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Synchronization of electrically coupled micromechanical oscillators with a frequency ratio of 3:1

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In this Letter, synchronization of micromechanical oscillators with a frequency ratio of 3:1 is reported. Two electrically coupled piezoresistive micromechanical oscillators are built for the study, and their oscillation frequencies are tuned via the Joule heating effect to find out the synchronization region. Experimental results show that the larger coupling strength or bias driving voltage is applied and a wider synchronization region is obtained. Interestingly, however, the oscillator's frequency tunability is dramatically reduced from –809.1 Hz/V to –23.1 Hz/V when synchronization is reached. A nearly 10-fold improvement of frequency stability at 1 s is observed from one of the synchronized oscillators, showing a comparable performance of the other. The stable high order synchronization of micromechanical oscillators is helpful to design high performance resonant sensors with a better frequency resolution and a larger scale factor. *Published by AIP Publishing.* https://doi.org/10.1063/1.5000786

Micro-/nanomechanical devices have been seen as alternatives that could be developed to reform time reference oscillators and frequency-shift based sensors for their many merits such as easier to be integrated with electronics, tiny size and power consumption, and low cost.[1,2] However, they are easily driven into the nonlinear regime and their performance is often degraded by large displacement instabilities due to the scaling effect and nonlinearity.[3–6] To address these challenges, techniques counter-intuitively using nonlinear behaviors to improve the oscillator's performance have attracted extensive interest such as modal coupling,[7–9] internal resonance,[10,11] and parametric feedback.[12]

Synchronization phenomenon first discovered in Huygens' clock shows that the rhythms of oscillating objects can be adjusted via an interaction,[13] and accordingly, it provides another approach to improve the frequency stability of micro-/nanomechanical oscillators.[14–17] When a micromechanical oscillator is synchronized to an external high precision perturbation, there will be a 30 dB phase noise reduction at a 0.5 Hz frequency offset and one order magnitude lifting of frequency stability.[18] Our previous study shows that the frequency stability can be improved nearly 10 folds at 2 s (integration time) and can be further improved by a larger perturbation.[19] This type of synchronization by an external high precision perturbation can be regarded as a special case of two interacting oscillators when the coupling is unidirectional. When a micromechanical oscillator is electrically coupled to an identical oscillator, its frequency stability can be improved up to 7-fold under 1:1 synchronization.[20] Theoretically, mutual synchronization at higher order can also be realized in coupled self-sustained micromechanical oscillators, but it is hardly reported yet, which is partially because it is less robust and has narrow Arnold tongues.[21]

In this Letter, synchronization of micromechanical oscillators with a 3:1 frequency ratio is studied. Two micromechanical Double-Ended Tuning Fork (DETF) resonators were fabricated by using a standard Silicon on Insulator process. Figure 1 shows an image of two electrically coupled micromechanical resonators and a schematic drawing of the piezoresistive oscillator circuits. The two DETFs were designed with a frequency ratio of 3. The low frequency oscillator (LFO) is built upon the long DETF resonator with a length of 550 μm,

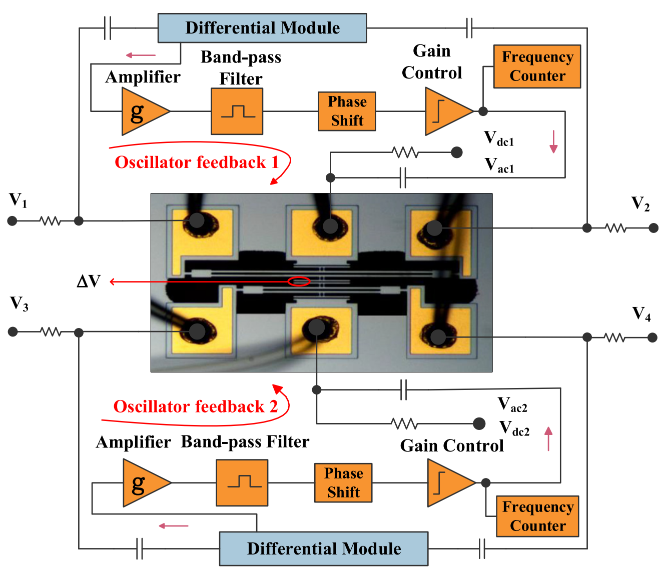

FIG. 1. Optical image of DETF micromechanical resonators and a schematic drawing of the piezoresistive oscillator circuit that consists of DETF, a differential transimpedance amplifier, a bandpass filter, and a phase shifter and comparator.

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and the high frequency oscillator (HFO) is built upon a short one with a length of 292 μm. The DETFs' width and thickness are 7 μm and 25 μm, respectively.

The device is tested in a vacuum chamber at a pressure below 0.03 Pa in order to minimize the air-damping loss. DC voltages of $V_{12}(=|V_1 - V_2|)$ and $V_{34}(=|V_3 - V_4|)$ are applied on each pair of the DETF's electrical pads, and thus, a DC current I_d is generated through the DETF body to facilitate a differential piezoresistive sensing of the transverse displacement.[22] After amplification, the sensed motional signal is filtered, phase shifted, and regulated. The output signal V_{ac} from the gain control element is then fed back to drive the DETF via a parallel-plate electrode with a length of 140 μm and a gap of $g = 2\,\mu$m. The electrostatic driving force f_d keeps DETF in oscillation by compensating the dissipation, and it can be expressed as $f_d = \frac{1}{2}\frac{\varepsilon A}{g^2} V_{ac}V_{bias}$, where ε is the dielectric coefficient and A and g are the area and gap of electrostatic plates, respectively. In this work, V_{ac} is fixed at 130 mV, and accordingly, f_d is proportional to the DC bias driving voltage V_{bias}, which is expressed as $V_{bias1} = V_{dc1} - (V_1 + V_2)/2$ for LFO and $V_{bias2} = V_{dc2} - (V_3 + V_4)/2$ for HFO. The electrical coupling voltage ΔV $[= (V_1 + V_2)/2 - (V_3 + V_4)/2]$ is the electrical potential difference across the coupling parallel-plates, and accordingly, the coupling force is calculated as $f_c = \delta x \frac{\varepsilon A}{g^3}\Delta V^2$, where δx represents the difference of DETFs' modal amplitudes.

Figure 2(a) shows the frequency spectra of individual free running oscillators without electrical coupling. The frequency ratio between HFO ($f_2 = 357.395$ kHz) and LFO ($f_1 = 119.023$ kHz) is 3.0027:1, which is close to but still different from the designed ratio of 3:1 due to unavoidable manufacturing errors. It makes difficult to entrain these two output frequencies because of a narrower Arnold tongue for the high order synchronization case.[21] Therefore, the frequency ratio is adjusted through the Joule heating induced thermal tuning of DETF, which is done by varying V_{12} in this case to adjust the self-oscillation frequency of LFO. The change in V_{12}, however, is negligible as compared to the large DC bias voltage $V_{bias} \sim 10$ V and the coupling voltage $\Delta V \sim 10$ V, and it hardly has an effect on the self-sustaining force and the coupling strength. Figure 2(b) shows the frequency spectra of coupled DETF oscillators, where the orange line clearly indicates the frequency components of HFO and three order harmonics of LFO. Note that an increase of V_{12} with a few steps of $\delta V_{12} \sim 0.01$ V is enough to reach the frequency ratio of 3:1, realizing a higher order harmonic frequency entrainment as indicated by the single peak (blue line) in Fig. 2(b).

To clearly visualize the transition into synchronization, the output frequencies of LFO and HFO are plotted against the drain voltage V_{12} that is varied to tune the frequency of LFO based on the Joule heating effect. When LFO and HFO are not coupled, the black dotted line in Fig. 3(a) shows a linear relationship between the frequency of free running LFO and its drain voltage V_{12}, which gives a slope of $k_{unsyn} = -809.108$ Hz/V representing the piezoresistive thermal frequency tunability.

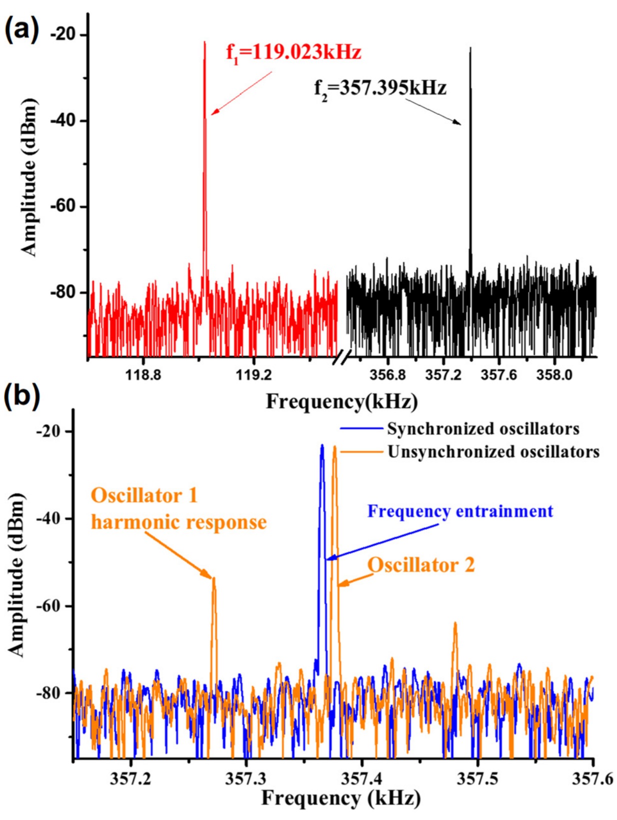

FIG. 2. Frequency spectra of DETF oscillators. (a) Free running oscillations of LFO and HFO without electrical coupling; (b) Frequency entrainment is observed (the blue line) when the frequency of LFO is tuned by varying drain current.

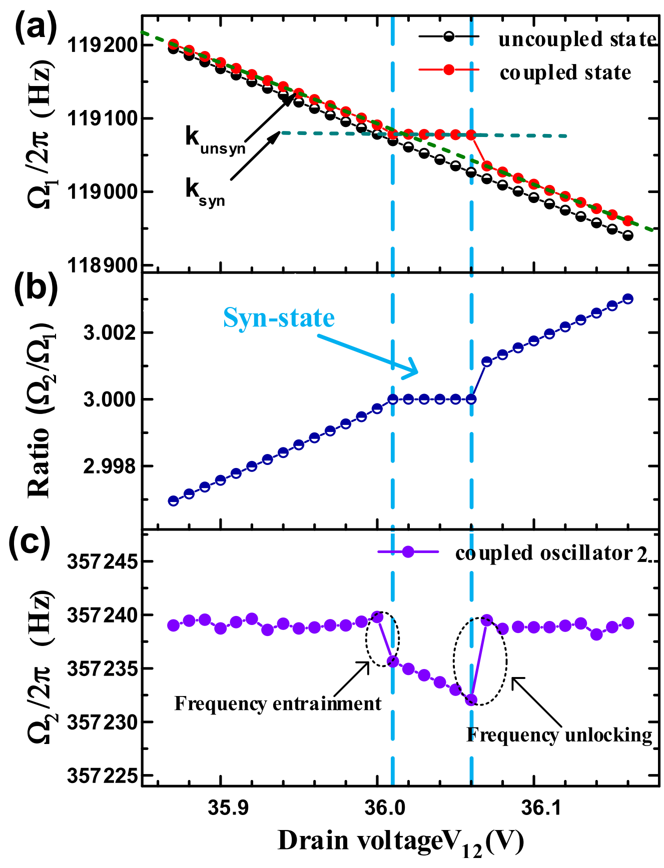

FIG. 3. (a) Frequency output of LFO with (red line) and without (black line) electrical coupling to HFO. (b) Frequency ratio of HFO to LFO. (c) Frequency output of HFO as V_{12} is tuned. The light blue dashed line indicates the window within which the two oscillators are synchronized.

When LFO is electrically coupled to HFO, however, the output frequency of LFO is perturbed and a linear range with a decreased slope of $k_{syn} = -23.307$ Hz/V appears when it is synchronized with HFO. Within the synchronization range, an exact frequency ratio of 3:1 is achieved and a plateau is observed from the frequency ratio curve as shown in Fig. 3(b). The state of on and off synchronization can be clearly observed from the frequency output of HFO as shown in Fig. 3(c). When V_{12} is 36.01 V, a clear frequency entrainment is emerged, while HFO frequency is unlocked from three times the frequency of LFO when V_{12} is increased to 36.06 V, which leads to a very narrow window of locked frequency.

The frequency of LFO was thermally tuned upwards and downwards around one third of HFO frequency via slightly varying the drain current of LFO, and the high order synchronization region was figured out accordingly based on the definition of the synchronization region for the mutual synchronized oscillators.[13] Figure 4(a) shows a comparison of synchronization regions under different coupling forces, while the other conditions were kept constant. It can be seen that the bandwidth of the synchronization region for LFO is increased from 81.6 Hz to 116.2 Hz when the coupling voltage ΔV is increased from 16 V to 32 V, which leads to a small tunability of about 2.16 ± 0.56 Hz/V. This limited tuning capability is further constrained by the maximum coupling voltage to avoid electrical latching of coupling parallel-plates. Figure 4(b) shows the effect of driving force on the synchronization region with a fixed coupling voltage ΔV at 32 V and fixed DC bias voltage of HFO V_{bias2} at 13 V. It can be seen that the bandwidth of synchronization is increased from 116.5 Hz to 225 Hz as V_{bias1} is increased from 10 V to 18 V, which gives a tunability of about 13.56 ± 2.64 Hz/V. This indicates that it is more effective to adjust the bandwidth of the synchronization region by increasing the DC bias driving voltage than increasing the electrical coupling voltage. This is mainly because LFO is driven into nonlinear vibration, which leads to an enhancement of the synchronization region.[6]

It is well known that frequency stability is an essential property of micromechanical oscillators. Previous work has shown a large improvement of frequency stability of a micromechanical oscillator when it is synchronized to a stable frequency source[18,19] or another coupled micromechanical oscillator[17,20] with a frequency ratio of 1:1 for both cases. Here, a series of experiments were performed to study the effect of higher order synchronization on the frequency stability of coupled DETF oscillators. The output frequencies were recorded at a fixed sample time of 0.1 s using a frequency counter for a duration time of 220 s. The Allan deviation of oscillation frequency was accordingly calculated to characterize the short term stability of DETF oscillators as shown in Fig. 5.

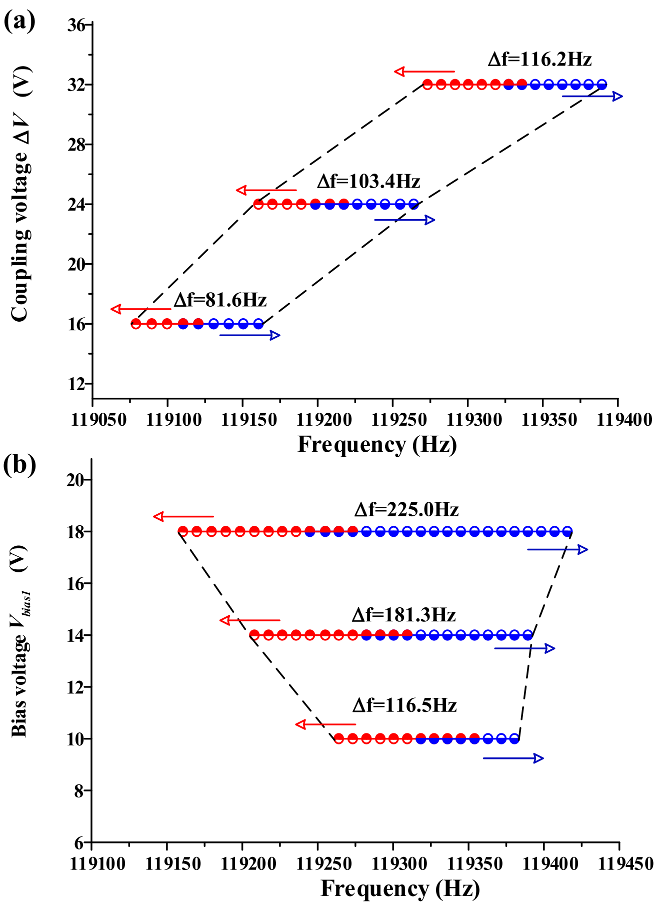

FIG. 4. The bandwidth of the synchronization region for LFO (a) at different ΔV values of 16, 24, and 32 V while the driving force is fixed; (b) at different V_{bias1} values of 10, 14, and 18 V while V_{bias2} is 13 V and ΔV is 32 V. The blue and red circled lines represent the synchronization region obtained when sweeping the LFO frequency upward and downward, respectively.

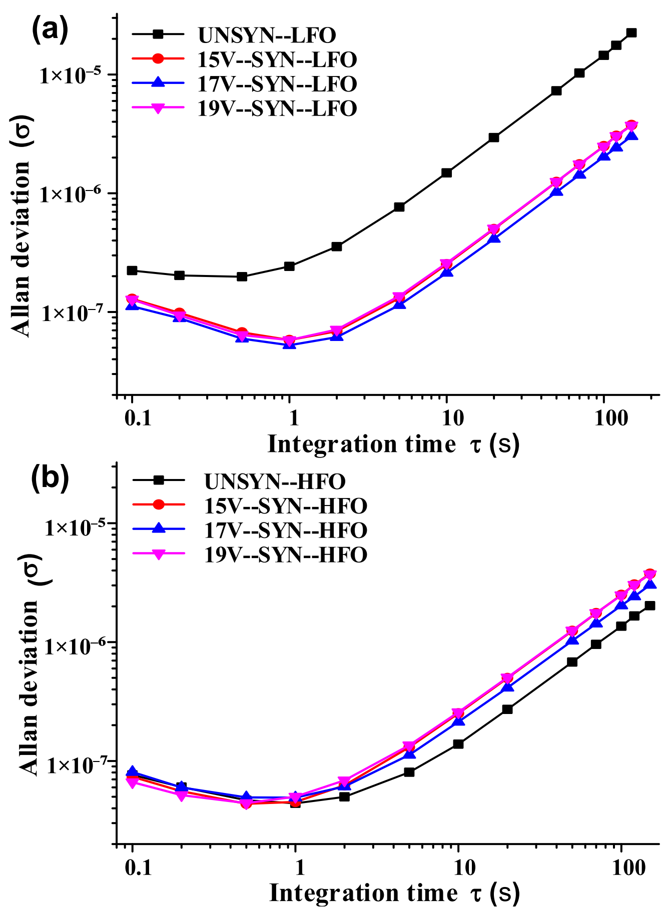

FIG. 5. Allan deviations of LFO (a) and HFO (b) obtained from synchronized cases (color lines) at different coupling voltages ΔV but fixed self-sustaining forces and fixed drain voltages compared to the unsynchronized case (black line). ΔV varies from 15 V to 19 V at a step of 2 V.

The black lines in Figs. 5(a) and 5(b) represent the case of LFO and HFO at free running without coupling. It is clear that the Allan deviation of LFO is about one order of magnitude larger than that of HFO at an integration time of 1 s. Once LFO and HFO are electrically coupled and become synchronized, the frequency stability of LFO is improved and close to that of HFO as shown in Fig. 5(a), and the frequency stability of HFO is not affected in the short term, which implies that the frequency stability of an array of synchronized oscillators depends on the one of best performance. Another interesting fact is that the frequency stability is not affected by the coupling strength even though it has an effect on the bandwidth of the synchronization region as shown in Fig. 4.

In this Letter, synchronization of electrically coupled micromechanical oscillators with a frequency ratio of 3:1 is reported. Two micromechanical oscillators are built, and their frequencies are thermally tuned to figure out the synchronization region. It is found that the larger the driving force or coupling strength is the wider the synchronization region is. Interestingly, however, the frequency tunability of the micromechanical resonator is dramatically reduced from –809.1 Hz/V to –23.1 Hz/V when synchronization is reached. A nearly 10-fold improvement of frequency stability at 1 s is observed from one of the synchronized oscillators, showing a comparable performance of the other. In the stable synchronization region, the frequency shift of LFO induced by physical measurands will be tripled compared to that of HFO, which leads to an amplified scale factor. This phenomenon of mutual synchronization of micromechanical oscillators at high order is helpful to design high performance resonant sensors with a better frequency resolution and a larger scale factor.

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