low-distortion optical system are optimized further. The relative distortion of the optimized system is less than $3.6 \times 10^{-3}\%$ in the full field of view, and the structure of the optimized optical system is shown in Fig. 6. Fig. 7 shows the Seidel diagram of the optimized optical system, and its scale is the same as that in Fig. 3.

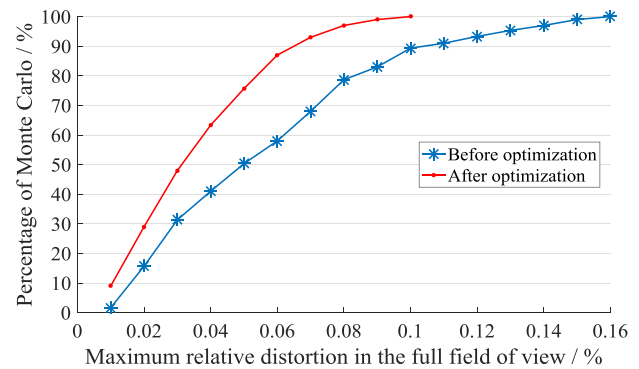


Fig. 8. Distortion distribution curves before and after optimization.

Comparing with the primary aberrations in Figs. 3 and 7, the primary astigmatism and the distortion of the eighth surface of the optical system are properly corrected, and the primary aberrations of other surfaces are also optimized to some extent. However, the primary spherical aberrations of the first surface, third surface, and sixth surface increased a little, but not to the extent to which they can influence distortion sensitivity. In order to ensure that MTF sensitivities are not affected, the primary spherical aberration cannot be changed abruptly.

Using the same tolerances to analyze the optimized optical system, in the process of Monte Carlo analysis, the RMS spot radius is used as the optimization function to ensure center symmetry of the imaging points, and the back length of the optical system is used as the compensation. The distortion sensitivities of the tolerances are shown in Fig. 5.

As shown in Fig. 5, the distortion sensitivity of the back length, which has the highest distortion sensitivity, was reduced from 0.0121% to 0.0061%, and the distortion sensitivity of the thickness of the fourth lens was reduced from 0.006% to 0.0029%. The distortion sensitivities of the other parameters are also optimized to some extent.

However, comparing with the distortion sensitivities of element spacing tolerances, the distortion sensitivities of surface radius tolerances are more than ten times smaller. High distortion sensitivity is more important to be optimized, but low distortion sensitivity is meaningless and difficult to be optimized. So the distortion sensitivities of the surface radius have barely changed.

The Monte Carlo analysis of the optimized optical system is performed 500 times, and the distortion distribution law is shown in Fig. 8.

Comparing the distortion distribution curves before and after optimization, it can be seen that the maximum distortion of the worst-quality lens of the optimized optical system after manufacturing and assembly is less than 0.1%, which is far less than 0.16% of the case without optimized lens, and the probability of having relative distortion less than 0.06% increased from 58% to 87%. Therefore, it is verified that the optimization method has a remarkable effect on the distortion optimization of optical system tolerances.

In order to further reduce the distortion of the lens, the back length can be used as compensation. When using the back length to compensate for the distortion, the size and center symmetry of the imaging points, and the imaging quality should be fully considered. However, the compensation of the back length is very limited. As can be seen from Fig. 8, the back length changes 0.02 mm, and the relative distortion changes 0.0061%. Therefore, the most effective distortion optimization method is to reduce the distortion sensitivities of the tolerances.

5. Conclusion

When manufacturing and assembling ultra-low-distortion optical systems, strict tolerances are required to meet the index requirements. In order to relax the tolerances, the distortion sensitivities of the tolerances

are analyzed by example, and the distortion sensitivities of sensitive tolerances are reduced by optimizing the primary aberrations on the lens surface of the optical system. Monte Carlo analysis shows that the distortion of the optimized optical system is controlled remarkably under the same tolerances, and the method has a significant effect on the distortion optimization of the optical system. At the same time, the distortion sensitivity analysis of the tolerances also provides a quantitative analysis of the tolerance formulation.

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