Contents lists available at ScienceDirect

# Optik

journal homepage: www.elsevier.com/locate/ijleo

# Original research article

# Study on application of model reference adaptive control in fast steering mirror system

Zhengxi Wang<sup>a,b</sup>, Bao Zhang<sup>a,\*</sup>, Xiantao Li<sup>a</sup>, Shitao Zhang<sup>a,b</sup>

<sup>a</sup> Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun 130033, China <sup>b</sup> University of Chinese Academy of Sciences, Beijing 100049, China

## ARTICLE INFO

Keywords: Fast steering mirror MRAC Lyapunov theory Feedback control Stability

## ABSTRACT

The control accuracy of traditional controllers for fast steering mirror system depends largely on the accuracy of the system model, which limits the performance of the controllers. Model reference adaptive controller (MRAC) is suggested for further improving the control accuracy of the fast steering mirror. Before designing the controller, we first introduced rate feedback to reduce the resonance peak of the fast steering mirror. Then a reference model which has good dynamic performance is selected. The MRAC controller is designed base on Lyapunov theory, which guarantees the asymptotic stability of the whole system and makes the output of the FSM system evolves asymptotically to the selected model. Experimental results show that FSM system with MRAC controller shows excellent dynamic performance.

#### 1. Introduction

Recently, the rapid development of adaptive optics, space laser communication, target detection, aerial imaging and other systems call for a high performance of beam pointing control technology [1–3]. To improve the precision of the beam pointing control system, the compound axis control system is introduced [4]. In the compound axis system, a fast steering mirror(FSM) which has high tracking accuracy, fast response speed, high control bandwidth and other outstanding advantages is mounted on a big inertia signal-axis gimbal and with this structure, performance of the whole tracking system is improved significantly [5,6].

FSM is the core component of the fine tracking sub-system, as the tracking ability and tracking accuracy of the whole system depends mostly on the precision of the FSM. Under the given structure and driven method of FSM, the key point to further improve the performance of FSM is to improve the control strategy. In actual operation process, the accuracy of FSM is affected by many factors, including time delay, mechanical resonance, random bias of states and un-modeled nonlinear uncertainty. How to perform high-precision control on the FSM system has received considerable attention. Due to the various nonlinear disturbances in the system, it is necessary to implement online real-time adjustment of the controller parameters to achieve a satisfactory control effect. The traditional proportional-integral-derivative (PID) controller is widely used in the real control system [7–10] due to its good performance and functional simplicity. However, it is very difficult to make real-time adjustment of the PID controller parameters, so the controller cannot meet the desired higher-level performance.

In recent years, scholars proposed large numbers of advanced control methods. Tang introduced feed-forward control into the FSM system. Thus, his closed-loop FSM system is composed of a feedback and a feedforward controller, and experimental results show that his control method can significantly improve the tracking performance and enhance the error attenuation of the system







<sup>\*</sup> Corresponding author. *E-mail address*: ldh\_wh@163.com (B. Zhang).

(1)

[11]. Then the acceleration control was applied to improve the robustness of the FSM system [5]. Further a predict control method was achieved [12]. By introducing a robust Smith predictor into the FSM control system, the tracking performance of the FSM system was improved effectively. Many other scholars have also proposed different control schemes. However, most of these controllers rely on the accurate parameters of the system models during the design process, which may not always be satisfied in the industrial practice [8]. Therefore, the parameter identification problem limits the performance of their control methods.

In order to further improve the control accuracy of FSM under the uncertainty of the model parameters, our group introduce Model Reference Adaptive Control (MRAC) into the FSM control system. The concept of MRAC was first proposed by Whitaker in 1958, and then, as a kind of adaptive control method it has been studied by many researchers [13–16].

The core idea of the MRAC control method is to make the output of the controlled object tracking a reference model, so as to improve the dynamic performance [17-19]. The online real-time adjustment of the adaptive parameters is achieved without using the accurate value of the model parameters of the controlled system. So, the control scheme is particularly suitable for controlled systems with unknown or time-varying parameters.

In fact, Ai [20] has already used the MRAC algorithm on servo control system, and the result shows that the servo system performance is satisfied. But the controller he designed is based on the MIT rule, which means his algorithm cannot guarantee the stability of the controlled system. Therefore, it is necessary to analyze the stability of the system after the designed MRAC controller is introduced into the servo system. To improve the situation, in this paper, the MRAC controller is designed base on Lyapunov theory. The controller obtained by this method can ensure the entire close-loop adaptive system is stable and it allows us to avoid the stability analysis problem [21–23]. What's more, the controller is designed without determining the accurate parameters of the controlled object. Even if the parameters are time-varying, this control strategy is applicable [13,15].

This paper is organized as follows: A general description of the whole FSM system is presented in Section 2, including a simplified system model. In Section 3, the MRAC controller is designed to ensure that the output of the FSM evolves asymptotically to the output of the selected reference model. Experiments are conducted and discussed in Section 4. Section 5 provides our conclusions.

### 2. System model and reference model

Generally, the FSM system is composed of a reflection mirror, an actuator, a base and a flexure hinge. The mirror is connected to the base by a flexure hinge which has the advantages of small size, simple structure, no mechanical friction, and high motion sensitivity.

FSM can be driven by voice coil actuators (VCA) or piezoelectric actuators (PZT) [24,25]. As FSM driven by VCA can bring larger scanning angle with smaller volume, in this paper, the FSM is driven by VCA. Eddy current sensor (ECS) which has the merits such as high precision, high operating frequency, strong anti-interference ability and high bandwidth is used to measure the rotation angle of the FSM as feedback amount. The structure of the FSM is shown in Fig. 1.

### 2.1. Description of the FSM model

Due to the machining error and assembling error are inevitable, the system analysis is very difficult. More serious is the inter-axis coupling problem is exist in the FSM system, which makes the system a MIMO system. Since the main purpose of this paper is to study the feasibility and advantage of the MRAC algorithm, we treat the FSM a SISO system and neglect the inter-axis coupling effect, which will simplify the controller design and analysis process.

To get the open loop response of the FSM system, sinusoidal sweep experiment is conducted and the rotation angle is measured by ECS of the FSM. The open-loop frequency response is shown in Fig. 2. The experimental curve can be fitted as a second-order model and the transfer function can be expressed as:

$$\frac{1}{\frac{1}{\omega_n^2}\mathbf{s}^2 + \frac{2\xi}{\omega_n}\mathbf{s} + 1}$$

where  $\omega_n$  is the natural frequency and  $\xi$  is the damping ratio of the FSM system.



Fig. 1. Structure of a FSM.



Fig. 2. Frequency responses of the FSM system.

From Fig. 2 we can also see that the resonance peak near 240 rad/sec severely limits the closed-loop performance of the FSM system. What's more, the existence of mechanical resonance can even cause system instability. So before designing the MRAC controller, the mechanical resonance should be suppressed first. Here we introduce rate feedback to improve the damping coefficient of the FSM system, so as to eliminate the resonance. This is accomplished by adding a derivative term into the feedback path of the FSM system, the block diagram of which is shown in Fig. 3.

After introducing rate feedback into the feedback loop, the new transfer function of the FSM becomes:

$$\frac{1}{\frac{1}{\omega_n^2} s^2 + (b + \frac{2\xi}{\omega_n}) s + 1}$$
(2)

Compare Eq. (1) with Eq. (2) we can find that the rate feedback provides the system electronic damping. By adjusting the rate feedback coefficient b, the FSM system can be made a critically damped second order system. Fig. 4 shows the open loop frequency responses of the FSM with and without rate feedback. The resonance peak in the frequency response curve disappeared after rate feedback is introduced. This will help us to design the closed-loop controller in the next step.

## 2.2. Selection of the reference model

After introducing the rate feedback into the FSM control system to eliminate the resonance, we design a classic PID controller and perform the closed-loop step response test on the system. The experimental result is shown in the Fig. 5. In practical applications, adjustment time of the step response should be less than 0.1 s and the maximum overshoot should be less than 10%. However, in Fig. 5, the practical situation is: the adjustment time of the step response is near 0.1 s and the maximum overshoot is near 40%, which means the dynamic performance of the FSM system should be further improved.

In order to satisfy the engineering demands, we chose a second-order model as our reference model, which is shown as follows:

$$G_{\rm m}(s) = \frac{360000}{s^2 + 850s + 360000} \tag{3}$$

of which the step response is shown in Fig. 6. From the figure we can see that the adjustment time of the reference model is about 10 ms and nearly non-overshoot. If our FSM output can track the output of the reference model accurately, then the servo accuracy of the FSM system could be improved.



Fig. 3. Block diagram of a second order system with rate feedback.



Fig. 4. Open loop frequency responses of the FSM with and without rate feedback.



Fig. 5. Step response curve of FSM with PID controller.



Fig. 6. Step response curve of the reference model.

# 3. Design of the MRAC controller

From part 2 we know that the real system can be seen a second-order system, so the state equation can be depicted as follows:

(4)

$$\dot{x}_p = A_p x_p + B_p u$$

where  $x_p$  is two-dimensional state vector, u is the control input of the FSM and  $A_p$ ,  $B_p$  are  $2 \times 2$  and  $2 \times 1$  matrixes, respectively. Normally the exact value of  $A_p$  and  $B_p$  are unknown and cannot be adjusted directly, which is why many control methods are limited



Fig. 7. The scheme of MRAC.

Accordingly, the state equation of the reference model can be shown in Eq. (5):

$$\dot{x}_m = A_m x_m + B_m y_r \tag{5}$$

where  $A_m = \begin{bmatrix} 0 & 1 \\ -360000 & -850 \end{bmatrix}$ ,  $B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ,  $x_m$  is two-dimensional state vector of the reference model and  $y_r$  is the reference input. In order to make the output of the FSM tracking the output of the reference model and improve the dynamic performance, we designed a state feedback controller F and a feedforward controller K to form an adjustable MRAC system as shown in Fig. 7.

Then, the control law of the control structure is calculated as:

$$u = Fx_p + Ky_r \tag{6}$$

When substituting Eq. (6) into Eq. (4) the result is the closed-loop form of the controlled system:

$$\dot{x}_p = (A_p + B_p F) x_p + B_p K y_r \tag{7}$$

Then the state error equation could be written as:

$$\dot{e} = \dot{x}_m - \dot{x}_p = A_m e + (A_m - A_p - B_p F) x_p + (B_m - B_p K) y_r$$
(8)

The final purpose is to generate K and F in such a way that  $e \rightarrow 0$  as  $t \rightarrow \infty$ , and the adjustable system parameters asymptotic converge to the parameters of the reference model, which can be descripted as follows:

$$\lim_{t \to \infty} e(t) = 0 \tag{9}$$

$$\begin{cases} \lim_{t \to \infty} [A_p(t) + B_p(t)F(e, t)] = A_m \\ \lim_{t \to \infty} [B_p(t)K(e, t)] = B_m \end{cases}$$
(10)

Suppose when  $F(e, t) = F_0$  and  $K(e, t) = K_0$ , we can get the complete match between the reference model and the adjustable system, which means:

$$\begin{cases}
A_p + B_p F_0 = A_m \\
B_p K_0 = B_m
\end{cases}$$
(11)

By substituting Eq. (11) into Eq. (8) the result is: ~ /

~ /

$$\dot{e} = A_m e + B_m K_0^{-1} F x_p - B_m K_0^{-1} K y_r, \tag{12}$$

where:

$$\begin{split} \vec{F} &= F_0 - F \\ \vec{K} &= K_0 - K \end{split} \tag{13}$$

As the reference model we chose is a stable model and  $A_m$  is a stable state matrix, there must exists a positive definite symmetric matrix P which is the solution of:

$$A_m^{\rm T}P + PA_m = -Q, \ Q = Q^{\rm T} > 0 \tag{14}$$

where Q is also a positive definite symmetric matrix.

Then we can choose a Lyapunov function as:

$$V = e^{\mathrm{T}}Pe + \mathrm{tr}(\widetilde{F}^{\mathrm{T}}\mathrm{P}_{F}^{-1}\widetilde{F}) + \mathrm{tr}(\widetilde{K}^{\mathrm{T}}\mathrm{P}_{K}^{-1}\widetilde{K})$$
(15)

where P,  $P^F$ ,  $P^K$  are  $n \times n$ ,  $m \times m$  and  $m \times m$  positive definite matrix respectively, and tr is the mathematical symbol of trace. The total time derivative of Eq. (15) is

$$\dot{V} = \dot{e}^{\mathrm{T}} P e + e^{\mathrm{T}} P \dot{e} + \mathrm{tr}(\ddot{F}^{\mathrm{T}} P_{F}^{-1} \widetilde{F} + \widetilde{F}^{\mathrm{T}} P_{F}^{-1} \dot{F}) + \mathrm{tr}(\ddot{K}^{\mathrm{T}} P_{K}^{-1} \widetilde{K} + \widetilde{K}^{\mathrm{T}} P_{K}^{-1} \dot{K})$$

$$= e^{\mathrm{T}} (A_{m}^{\mathrm{T}} P + P A_{m}) e + 2\mathrm{tr}(\ddot{F}^{\mathrm{T}} P_{F}^{-1} \widetilde{F} + x_{p} e^{\mathrm{T}} P B_{m} K_{0}^{-1} \widetilde{F}) + 2\mathrm{tr}(\ddot{K}^{\mathrm{T}} P_{K}^{-1} \widetilde{K} + y_{r} e^{\mathrm{T}} P B_{m} K_{0}^{-1} \widetilde{K})$$

$$(16)$$

In order to make  $\dot{V}$  a negative definite function, which means the entire system is stable, we choose:

$$\begin{cases} \widetilde{F} = -P_F K_0^{-T} B_m^{T} P e x_p^{T} \\ \widetilde{K} = -P_K K_0^{-T} B_m^{T} P e y_r^{T} \end{cases}$$
(17)

Considering Eq. (13), the adaptive laws can be expressed as:

$$\begin{cases} \dot{F} = P_F K_0^{-T} B_m^{T} P e x_p^{T} \\ \dot{K} = P_K K_0^{-T} B_m^{T} P e y_r^{T} \end{cases}$$
(18)

Generally, the parameters  $A_p$  and  $B_p$  are unknown, and the parameters  $F^0$  and  $K^0$  are also difficult to obtain. Considering that the value of  $P^F$  and  $P^K$  can be selected flexibly, Eq. (18) can be written as:

$$\begin{cases} \dot{F} = R_1 B_m^{\mathrm{T}} P e x_p^{\mathrm{T}} \\ \dot{K} = R_2 B_m^{\mathrm{T}} P e y_r^{\mathrm{T}} \end{cases}$$
(19)

where  $R_1$  and  $R_2$  are parameters can be obtained by experiment. Then Eq. (19) is the adaptive control law we need to make the output of the FSM track the output of the reference model. And when the control law is applied to the FSM system, the entire servo system is globally asymptotically stable. Which means Eqs. (9) and (10) are satisfied.

# 4. Experimental results and discussions

To investigate the performance of the FSM system with the MRAC controller designed in part 3, we studied the dynamic performance of the FSM response. The experimental setup is shown in Fig. 8. The FSM is fixed to vibrating table and is driven by voice coil motor. The eddy current sensor is used to measure the rotation angle of the mirror for closed-loop operation. All data processing, analysis processes, control algorithm and the generation of control signals are completed by the data processing module.

## 4.1. Set-point trajectory tracking

First, we test the set-point tracking performance of the closed-loop system. The experiment result is shown in Fig. 9. From Fig. 9,



Fig. 8. Photo of the experimental setup.



Fig. 9. Closed-loop step response of the FSM.

we can see that the system with the MRAC controller shows only an overshoot less than 10% and a significantly short setting time of less than 50 ms. Whereas the FSM system without MRAC controller shows an overshoot about 40% and a setting time of about 100 ms. After adding MRAC controller, the performance of the FSM has been greatly improved: the overshoot of the step response is almost disappeared, and the setting time is decreased by 50%.

# 4.2. Sinusoidal trajectory tracking

Then the sinusoidal trajectory tracking experiments are conducted to further test the tracking ability of the FSM with MRAC controller, and the result is shown in Fig. 10. It can be seen that the values of peak-valley errors of the FSM system with and without MRAC controller are respectively about 98 µrad and 340 µrad. Obviously, compared with the traditional control method, the system with MRAC controller showed excellent tracking performance.

## 4.3. Sinusoidal trajectory tracking with disturbance

Vibrations often occur within the working environment of FSM, so again we conduct the Sinusoidal trajectory tracking experiment. But unlike the previous experiment, this time the vibrating table will produce random vibration to simulate real working condition. The seismic excitations peak acceleration of shaking table is 1.0 g and the tracking error is shown in Fig. 11.

From Fig. 11, we can find that the values of peak-valley errors of the FSM system with and without MRAC controller are respectively about 140 µrad and 900 µrad. Comparing Figs. 10 and 11 we can conclude that even in the presence of random disturbances, the MRAC controller can still show good performance. However, under the traditional control scheme, the FSM performance is seriously degraded.



Fig. 10. Angle tracking performance and tracking error.



Fig. 11. Angle tracking performance and tracking error.

#### 4.4. Discussion

The results show that the presented MRAC controller improves the dynamic performance of the FSM system effectively. This is because the reference model we selected has good dynamic performance, and the reference model reflects the ideal output of the close-loop system. Then the MRAC controller adjusts the adjustable parameters in the controller according to the difference between the output of the FSM and the output of the reference model, which will make the output of the FSM tracking the output of the reference model.

# 5. Conclusion

In the paper a high-performance FSM control strategy is prosed. First, rate feedback is introduced to reduce the resonance peak in the frequency response. Then a reference model is selected based on the system's performance indicators. In order to guarantee the stability of the FSM system, the controller is designed base on Lyapunov theory, in this method we can be free from the discussion of stability problem. In the controller design process, the accurate parameters of the FSM model are not required, which means the effect of this control method does not depend entirely on the specific parameters of the system model. The results of the step response and angle tracking experiments indicate that the MRAC controller achieves outstanding performance. The FSM system with MRAC controller satisfies practical engineering applications.

#### Acknowledgement

This work is supported by National Natural Science Foundation of China (Grant No. 61705225).

# References

- [1] N. Sweeney M, E. Erdelyi, M. Ketabchi, et al., Design Considerations for Optical Pointing and Scanning Mechanisms, (2003).
- [2] Shuxin Zhang, Jingli Du, Wei Wang, Xinghua Zhang, Yali Zong, Two-step structural design of mesh antennas for high beam pointing accuracy, Chin. J. Mech. Eng. 30 (2017) 604–613.
- [3] Y. Juqing, W. Dayong, Z. Weihu, Precision laser tracking servo control system for moving target position measurement, Opt. Int. J. Light Electron Opt. 131 (2017) 994–1002.
- [4] C. Lv, H. Jiang, S. Tong, Optical-path design and study of fine tracking assembly in space optical communication. City: Year. (2012) 1162–1165.
- [5] J. Tian, W. Yang, Z. Peng, et al., Application of MEMS gyroscopes and accelerometers in FSM stabilization. City: SPIE, Year. (2016) 6.
- [6] G. Wang, F. Bai, Robust tracking control of piezoelectric fast steering mirror with hysteresis and disturbances correction. City: Year. (2015) 6.
- [7] K.J. Åström, T. Hägglund, The future of PID control, IFAC Proceedings Volumes 33 (2000) 19–30.
- [8] J. Han, Z. Zhu, Z. Jiang, et al., Simple PID parameter tuning method based on outputs of the closed loop system, Chin. J. Mech. Eng. 29 (2016) 465–474.
- [9] A. Kiam Heong, G. Chong, L. Yun, PID control system analysis, design, and technology, IEEE Trans. Control. Syst. Technol. 13 (2005) 559–576.
- [10] B. Kristiansson, B. Lennartson, Robust tuning of PI and PID controllers: using derivative action despite sensor noise, IEEE Control Syst. 26 (2006) 55-69.
- [11] T. Tang, R. Ge, J. Ma, et al., Compensating for some errors related to time delay in a charge-coupled-device-based fast steering mirror control system using a feedforward loop. City: SPIE, Year. (2010) 7.
- [12] Z. Cao, J. Chen, C. Deng, et al., Improved Smith predictor control for fast steering mirror system, IOP Conference Series: Earth and Environmental Science 69 (2017) 012085.
- [13] R.M. Dressler, An approach to model-referenced adaptive control systems. City: Year. (1967) 75-80.
- [14] R.R. Gupta, V.V. Chalam, Lyapunov redesign of model reference adaptive control systems (Part I), IETE J. Res. 25 (1979) 456–458.
- [15] R.V. Monopoli, Model reference adaptive control with an augmented error signal, IEEE Trans. 19 (1974) 474–484.
- [16] Z. Ali, D. Wang, M. Aamir, Fuzzy-based hybrid control algorithm for the stabilization of a tri-rotor UAV, Sensors 16 (652) (2016).
- [17] R. Kamnik, D. Matko, T. Bajd, Application of model reference adaptive control to industrial robot impedance control, J. Intell. Robot. Syst. 22 (1998) 153–163.
- [18] G.-Q. Wu, S.-N. Wu, Y.-G. Bai, et al., Experimental studies on model reference adaptive control with integral action employing a rotary encoder and tachometer sensors, Sensors 13 (4742) (2013).
- [19] M.A. Demetriou, S. Reich, I.G. Rosen, Model reference adaptive control of distributed parameter systems, SIAM J. Control. Optim. 36 (1995) 33-81.
- [20] A. Xiong, Y. Fan, Application of a PID Controller using MRAC Techniques for Control of the DC Electromotor Drive. City: Year. (2007) 2616–2621.
- [21] D.J.G. James, Stability analysis of a model reference adaptive control system with sinusoidal inputs, Int. J. Control 9 (1969) 311–321.
- [22] G. Luders, K. Narendra, An adaptive observer and identifier for a linear system, IEEE Trans. Autom. Control 18 (1973) 496-499.
- [23] V.M. Hung, I. Stamatescu, C. Dragana, et al., Comparison of model reference adaptive control and cascade PID control for ASTank2. City: Year. (2017) 1137–1143.
- [24] M. Hafez, T.C. Sidler, R.P. Salathé, et al., Design, simulations and experimental investigations of a compact single mirror tip/tilt laser scanner, Mechatronics 10 (2000) 741–760.
- [25] M.N. Sweeney, G.A. Rynkowski, M. Ketabchi, et al., Design considerations for fast-steering mirrors (FSMs). City: SPIE, Year. (2002) 11.