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The effect of beam quality factor for the laser beam propagation through turbulence

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ABSTRACT

Beam quality factor is an important parameter for evaluating the laser beam when the beam propagates through turbulence. In this paper, analytical expressions of lowest-Gaussian beam, Hermite-Gaussian beam and Laguerre-Gaussian beam propagating in free space are derived. The relationship between the beam quality factor and the peak intensity at the target is discussed. Besides, the influences induced by other parameters like beam width and propagation distance are also analyzed in detail. It can be concluded that with the increase of the turbulence strength, the beam quality factor loses its place of domination in beam propagation. Finally, a simulation of six laser beams combining is carried out. The result shows that the selection of the light source's beam quality can be extended for the applications under the atmospheric turbulence.

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1. Introduction

High-energy, high-quality laser beam output has always been an important subject for laser beam communications, directed energy applications as well as laser manufacture. Many researchers are willing to make the commitment to this field and have acquired huge amounts of achievements[1,2]. Nevertheless, a single-mode laser output is merely in an order of around 10 kW limited to the thermal effects and optical nonlinearities[3]. Therefore, laser beam combining is proposed as a technology that can achieve high laser output and near-diffraction-limited beam quality at the same time[4]. Laser beam combining can be categorized into coherent beam combining (CBC) and incoherent beam combining (ICBC). The main difference between these two configurations is that the CBC architecture is actively phase-locked. On the contrary, an array of lasers is superimposed in the far field without the control of the relative phases of the different elements in the ICBC architecture[5]. The atmospheric turbulence has always been a big challenge for the application of laser beam combining[6] for it would induce the beam expand, beam wander as well as scintillation[7]. Beam quality is a practical metric for evaluating the laser beam[8,9]. There are many definitions of the beam quality, like the power in the bucket (PIB), beam parameter product (BPP) and the Strehl ration[9]. Among all of the metrics, the M² factor is one of the most popular laser beam metrics used by the scientific community. The M² factor, defined by the beam size and the beam divergence, can describe the focusability, mode content and other important characters of the laser beam. A bunch of laser beams with different M² factor that propagates through the atmospheric turbulence would experience different beam spreading and beam wander at the target plane. An army of researches have been carried on to investigate the effects of laser beam propagation through

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atmospheric turbulence [6,10]. However, to our knowledge, the questions of how the M^2 factor influences the peak intensity and the selection of applicable M^2 factor for laser beam combining system have not been studied by far. In this paper, the beam quality is chosen as the main character of a laser beam to see how the atmospheric turbulence would influence the beam.

2. The beam quality factor and the atmospheric turbulence

The M^2 factor of a laser beam is defined as below:

$$M^2 = \frac{\Theta W}{\Theta_0 W_0} \quad (1)$$

in which Θ_0 and W_0 are the beam divergence and spot size of an ideal Gaussian reference beam separately. Θ and W are the beam divergence and spot size of an actual beam. The equation above can be simplified using the relationship of the beam divergence and spot size of a Gaussian reference beam as listed below:

$$\Theta_0 = \frac{\lambda}{\pi W_0} \quad (2)$$

where λ is the wavelength of the laser beam. Substitute equation (2) into equation (1), the relationship between the beam divergence and the spot radius can be deduced:

$$\Theta = \frac{\lambda}{\pi W} M^2 \quad (3)$$

The atmospheric turbulence can be described using the power spectrum. The most popular model of the turbulence is the Kolmogorov power spectrum as showed below[7]:

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3}, \quad \gamma_{l_0} \ll \kappa \ll \gamma_{l_0} \quad (4)$$

The symbol κ in Eq. (4) represents the scalar spatial frequency, C_n^2 is the index-of-refraction structure constant which reflects the strength of the turbulence, L_0 and l_0 are the outer scale and inner scale of the turbulence separately. Because of the limitation in calculating the effects beyond the inner scale and outer scale using the Kolmogorov spectrum model, other spectrum models have been proposed, like the Tatarskii spectrum model and von Kármán spectrum [7].

$$\begin{aligned} \Phi_n(\kappa) &= 0.033 C_n^2 \kappa^{-11/3} \exp\left(-\frac{\kappa^2}{\kappa_m^2}\right), \quad \kappa \gg \gamma_{l_0}; \quad \kappa_m = \sqrt[5.9]{l_0} \\ \Phi_n(\kappa) &= 0.033 C_n^2 \frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2 + \kappa_0^2)^{11/6}}, \quad 0 \leq \kappa < \infty; \quad \kappa_m = \sqrt[5.9]{l_0} \end{aligned}$$

The Gaussian beam can be written as:

$$U_0(r_0, 0) = \exp\left(-\frac{r^2}{W_0^2} - \frac{i k r^2}{2 F_0}\right) \quad (6)$$

in which the amplitude of the beam is assumed to be normalized. F_0 is the radius of the curvature. The cases of $F_0 = \infty$, $F_0 > 0$ correspond to collimated, convergent beam forms. When there is no turbulence in the propagation path, the optical wave at the distance L from the source can be deduced from the Huygens-Fresnel integral as the equation below.

$$U_0(r, L) = \frac{1}{1 + i \alpha_0 L} \exp\left[i k L + \frac{i k}{2L} \left(\frac{i \alpha_0 L}{1 + i \alpha_0 L}\right) r^2\right] \quad (7)$$

in which

$$\alpha_0 = \frac{2}{k W_0^2} + i \frac{1}{F_0} \quad (8)$$

Substituting Eq. (3) into Eq. (8):

$$\alpha_0 = \frac{2}{k W_0^2} + i \frac{\tan(\frac{\lambda}{\pi W_0} M^2)}{W_0} \quad (9)$$

The form of the wave discussed above is the lowest-order Gaussian beam, other higher-mode Gaussian beams like Hermite-Gaussian beam and Laguerre-Gaussian beam[10] are usually used in the real applications. The form of the higher-mode Gaussian beam can be written as:

Hermite-Gaussian beam:

$$U_{mn}(x, y, L) = (\Theta - i \Lambda) \left(\frac{\Theta - i \Lambda}{\Theta + i \Lambda} \right)^{(m+n)/2} H_m\left(\frac{\sqrt{2}x}{W}\right) H_n\left(\frac{\sqrt{2}y}{W}\right) \exp\left[i k L + \frac{i k}{2L} (1 - \Theta + i \Lambda)(x^2 + y^2)\right] \quad (10)$$

Laguerre-Gaussian beam:

$$U_{pl}(r, \theta, L) = \frac{C_p^l}{W} \exp \left[-\frac{r^2}{W^2} - i \frac{kr^2}{2R} + i\Phi_p^l + il\theta \right] \left(\frac{\sqrt{2}r}{W} \right)^l L_p^l \left(\frac{2r^2}{W^2} \right) \quad (11)$$

Where H_m and H_n represent the Hermite polynomials, L_p^l is the Laguerre polynomial, Θ and Λ are the output plane beam parameters, W is beam radius in the output plane. C_p^l is an unimportant constant. The representations of the parameters are listed below.

$$\Theta = 1 + \frac{l}{\lambda}$$

$$\Lambda = \frac{2l}{\lambda k w^2}$$

$$W^2 = W_0^2 (1 + \frac{l^2}{z_R^2})$$

$$z_R = \pi w_0^2 / \lambda$$

$$R = \frac{(z_R^2 + l^2)}{\lambda}$$

$$\Phi_p^l = (2p + l + 1) \arctan(\frac{l}{z_R})$$

3. Propagation in free space and turbulence

The intensity on the axis at the target plane is relevant to the beam quality factor M^2 . It can be inferred from the figure that the intensity on the axis decreases as the M^2 factor increases for each type of the Gaussian beam mentioned above. Additionally, the intensity decreases much faster when M^2 is near 1. This indicates that when the laser propagates in free space, the beam quality factor dominates the propagation efficiency when it is near the ideal Gaussian beam. It can also be inferred that the laser with excellent beam quality is preferred when there is no distribution. Compared Fig. 1 (a) with Fig. 1 (c), the higher mode laser beam has smaller intensity on axis. Therefore, the lowest-Gaussian beam with M^2 equal 1 is preferred when the system is placed in free space.

When the laser beam propagations through the turbulence, the optical wave can be written under the assumption of the Rytov approximation[7].

$$U(r, L) = U_0(r, L) \exp [\psi(r, L)] \quad (13)$$

Where $U_0(r, L)$ denotes the field in the absence of turbulence, ψ is a complex phase perturbation induced by the atmospheric turbulence.

$$\psi(r, L) = \psi_1(r, L) + \psi_2(r, L) + \dots \quad (14)$$

ψ_1 and ψ_2 in the equation above are the first-order and second-order complex phase perturbations respectively. According to the published literature, the ensemble average of the optical field can be represented using the second-order statistical moments:

$$\begin{aligned} \langle U(r, L) \rangle &= U_0(r, L) \langle \exp [\psi_1(r, L) + \psi_2(r, L)] \rangle \\ &= U_0(r, L) \exp [E_1(0, 0)] \end{aligned} \quad (15)$$

in which $E_1(0, 0)$ is the second-order statistical moments which has the form:

$$E_1(0, 0) = -2\pi^2 k^2 \int_0^L \int_0^\infty \kappa \Phi_n(\kappa, s) d\kappa ds \quad (16)$$

The von Kármán spectrum is used in the calculations. Substituting equation 5 and equation 16 into equation 15, the mean field is given by

$$\langle U(r, L) \rangle = U_0(r, L) \exp(-0.39 C_n^2 k^2 L \kappa_0^2) \quad (17)$$

It can be inferred from above equation that when there exists turbulence, the ensemble average of the optical field is the unperturbed portion times a factor of attenuation. Therefore, it can be predicted that the propagation efficiency decreases more slowly than that in free space, which means that in the applications of laser beam propagates through turbulence, the beam quality factor can be extended appropriately.

The decreasing rate for intensity in the condition of atmospheric turbulence becomes smaller compared to the situation in free space as predicted above. The shift of intensity would decrease as the strength of turbulence increases. It can be concluded that when the turbulence is strong, the beam width is small and the propagation distance is long enough, the beam quality factor loses its place of domination. Therefore, the laser with not so perfect beam quality can be chosen as the source and won't affect the efficiency (Fig. 2).

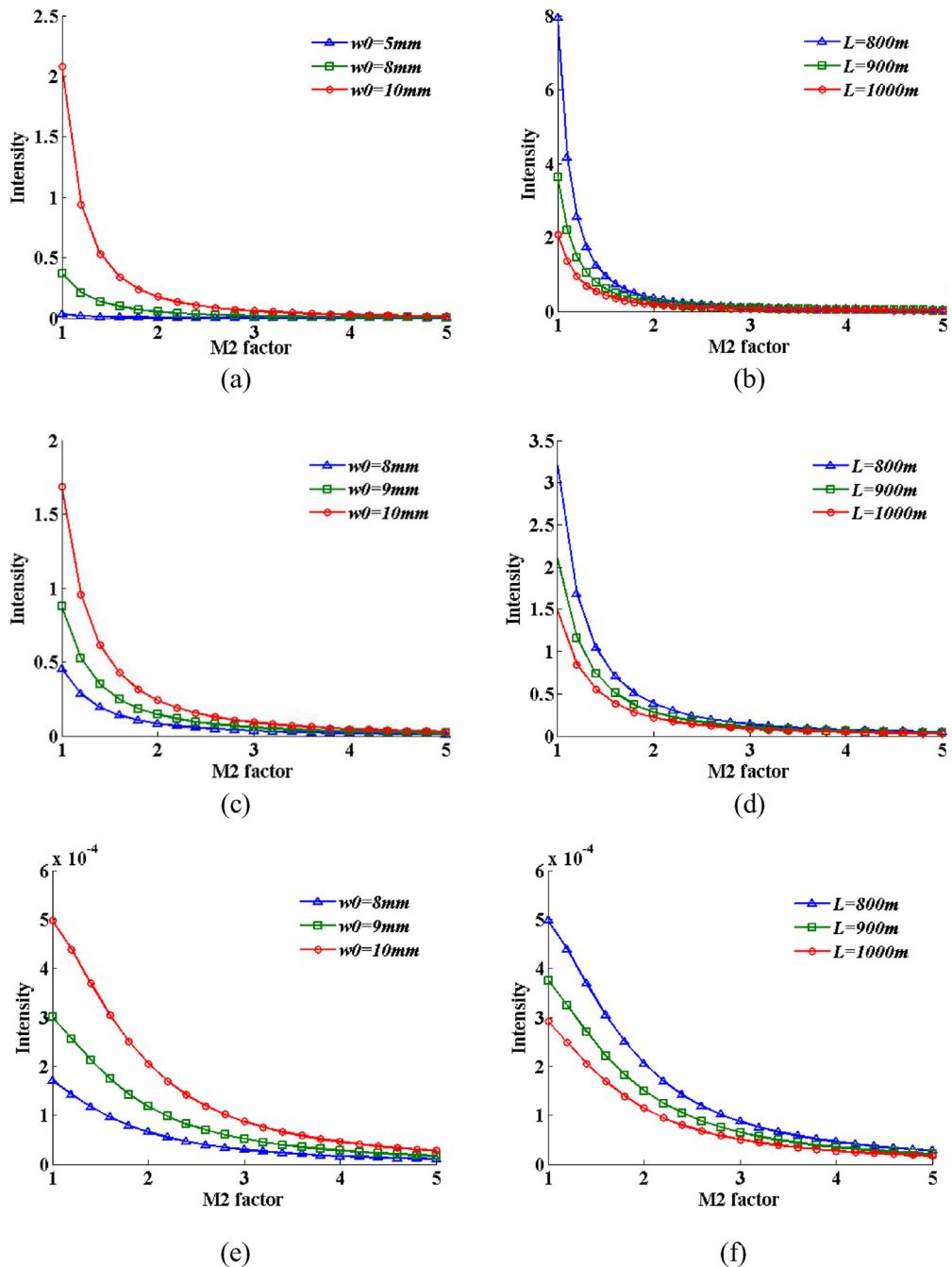


Fig. 1. The intensity on axis, at the target; (a), (b) The intensity on the axis of the lowest-Gaussian beam with different beam radius and different propagation distance; (c), (d) The intensity on the axis of the Hermite-Gaussian beam HG_{11} with different beam radius and different propagation distance; (e), (f) The intensity on the axis of the Laguerre-Gaussian LG_{11} beam with different beam radius and different propagation distance.

4. Six beam combining

The discussions above only take the one laser beam into account. In the application of beam combining, a bunch of laser beams participate in the system. In order to testify the consequences deduced above, a serials of simulations of beam combining system are carried out. The parameters used in the simulations are listed in Table 1.

The coherent length of atmospheric turbulence is another index that measures the strength of the turbulence. The larger the coherent length becomes, the weaker the turbulence becomes. For simplicity of the simulations, the coherent length of

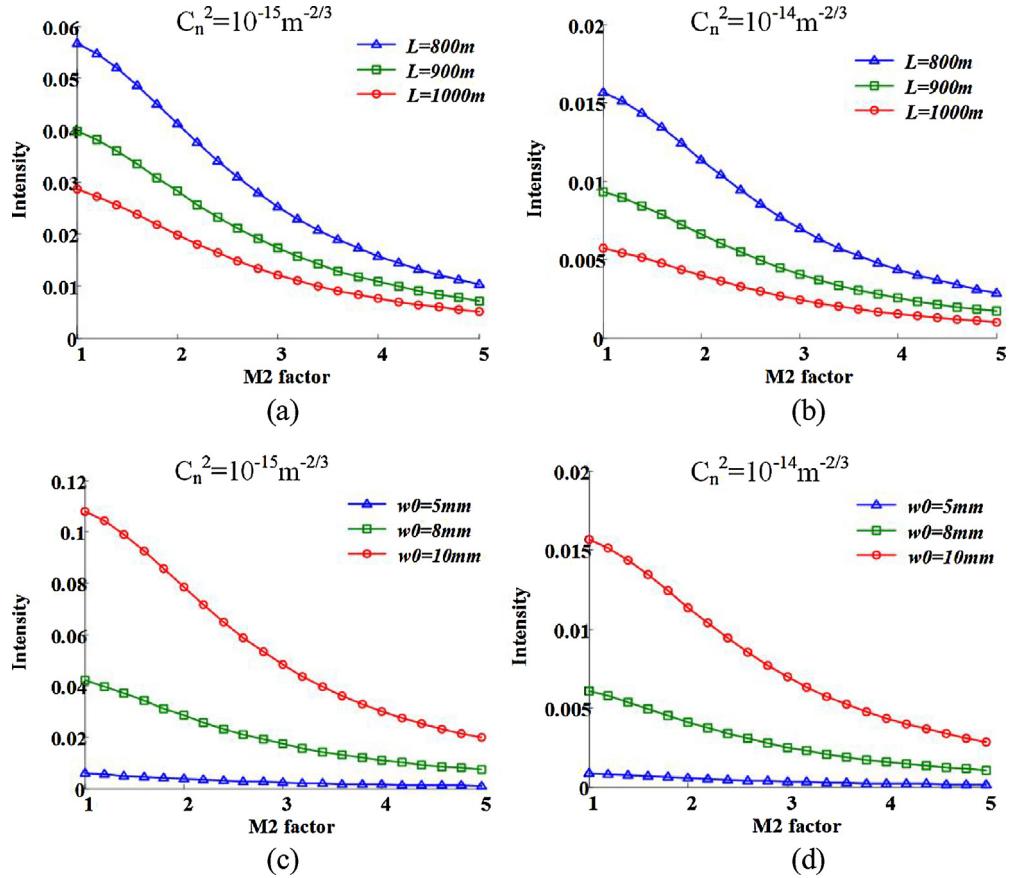


Fig. 2. The intensity on the axis of the lowest-Gaussian beam with different propagation distance.

Table 1

The main parameters used in simulations.

Parameter	Value
Propagation distance: L	400 m
Wavelength: λ	532 nm
Beam waist: w_0	10 mm
Radius of the FSM: r	0.007 m
Sampling number: N	512
Coherent length of atmosphere: r_0	0.1/0.05 m

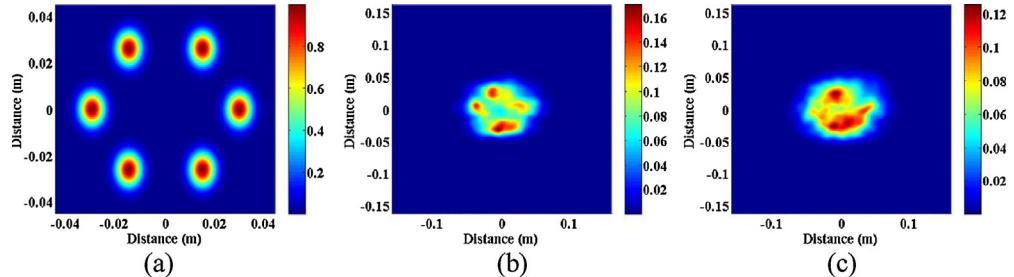


Fig. 3. (a) The intensity distribution at the source; (b) The intensity distribution at the target with the M^2 factor equal 1; (c) The intensity distribution at the target with the M^2 factor equal 1.2.

atmospheric turbulence is chosen to take the place of the index-of-refraction structure constant. The intensity distribution at the source is illustrated in Fig. 3 (a). Fig. 3 (b) and Fig. 3 (c) are the intensity distributions at the target without correction.

It can be inferred from the above figures that the light spot would be blurred due to the turbulence. The peak intensity would decrease as the increase of the M^2 factor as the theory predicts. Therefore, the SPGD algorithm is applied to correct the pointing of the laser beam. The results of correction are showed in Fig. 4.

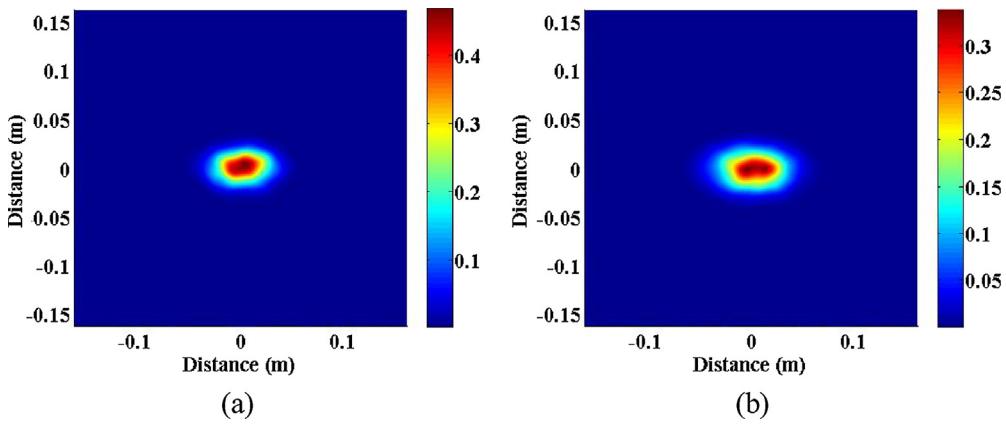


Fig. 4. The intensity distribution at the target with correction; (a) The M^2 factor equals 1; (b) The M^2 factor equals 1.2.

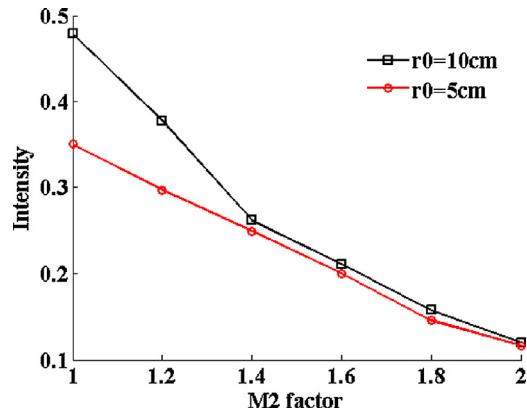


Fig. 5. The intensity on the axis under different strengths of turbulence.

The correction is efficient as showed in above figures. However, the beam quality factor still limited the peak intensity at the target. Fig. 5 is the peak intensity at the target with different strengths of the turbulence.

The result of simulation shown above is in accords with the result derived from theory. The peak intensity falls when the beam quality decreases, however, this phenomenon would be reduced when there exists turbulence in the propagation path. This gives the principle of selecting the light source for the application of beam combining. When the system is placed in free space, the M^2 factors of the lasers should be near 1 in order to achieve a promising high-energy output. However, the selection of the M^2 factors of the lasers can be extended when the system is utilized in turbulence environment. The actual M^2 factor for the system can be deduced using the above principle.

5. Conclusion

In this paper, we investigate the propagation of lowest-Gaussian beam, Hermite-Gaussian beam and Laguerre-Gaussian beam in the free space as well as in the turbulence. The beam quality factor, as one of the most important parameters, has been analyzed in detail. Analytical expressions reveal that the beam quality factor would have a huge influence on the peak intensity when the laser beam propagates in the free space. However, when the beam propagates in the turbulence, the beam quality factor becomes less important. This phenomenon indicates that lasers with not so perfect beam quality can be chosen for the application of beam combining system. Besides, the influences of the beam width as well as the propagation distance are also discussed in this paper. Lastly, a simulation of six beams combining is carried out to testify the influence of the beam quality factor and the turbulence. The results of simulation also show the same rules with the analytical expression. The conclusion gives a method of choosing the type of lasers for the beam combining system whether the system is placed in free space or in the turbulence.

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