# Significantly enhanced infrared absorption of graphene photodetector under surfaceplasmonic coupling and polariton interference 

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#### Abstract

Here, we present a graphene-based long-wavelength infrared photodetector, for enhancing the infrared absorption of which the design consists of magnetic- and electricplasmon resonators of metasurface to excite the graphene surface-plasmonic polaritons (SPPs). Through tuning the graphene Fermi energy to achieve the distinct resonances in a matching frequency, peak graphene absorbance exceeding $67.2 \%$ is confirmed, even when a lossy dielectric is used, and the field angle of view is up to $90^{\circ}$. If the graphene is of a different carrier mobility, then the absorption frequency is lockable, and the device always can keep the system absorbance close to 100 percent. The significantly enhanced graphene absorbance, up to $\sim 29$-fold that of a suspended graphene (general $2.3 \%$ ), is attributed to the surface-plasmonic coupling between the magnetic and the electric resonances, as well as Fabry-Pérot interference of the coherent SPPs. The plasmonic cavity-mode model and equivalent-circuit method developed in this study will also be useful in guiding other optoelectronic device design.


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## 1. Introduction

Graphene, a two-dimensional semiconductor with zero bandgap, has many intriguing properties [1-4]. With its high carrier mobility, the material shows great promise for applications in high-speed transistors and sensitive photodetectors. For the latter, high optical absorption of the graphene or the system of device is always desired. The opacity of a suspended graphene is determined solely by the fine structure constant, $\alpha=e^{2} / \hbar c$ (where $e$ is the electron charge, $\hbar$ the reduced Planck's constant, and $c$ the speed of light in vacuum). Its
 for normal incidence of electromagnetic (EM) waves over a broad frequency range from ultraviolet to infrared regime [5]. Such an absorbance is significant for only one-atomic thickness of graphene; however, it is insufficient for a practical photodetector. The low absorption limits its photoelectric responsivity, while the lack of spectral selectivity potentially is another problem for selective detection.

Since surface plasmons (SPs) or surface-plasmonic polaritons (SPPs) in shorter wavelength than incident light $\left(\lambda_{0}\right)$ are capable of stronger near-field localization or much tighter spatial confinement, the SP technology has rapidly emerged as an effective tool to pursue extreme light-matter interactions. Graphene SPs, the collective oscillations of charge carriers in extrinsic graphene, are available through intraband transitions excited by the optical field of infrared radiation. Comparing with noble metals, graphene has the highly tunable real part and much smaller imaginary part of permittivity [6-8]. Recently, the
methods to manipulate the graphene SPs by tuning the permittivity have attracted much attention [9-12]. As one of the metasurfaces, the structure of graphene integrated with subwavelength gold gratings was used to activate the graphene SPs directly by electric-fielddriven LC (inductor-capacitor) resonances (electric SPs) [13]. Magnetic-field-induced LC resonances (magnetic SPs) occur in similar structures, where the graphene SPs or SPPs are ascribed to a strong EM coupling from the magnetic SPs of the metals. The graphene absorbance at $\lambda_{0}=9.09 \mu \mathrm{~m}$ was boosted to $72 \%$ for a hybrid-SP system with zero dielectric loss [14], and multi-band perfect absorption was demonstrated for another system [15]. On the contrary, the graphene SPs generally are not activated in the spectral range from ultraviolet to near infrared. Owing to larger absorption coefficient from interband transitions in combination with strongly localized field from magnetic SPs of the metals, the optical absorption of graphene will be much higher, and those pure metallic-SP structures show great potential for the applications of perfect absorbers [16-21] and metal-insulator-metal (MIM) waveguide [22]. Furthermore, the magnetic-SP structure tolerates a wide range of spatial incident angle [23]. With these progresses, achieving the graphene SP coupling and absorption enhancement in long-wavelength infrared (LWIR) remains a great challenge for graphene-based optoelectronics, especially photodetector [24-27], where the graphene absorption is dominated by the intraband transitions.

In this article, we present a tunable graphene-based LWIR photodetector design using metasurface structure, which can dramatically enhance the optical absorption. The design is based on the EM-coupling effect between two distinct resonance elements of magnetic and electric SPs and the standing-wave effect of coherent SPPs in Fabry-Pérot (F-P) cavity modes. It is found that by tuning the graphene Fermi energy $\left(E_{\mathrm{F}}\right)$ to achieve the frequency matching for the electric and the magnetic SPs, EM-wave scattering is suppressed, while the infrared absorption of the graphene, even in different carrier mobility, is significantly enhanced at the fixed frequency. The underlying mechanisms revealed here will be helpful in guiding the design of high-performance graphene photodetector, and perhaps also be relevant to the design of other optoelectronic devices.

## 2. Device and method

Figure 1 illustrates the schematic metasurface structure of designed graphene LWIR photodetector. It consists of top-metal pattern, single layer of graphene, low-loss insulator layer of $\mathrm{Al}_{2} \mathrm{O}_{3}$, and bottom-metal film. The top-metal pattern works as subwavelength shallow gratings and interdigital electrodes. The graphene sheet acts as the photosensitive semiconductor. The insulator layer is the dielectric to compose the micro capacitor in LC resonators. The bottom-metal film takes a role as the optical reflector and also as the backgated electrode. When worked as the source/drain electrodes, the interdigitated grating has spectral selectivity and also contributes to high photoconductive gain. The dielectric layer interposes between the bottom metal and the grated graphene. This arrangement is used to build MIM and metal-insulator-semiconductor (MIS) configurations. Under TM-polarized infrared illumination, the MIM configuration constitutes arrayed magnetic-SP resonators. On the contrary, if the bottom metal is excluded, the graphene sheet conducting with the gratings forms a simple electric-SP resonator array. Two types of the resonator arrays are embedded together to enhance the graphene SPPs (if supported) through strong SP coupling. Moreover, if F-P interference condition of the SPPs is satisfied between the two neighboring gratings (slotted section), the coherent SPPs in cavity modes will confine more EM energies at the Air/Graphene $/ \mathrm{Al}_{2} \mathrm{O}_{3}$ interfaces within the grating slots. As another function, the MIS configuration is used to electrostatically tune the graphene Fermi energy by the gate field.

Optical reflection of the device system is simulated by the method of finite differential time domain (FDTD solutions 8.15, Lumerical). In simulation, the TM-polarized infrared radiation of $\left|\boldsymbol{E}_{0}\right|=1.0 \mathrm{~V} / \mathrm{m}$ is always considered at normal incidence (along the $y$ axis), except for determining the field angle of view $\left(\theta_{\mathrm{FV}}\right)$. The polycrystalline dielectric film of $\mathrm{Al}_{2} \mathrm{O}_{3}$ is
assumed to be isotropic, and the permittivity ( $\varepsilon_{\mathrm{D}}$ ) values in Ref [28] are used. The graphene is an anisotropic material with in-plane permittivity, $\varepsilon_{G}=1+i \sigma /\left(\omega \varepsilon_{0} 4\right)$, and out-of-plane permittivity $\varepsilon_{\perp}=2.5$ [29], where $\omega$ is the radiation angle frequency ( $f=\omega / 2 \pi$ is frequency), $\sigma$ and $\Delta$ are the optical conductivity and the thickness of graphene, respectively, and $\varepsilon_{0}$ is the vacuum permittivity. The conductivity used here is the calculated results from the Kubo formula, and it is attributed to intraband transitions in extrinsic graphene ( $E_{\mathrm{F}}>77.5 \mathrm{meV}$ ) under the LWIR illumination. Then, the conductivity can be written as [30]:

$$
\begin{equation*}
\sigma(\omega)=i \frac{2 e^{2} k_{\mathrm{B}} T}{\pi \hbar^{2}\left(\omega+i \tau^{-1}\right)} \ln \left[2 \cosh \left(\frac{E_{\mathrm{F}}}{2 k_{\mathrm{B}} T}\right)\right], \tag{1}
\end{equation*}
$$



Fig. 1. Schematic structure of the graphene-based photodetector under normal incidence of TM-polarized radiation: incident electric field $\left(\boldsymbol{E}_{0}\right)$, magnetic field $\left(\boldsymbol{H}_{0}\right)$, and longitudinal wave vector $\left(\boldsymbol{K}_{\mathrm{int}}\right)$ in a coordinate system. Structural parameters: grating period (p), width (w), and height $(h) ; \mathrm{Al}_{2} \mathrm{O}_{3}$ dielectric thickness $(t)$; bottom metal thickness $\left(t^{\prime}\right) . h=100 \mathrm{~nm}$ is selected for the height larger than EM-wave penetration depths $\left(\delta_{\mathrm{M}}\right), t=300 \mathrm{~nm}$ for nearly $t$-independence of the electric-SP frequency $\left(f_{\mathrm{SP}, \mathrm{e}}\right)$, and $t^{\prime}=200 \mathrm{~nm}$ for near-zero transmission. For the magnetic SPs, EM dissipation in the low-loss dielectric increases with the dielectric thickness, and the graphene absorption is thus reduced. The selected thickness is a compromise with the low dielectric loss.
where $k_{\mathrm{B}}$ is Boltzmann's constant, $T$ is temperature, and $\tau=\mu E_{\mathrm{F}} /\left(e v_{\mathrm{F}}{ }^{2}\right)$ is the relaxation time of carrier. The absorption spectra are obtained on the assumptions of $T=300 \mathrm{~K}$, the Fermi velocity $v_{\mathrm{F}} \approx 10^{6} \mathrm{~m} / \mathrm{s}$, and the carrier mobility $\mu \approx 2 \times 10^{3} \mathrm{~cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$ [30]. On a case by case basis, we also consider the absorption spectra to change with the mobility. The optical absorbance of graphene is calculated by the equation [17]:

$$
\begin{equation*}
A_{\mathrm{G}}=\frac{2 \pi}{\lambda_{0}\left|E_{0}\right|^{2} S} \mathcal{E}_{\mathrm{G}}^{\prime \prime} \iint_{V}\left|E_{\mathrm{G}}\right|^{2} d V . \tag{2}
\end{equation*}
$$

Here, $V$ is the graphene volume, $\boldsymbol{E}_{\mathrm{G}}$ is the local electric field in the graphene, and $S$ is the irradiated area of the device system.

## 3. Results and discussion

Magnetic and electric SPs can be discriminated by the distributions of field and surface current density in resonance. Distinct from the electric SPs, the magnetic SPs are generally observed for the MIM configuration, where anti-parallel currents are sustained in the two layers of metal to produce an induced magnetic field confined mainly in the dielectric spacer.

Figure 2 shows the equivalent circuits and the distributions of surface current density; here, the illustrated electric SPs are the 4th-order $(n=4)$ mode along the Air/Graphene $/ \mathrm{Al}_{2} \mathrm{O}_{3}$ interfaces. The different grating parameters, $(p, w)=(3.0,1.0)$ and $(7.0,5.0)$ (hereinafter in $\mu \mathrm{m})$, are selected to detune the magnetic-SP frequency $\left(f_{\text {SP, }}\right)$ off the electric one $\left(f_{\text {SP, }, \mathrm{e}}\right)$. Note that the different gratings turn out to be in an identical slot width $\left(w^{\prime}{ }_{\mathrm{G}}\right)$, i.e., $(p-w)=2.0 \mu \mathrm{~m}$. The following results of equivalent circuit analysis verify that $f_{\mathrm{SP}, \mathrm{e}}$ is independent of $w$, and in
both cases it corresponds to the free-space wavelengths of $\lambda_{0, \mathrm{e}}=10.15 \mu \mathrm{~m}$. Generally, $f_{\mathrm{SP}, \mathrm{m}}$ is inversely proportional to $w$ [31]. We find that for the structure of $(p, w)=(7.0,5.0)$, fundamental magnetic SPs in dipole mode also locates at $\lambda_{0, \mathrm{~m}} \approx 10.15 \mu \mathrm{~m}$, well matched with the electric SPs; while for $(p, w)=(3.0,1.0)$, the magnetic-SPs shift out from LWIR spectral range. As shown in Fig. 2(a), the current densities for the magnetic and the electric SPs, both at $\lambda_{0, \mathrm{e}} ; \lambda_{0, \mathrm{~m}}=10.15 \mu \mathrm{~m}$, are obviously to be high. Maximum current densities along the $x$ axis are extracted to be $J_{x, \mathrm{G}}=7.77 \times 10^{6} \mathrm{~A} / \mathrm{m}^{2}$, distributed in the slotted graphene, and $J_{x, \mathrm{M}}=5.72$ $\times 10^{5} \mathrm{~A} / \mathrm{m}^{2}$ on the bottom metal. In contrast, we observe that $J_{x, \mathrm{G}}$ significantly decreases with large detuning of $f_{\mathrm{SP}, \mathrm{m}}$ and $f_{\mathrm{SP}, \mathrm{e}}$. If the graphene is removed, only the magnetic-SP mode is retained with its location shifted to $\lambda_{0, \mathrm{~m}}=10.32 \mu \mathrm{~m}$, and $J_{x, \mathrm{M}}$ increases to $1.27 \times 10^{6} \mathrm{~A} / \mathrm{m}^{2}$. The red-shift of $f_{\text {SP,m }}$ results from change of the equivalent circuit in the absence of graphene. On the contrary, only the electric-SP mode at $\lambda_{0, \mathrm{e}}=10.15 \mu \mathrm{~m}$ is observable for $(p, w)=(3.0$, 1.0), as shown in Fig. 2(b), and $J_{x, \mathrm{G}}$ decreases to $1.03 \times 10^{6} \mathrm{~A} / \mathrm{m}^{2}$ as $\lambda_{0, \mathrm{~m}}$ is detuned off. The changes of $J_{x, \mathrm{G}}$ and $J_{x, \mathrm{M}}$ confirm a strong EM-coupling effect occurred between the magnetic and the electric SPs. Excluding the effect of grating fill-factor, we estimate that $\sim 24.4$-fold EM energy $\left(\Phi \propto J_{x, \mathrm{G}}{ }^{2}\right)$ is further confined in the graphene by the electric- and magnetic-SP coupling.


Fig. 2. When $E_{\mathrm{F}}=0.6 \mathrm{eV}$, distributions of surface current density at: (a) $\lambda_{0, \mathrm{~m}} ; \lambda_{0, \mathrm{e}}=10.15 \mu \mathrm{~m}$ for the structure of $(p, w)=(7.0,5.0)$ with the magnetic- and electric-SP modes both involved; (b) $\lambda_{0, \mathrm{e}}=10.15 \mu \mathrm{~m}$ for $(p, w)=(3.0,1.0)$, where $f_{\mathrm{SPP}, \mathrm{m}}$ is detuned. Color bar: $J_{x, \mathrm{G}(\mathrm{M})}$ in $\mathrm{A} / \mathrm{m}^{2}$; to guide the eyes, the boundaries of grating and bottom metal (in white solid line) are highlighted. RLC equivalent circuits: (c) the magnetic SPs in dipole mode for $(p, w)=(7.0,5.0)$; (d): the electric SPs in $n^{\text {th }}$-order mode for $(p, w)=(3.0,1.0)$, of which the bottom-metal inductance $\left(L_{\mathrm{e}}{ }^{\prime}\right)$ is negligible if the dielectric is thick enough to satisfy $\left(\omega C_{\mathrm{D}}\right)^{-1} \gg \omega L_{\mathrm{e}}{ }^{\prime}$.

To understand the underlying mechanism of absorption enhancement, an equivalent circuit model is used to interpret the EM-coupling effect between the magnetic and the electric SPs. First of all, for the electric-field response, the graphene or the metals with negative $\varepsilon^{\prime}{ }_{\mathrm{G}(\mathrm{M})}$ is equivalent to a series connection of resistor and self-inductor, and the resistance ( $R_{\mathrm{G}}$ or $R_{\mathrm{M}}$ ) and the inductance ( $L_{\mathrm{e}, \mathrm{G}}$ or $L_{\mathrm{e}, \mathrm{M}}$ ) are given by [32,33]:

$$
\begin{equation*}
R_{\mathrm{G}(\mathrm{M})}+i \omega L_{\mathrm{e}, \mathrm{G}(\mathrm{M})}=\gamma_{1} \frac{w_{\mathrm{G}(\mathrm{M})}^{\prime}}{\varepsilon_{0} \omega l \delta_{\mathrm{G}(\mathrm{M})}}\left(\frac{\varepsilon_{\mathrm{G}(\mathrm{M})}^{\prime \prime}}{\varepsilon_{\mathrm{G}(\mathrm{M})}^{\prime 2}+\varepsilon_{\mathrm{G}(\mathrm{M})}^{\prime \prime 2}}-\frac{i \varepsilon_{\mathrm{G}(\mathrm{M})}^{\prime}}{\varepsilon_{\mathrm{G}(\mathrm{M})}^{\prime 2}+\varepsilon_{\mathrm{G}(\mathrm{M})}^{\prime \prime 2}}\right), \tag{3}
\end{equation*}
$$

where $\gamma_{1}=0.2$ or 1.0 (fitting values) is a coefficient to calculate the effective cross section of surface current for the graphene or the metals, $w_{\mathrm{M}}^{\prime}$ and $l$ are width $\left(w_{\mathrm{M}}^{\prime}=w\right)$ and length of the
gratings along the $x$ and $z$ axes, respectively, and $\varepsilon_{G(M)}=\varepsilon^{\prime}{ }_{\mathrm{G}(\mathrm{M})}+i \varepsilon^{\prime \prime}{ }_{\mathrm{G}(\mathrm{M})}$ is the complex permittivity of graphene or the metals. The EM-wave penetration depth is $\delta_{\mathrm{G}}=\Delta$, for the graphene, or $\delta_{\mathrm{M}}=\lambda_{0} /\left[4 \pi \operatorname{Im}\left(\varepsilon_{\mathrm{M}}^{1 / 2}\right)\right]$ for the metal; obviously, $\delta_{\mathrm{G}} \ll \delta_{\mathrm{M}}$. In the spectral region of interest, $R_{\mathrm{G}} \gg R_{\mathrm{M}}$ and $L_{\mathrm{e}, \mathrm{G}} \gg L_{\mathrm{e}, \mathrm{M}}$ can be derived if $w_{\mathrm{G}}^{\prime}$ is close to $w_{\mathrm{M}}^{\prime}$. Hence, $R_{\mathrm{M}}$ and $L_{\mathrm{e}, \mathrm{M}}$ are negligible, and $f_{\mathrm{SPP}, \mathrm{e}}$ is thus independent of $w_{\mathrm{M}}^{\prime}$. Secondly, the arrayed MIM configuration exhibits both inductive and capacitive behavior when it responds to the transversal magnetic field. The inductance $\left(L_{\mathrm{m}}\right)$ is a sum of self-inductance $\left(L_{\mathrm{e}, \mathrm{M}}\right)$ and mutual inductance $\left(L_{\mathrm{m}, \mathrm{M}}\right)$ of the metals, i.e., $L_{\mathrm{m}}=L_{\mathrm{e}, \mathrm{M}}+L_{\mathrm{m}, \mathrm{M}}$. Approximately, $L_{\mathrm{m}, \mathrm{M}}=0.5 \mu_{0} w t / l$, where $\mu_{0}$ is vacuum permeability. The capacitance from the dielectric contribution is written as parallel-plated capacitor formula, $C_{\mathrm{D}}=\gamma_{2} \varepsilon_{0} \varepsilon_{\mathrm{D}} w l / t$, where $\gamma_{2}=0.222$ is an effective area coefficient for uneven electric-field distribution in the dielectric [34].

Figures 2(c) and 2(d) illustrate the RLC circuits for responses to the externally electric and magnetic fields of radiation, respectively. Here, $L_{\mathrm{e}}=L_{\mathrm{e}, \mathrm{M}}$ and $L_{\mathrm{G}}=L_{\mathrm{e}, \mathrm{G}}$. In the case $l \gg w_{\mathrm{G}}^{\prime} \gg h$, the parallel-wired capacitance to the magnetic response that accounts for the slotted-air contribution can be expressed as $C_{\mathrm{m}}=\pi \varepsilon_{0} l / \ln \left(w_{\mathrm{G}}^{\prime} / h\right)$; while the electric-response capacitance of the slotted graphene, $C_{\mathrm{e}}$, is mode-dependent, and can be determined by the grating structure and an effective permittivity of the interfaces. Therefore, the fundamental frequency of magnetic SPs is written as:

$$
\begin{equation*}
f_{\mathrm{SPP}, \mathrm{~m}}=\frac{1}{2 \pi}\left[\left(\frac{1}{L_{\mathrm{m}}}+\frac{1}{2 L_{\mathrm{G}}}\right) \frac{1}{C_{\mathrm{m}}}+\frac{1}{L_{\mathrm{m}} C_{\mathrm{D}}}-\frac{\sqrt{-8 L_{\mathrm{m}}^{2} L_{\mathrm{G}}\left(L_{\mathrm{m}}+L_{\mathrm{G}}\right) C_{\mathrm{m}} C_{\mathrm{D}}+\left(L_{\mathrm{m}}^{2} C_{\mathrm{D}}+2 L_{\mathrm{m}} L_{\mathrm{G}} C_{\mathrm{m}}+2 L_{\mathrm{m}} L_{\mathrm{G}} C_{\mathrm{D}}\right)^{2}}}{2 L_{\mathrm{m}}^{2} L_{\mathrm{G}} C_{\mathrm{m}} C_{\mathrm{D}}}\right]^{\frac{1}{2}} \tag{4}
\end{equation*}
$$

Assuming $L_{\mathrm{m}}$ is close to $L_{\mathrm{G}}$ and $C_{\mathrm{D}} \gg C_{\mathrm{m}}$, this frequency approaches to $f_{\mathrm{SP}, \mathrm{m}}=0$, suggesting that the magnetic SPs are no longer supported. For the electric SPs, only the even-order modes can be excited by the radiation, and the mode-dependent frequency is:

$$
\begin{equation*}
f_{\mathrm{SP}, \mathrm{e}}=\frac{1}{2 \pi}\left[\left(L_{\mathrm{G}}+L_{\mathrm{e}}\right)\left(C_{\mathrm{e}}+\frac{C_{\mathrm{D}}}{2}\right)\right]^{-\frac{1}{2}} \tag{5}
\end{equation*}
$$

From Eq. (5), $f_{\mathrm{SP}, \mathrm{e}}$ is certain to be $w$-independent in the case $L_{\mathrm{e}} \ll L_{\mathrm{G}}$, and nearly keeps unchanged for different dielectric thickness when the dielectric is thick enough to satisfy $C_{\mathrm{D}} \ll C_{\mathrm{e}}$, too.

If $f=f_{\mathrm{SPP}, \mathrm{e}}$ or $f=f_{\mathrm{SP}, \mathrm{m}}$, more EM-radiation energies are absorbed by the intraband transitions of graphene after coupled to the graphene SPs and confined at the $\mathrm{Air} / \mathrm{Graphene} / \mathrm{Al}_{2} \mathrm{O}_{3}$ interfaces; otherwise, the energy of graphene SPs is attenuated due to non-zero imaginary impedance of the RLC circuit. Furthermore, when the electric and the magnetic SPs are matched in frequency, the absorption can reach a maximum at $f=f_{\mathrm{SP}, \mathrm{e}}=$ $f_{\mathrm{SP}, \mathrm{m}}$.

Both the grating parameters and the graphene Fermi energy decide the electric-SP frequency, while the magnetic-SP frequency is dominated only by the grating parameters. Thus, an appropriate grating structure in combination with the tunable $E_{\mathrm{F}}$ can be used to guarantee $f_{\mathrm{SP}, \mathrm{m}}$ and $f_{\mathrm{SP}, \mathrm{e}}$ matching. In reality, $f_{\mathrm{SP}, \mathrm{m}}$ can be exactly calculated, and the magnetic SPs in fundamental mode can lead to lower EM dissipations than higher-order modes when the frequency keeps unchanged; instead, $f_{\mathrm{SP}, \mathrm{e}}$ can be located on the basis of the following F-P model of SPPs, owing to an uncertain value of $C_{\mathrm{e}}$ when the electric SPs are in higher-order modes. Figures 3(a) and 3(b) illustrate the magnetic-SP frequency dependence of the grating structure. It is confirmed that for a fixed period of $p=7.0 \mu \mathrm{~m}, \lambda_{0, \mathrm{~m}}$ blue-shifts from 10.47 to $8.99 \mu \mathrm{~m}$ as the grating width decreases from $w=6.0$ to $3.0 \mu \mathrm{~m}$; while for a fixed grating width of $w=5.0 \mu \mathrm{~m}$, the magnetic SPs keep the wavelength at $\lambda_{0, \mathrm{~m}}=10.23 \mu \mathrm{~m}$ nearly unchanged when the period is varied. $f_{\mathrm{SPP}, \mathrm{m}}$ is off proportional to $1 / w$, but does not depend on $p$ to a large extent [35]. It is observed that $\lambda_{0, \mathrm{~m}}$ calculated from Eq. (4) are in good agreement
with the simulated. Figure 3(c) shows the graphene absorption spectra at different $E_{\mathrm{F}}$ when the magnetic-SP frequencies are detuned. Indeed, only the even-order electric SPs are observable. For each order of the electric-SP modes, $\lambda_{0, \mathrm{e}}$ is blue-shifted and the graphene absorbance increases with the Fermi energy. In LWIR spectral region, $\left|\varepsilon_{\mathrm{G}}^{\prime}\right|$ increases but $\varepsilon^{\prime \prime}{ }_{\mathrm{G}}$ decreases with the increased $E_{\mathrm{F}}$ when $E_{\mathrm{F}}>77.5 \mathrm{meV}$. As a consequence of Eq. (3), the equivalent impedances, $R_{\mathrm{G}}$ and $\omega L_{\mathrm{G}}$, decrease, leading to the decreasing $\lambda_{0, \mathrm{e}}$. However, the increasing graphene absorption is still insufficient for a practical device if only electric SPs are involved. Figure 3(d) shows how the spectra in absorbance change with the involvement of magnetic-SPs. With a constant slot width, all electric SPs in the same mode have nearly the same frequency even though $w$ is altered. Therefore, to match with those invariable $\lambda_{0, \mathrm{e}}$, different $\lambda_{0, \mathrm{~m}}$ of the fundamental magnetic SPs can be easily obtained by altering $w$ instead of $w^{\prime}{ }_{\mathrm{G}}$ or $p$. The electric-SP modes of $n=4,6$, and 8 are found at $\lambda_{0, \mathrm{e}}=10.15,9.38$, and $8.76 \mu \mathrm{~m}$, respectively. If the varied $f_{\mathrm{SP}, \mathrm{m}}$ approach those $f_{\mathrm{SP}, \mathrm{e}}$ individually, significant enhancement of the absorption is always observed, attributable to the strong EM-coupling effect between the magnetic and the electric SPs. By the way, the SP coupling can be applied to absorption-band broadening or splitting. Generally, the latter is accompanied by a phenomenon of EM-induced reflection or transparency.


Fig. 3. At $E_{\mathrm{F}}=0.4 \mathrm{eV}$, the device reflectance ( $R$ ) contours: (a) dependence of $w$ when the period of $p=7.0 \mu \mathrm{~m}$ is fixed; on the contrary, (b) dependence of $p$ when the grating width of $w$ $=5.0 \mu \mathrm{~m}$ is fixed. Dash line: the electric SPs along Air/Metal interface of the gratings; scattered symbol: calculated $\lambda_{0, \mathrm{~m}}$. Spectra in absorbance of the graphene $\left(A_{\mathrm{G}}\right)$ : (c) at the different $E_{\mathrm{F}}$ and for the device in the structure of $(p, w)=(3.0,1.0)$, where the electric SPs in the different mode are solely involved in LWIR region; (d) at $E_{\mathrm{F}}=0.6 \mathrm{eV}$ and under the magnetic- and electric-SP coupling when the slotted-graphene width of $w_{\mathrm{G}}{ }^{\prime}=2.0 \mu \mathrm{~m}$ is fixed. To approach $f_{\mathrm{SP}, \mathrm{e}}$ in the modes of $n=4,6$, and 8 individually, the various structures in $(p, w)$ are selected to alter the fundamental $f_{\mathrm{SP}, \mathrm{m}}$.

In contrast, $f_{\mathrm{SPP,}}$ can be determined by the F-P model of coherent SPPs in cavity modes. If the surface-wave-vector condition is satisfied, the graphene SPPs are supported to propagate along the Air/Graphene $/ \mathrm{Al}_{2} \mathrm{O}_{3}$ interfaces, and their dispersion relation can be expressed as [36]:

$$
\begin{equation*}
\frac{1}{\sqrt{k_{\mathrm{SPP}}^{2}-k_{0}^{2}}}+\frac{\varepsilon_{\mathrm{D}}}{\sqrt{k_{\mathrm{SPP}}^{2}-\varepsilon_{\mathrm{D}} k_{0}^{2}}}=-\frac{i \sigma}{\omega \varepsilon_{0}}, \tag{6}
\end{equation*}
$$

where $k_{\text {SPP }}=k_{\text {SPP }}^{\prime}+i k_{\text {SPP }}^{\prime \prime}$ is the complex wave vector of graphene SPPs, and $k_{0}=\omega / c$ is the vacuum wave vector. For $k_{\text {SPP }}^{\prime} \gg k_{0}$, Eq. (6) simplifies to:

$$
\begin{equation*}
k_{\mathrm{SPP}}^{\prime} \approx \frac{\varepsilon_{0}\left(1+\varepsilon_{D}\right)}{2} \frac{2 \omega \sigma^{\prime \prime}}{\sigma^{\prime 2}+\sigma^{\prime 2}} \tag{7}
\end{equation*}
$$

In the model, F-P cavities of the interfaces are formed between the two neighboring gratings, and the coherent SPPs propagating in opposite directions along the interfaces are interfered. The well-known resonance condition is given by $k_{\text {SPP }}^{\prime}=(n \pi-\Delta \varphi) / w_{\mathrm{G}}^{\prime}$, where $\Delta \varphi$ is the phase pickup upon grating edge reflection. To substitute the $k_{\text {SPP }}^{\prime}$ in Eq. (7), we obtain the electricSP frequency

$$
\begin{equation*}
f_{\mathrm{SP}, \mathrm{e}}=\frac{1}{2 \pi} \frac{n \pi-\Delta \varphi}{\varepsilon_{0}\left(1+\varepsilon_{\mathrm{D}}\right) w_{\mathrm{G}}^{\prime}} \frac{\sigma^{2}+\sigma^{\prime \prime 2}}{\sigma^{\prime \prime}} ; n=2,4,6 \ldots \tag{8}
\end{equation*}
$$

As the antisymmetric odd modes are restrained, only even modes of the electric SPs are symmetric to be excited [to see Fig. 5(b)]. Different from $\Delta \varphi \approx \pi / 4$ for the graphene ribbons within an open boundary [37,38], the phase is $\Delta \varphi \approx-3 \pi / 4$ for the graphene SPPs confined between conductive boundaries upon the grating edge reflection. It is due to the absence of tangential electric field at the boundary as the high conductivity of Au is used. As shown in Fig. 4, $\lambda_{0, \mathrm{e}}$ calculated from Eq. (8) is also in good agreement with the simulated, especially for the mode of $n=4$. Larger errors occurred in the other modes are probably resulted from: i) the mode dependence of the reflection phase in calculation; ii) effects of the metal inductance and the dielectric capacitance in simulation; in addition, iii) the longer SP wavelength ( $\lambda_{\mathrm{SP}}=$ $2 \pi / k_{\text {SPP }}^{\prime}$ ), the weaker confinement. The reflection phase for the lower-order mode may exhibit stronger $\lambda_{0, \mathrm{e}}$-dependence than the higher-order modes.


Fig. 4. For the device in the structure of $(p, w)=(3.0,1.0), E_{\mathrm{F}}$ dependence of $\lambda_{0, \mathrm{e}}$ in different mode. $\lambda_{0, \mathrm{e}}$ : calculated (in line); simulated (in symbol).

After the structural parameters are fixed, any deviation of $f_{\mathrm{SP}, \mathrm{e}}$ from $f_{\mathrm{SP}, \mathrm{m}}$ can be corrected by tuning the graphene Fermi energy. Figure 5(a) shows $E_{\mathrm{F}}$ dependence of the graphene absorption for the finally determined structure of $(p, w)=(5.5,3.5)$. As the magnetic SPs are solely involved in LWIR region, only one absorption peak is observable for the graphene at $E_{\mathrm{F}}=0.2 \mathrm{eV}$, and the absorbance has a maximum of $A_{\mathrm{G}}=25 \%$ at $\lambda_{0}=9.50 \mu \mathrm{~m}$. If the Fermi energy increases to $E_{\mathrm{F}}=0.6 \mathrm{eV}$, the peak at $\lambda_{0}=9.38 \mu \mathrm{~m}$ that corresponds to the electric-SP mode of $n=6$ is significantly enhanced. Peak absorbance of the graphene is confirmed to exceed $67.2 \%$, i.e., an enhancement over 29 -fold the absorbance of a suspended graphene (general $2.3 \%$ ). In addition, with the ohmic dissipation of $12.3 \%$ from all the metals and the dielectric loss of $19.5 \%$, the system of device has an absorbance close to $A_{\mathrm{SYS}}=100 \%$. If a
lossless dielectric is applied, it is anticipated that the graphene absorbance can be further boosted to $83.7 \%$. However, other four peaks observed at $\lambda_{0}=10.96,10.15,8.76$, and 8.25 $\mu \mathrm{m}$ are much lower, and correspond to the electric-SP mode of $n=2,4,8$, and 10 , respectively. As illustrated in Fig. 5(b), the lower absorptions are attributable to the weaker electric-field intensities in the graphene; here, the distributions for the modes of $n=8$ and 10 are not shown because their maximum intensities are close to the modes of $n=4$ and 2 , respectively. Detuning of the frequency results in the decrease of EM-coupling efficiency between the magnetic and the electric SPs. In addition, the standing-wave effect from the coherent interference of SPPs further restrains the energy from flowing between the two neighboring electric- or magnetic-SP elements.


Fig. 5. (a) When $(p, w)=(5.5,3.5)$, absorbance spectra of the graphene $\left(A_{\mathrm{G}}\right)$ at different $E_{\mathrm{F}}$. When $E_{\mathrm{F}}=0.6 \mathrm{eV}$, three intense peaks, located at $\lambda_{0}=10.15,9.38$, and $8.76 \mu \mathrm{~m}$, correspond to the electric SPs in the modes of $n=4,6$, and 8 , respectively. (b) At the three peaks, normalized field of $\left|\boldsymbol{E}_{x}\right| /\left|\boldsymbol{E}_{0}\right|$ along the $x$ axis. Boundaries of the grating and bottom metal (in white solid line) and graphene (in black dash) are highlighted.
In case the carrier mobility is different, enhancing the graphene absorption still can be attained, and the center frequency is fixed when the Fermi energy keeps unchanged. Figure 6(a) shows the graphene absorbance dependence of the carrier mobility. There seems no difference in the maximum absorbance of the graphene between $\mu=1 \times 10^{3}$ and $2 \times 10^{3}$ $\mathrm{cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$. As the mobility increases, the absorption peak slightly drops in height and width, while the frequency is always locked at $\lambda_{0}=9.38 \mu \mathrm{~m}$ by fixing the Fermi energy at $E_{\mathrm{F}}=0.6$ eV . The maximum absorbance is reduced to $48.4 \%$ till $\mu=1 \times 10^{4} \mathrm{~cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$. Since $E_{\mathrm{F}} \gg k_{\mathrm{B}} T$, from Eqs. (1) and (7) it is straightforward to obtain [34]

$$
\begin{equation*}
k_{\mathrm{SPP}}^{\prime} \approx \frac{\pi \hbar^{2} \varepsilon_{0}\left(1+\varepsilon_{D}\right)}{e^{2} E_{\mathrm{F}}} \omega^{2}, \tag{9}
\end{equation*}
$$

indicating that the $k_{\text {SPP }}^{\prime}$ is $\mu$-independent. If $E_{\mathrm{F}}>\sim 0.2 \mathrm{eV}$, the optical phonon energy of graphene [34], $\varepsilon_{\mathrm{G}}^{\prime} \ll 0$, in the spectral regime of interest, and from Eqs. (1) and (3) the inductance thus can be expressed as

$$
\begin{equation*}
L_{\mathrm{e}, \mathrm{G}} \approx \gamma_{1} \frac{w_{\mathrm{G}}^{\prime}}{l} \frac{\pi \hbar^{2}}{e^{2} E_{\mathrm{F}}} \tag{10}
\end{equation*}
$$



Fig. 6. The device is designed in the structure of $(p, w)=(5.5,3.5)$. The carrier mobility dependence of the absorbance spectra: for the graphene $\left(A_{\mathrm{G}}\right)$ (a) at $E_{\mathrm{F}}=0.6 \mathrm{eV}$ and (b) around $E_{\mathrm{F}}=0.6 \mathrm{eV}$, as well as (c) for the system of device $\left(A_{\mathrm{SYS}}\right)$ around $E_{\mathrm{F}}=0.6 \mathrm{eV}$.

Only if $E_{\mathrm{F}}<0.75 \mathrm{eV}$ to satisfy $L_{\mathrm{e}, \mathrm{M}} \ll L_{\mathrm{e}, \mathrm{G}}$, from Eqs. (4) and (5), $f_{\mathrm{SP}, \mathrm{e}}$ and $f_{\mathrm{SP}, \mathrm{m}}$ also become $\mu$-independent. Figures 6 (b) and 6 (c) show how the absorption spectra change with the mobility and the Fermi energy. The graphene and the system absorbance can be further increased by fine tuning the Fermi energy to shift the center frequency of EM-induced reflection, which is more clearly observed for the higher mobility. At $E_{\mathrm{F}}=0.65 \mathrm{eV}$, the graphene absorbance is increased to $A_{\mathrm{G}}=54.3 \%$ for $\mu=1 \times 10^{4} \mathrm{~cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$; especially, the device always can keep the system absorbance close to $A_{\mathrm{SYS}}=100 \%$ even though the graphene exhibits the different mobility. To an absorber, the improvement is significant, too.


Fig. 7. For the device in the structure of $(p, w)=(5.5,3.5)$, the absorbance of graphene $\left(A_{\mathrm{G}}, \theta\right)$ and system $\left(A_{\mathrm{SYS}, \theta}\right)$ at $\lambda_{0}=9.38 \mu \mathrm{~m}$ changes with the radiation incident angle $(\theta)$. The graphene: $E_{\mathrm{F}}=0.6 \mathrm{eV} ; \mu=2 \times 10^{3} \mathrm{~cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$. The field angle is determined: at $\theta=\theta_{\mathrm{FV}} / 2$, the graphene absorbance is dropped by 0.37 times that at normal incident.

After the working wavelength is locked, the field angle of view can be determined for the device. Figure 7 shows the graphene and system absorptions at $\lambda_{0}=9.38 \mu \mathrm{~m}$ dependence of the radiation incident angle $(\theta)$. Although the graphene absorbance decreases with the increasing angle, the system one keeps nearly unchanged when $\theta \leq 40^{\circ}$. From the decreasing absorbance of graphene, we determine the field angle up to $\theta_{\mathrm{FV}}=90^{\circ}$.

## 4. Conclusions

In summary, we have designed the graphene LWIR photodetector with a metasurface structure capable of significantly enhancing the optical absorption in the graphene. Peak absorbance of the graphene is confirmed to exceed $67.2 \%$, and can be anticipated to reach $83.7 \%$ if a lossless dielectric is applied. With electric and magnetic resonances in matching frequencies, the absorption enhancement is attributed to the strong EM-coupling effect between the magnetic and the electric SPs, as well as the coherent interference of the graphene SPPs. In case of the graphene in different mobility, significantly enhancing the absorption always can be realized at the fixed frequency, due to the grating and the slot widths designed in the same order of magnitude, and the system absorbance keeps close to $100 \%$. In addition to revealing the underlying coupling mechanism, the equivalent circuit method and the F-P model of coherent SPPs used in this work can also be used to accurately predict the wavelengths of magnetic and electric SPs. This study offers a new approach to design high-performance graphene photodetector and other relevant optoelectronic devices.

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