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Nonrotationally symmetric aberrations of off-axis two-mirror astronomical telescopes induced by axial misalignments

GUOHAO JU,* CHANGXIANG YAN, ZHIYUAN GU, AND HONGCAI MA

Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun, Jilin 130033, China
**Corresponding author: juguohao123@163.com*

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In unobscured off-axis astronomical telescopes with an offset pupil, the effects of axial misalignments are very different from those in on-axis ones. Specifically, a series of nonrotationally symmetric aberrations with characteristic field dependence will be induced by axial misalignments. This paper takes off-axis two-mirror astronomical telescopes as an example to discuss the field characteristics of several important nonrotationally symmetric aberrations (including astigmatism, coma, and trefoil aberration) induced by axial misalignments in off-axis astronomical telescopes. The expressions of these aberrations are derived under some approximations. The accuracy of the proposed expressions is demonstrated. The specific field characteristics of these aberrations are presented and explicated. It is shown that the effects of axial misalignments bear strong similarities to the effects of the lateral misalignments in the symmetry plane of the off-axis system. On the other hand, the inherent relationships between astigmatism and coma induced by axial misalignments are further revealed, which are different from those induced by lateral misalignments. This fact presents the possibility of separating the effects of axial misalignments and lateral misalignments. Most of this work can be extended to other off-axis astronomical telescopes with more freedom.

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1. INTRODUCTION

Despite the advantages in scattering property, emissivity throughput, dynamic range, and ellipticity performance over on-axis astronomical telescopes [1–3], to date only a few large unobstructed off-axis astronomical telescopes have been constructed, including the new solar telescope (NST) [4] and new planetary telescope (NPT) [5]. The main reasons for this lie in the difficulties with the optical surfaces' fabrication and alignment for this class of telescopes [6,7]. While the optical fabrication of the large off-axis surfaces is still challenging, the relevant theory is comparatively mature. On the other hand, while numerical methods have been used for the alignment as well as the active alignment of off-axis astronomical telescopes [8–10], we still lack a deep and comprehensive understanding of the effects of misalignments on the net aberration fields in large off-axis astronomical telescopes, which is not conducive to the construction of large off-axis astronomical telescopes.

In general, there are mainly two kinds of misalignments in astronomical telescopes, i.e., axial misalignments (also called longitudinal misalignments) and lateral misalignments (also called transverse misalignments). Axial misalignments represent the dislocation of optical surfaces along the axial direction, and lateral misalignments represent the decenter and tip-tilt of optical surfaces in the lateral direction. In on-axis astronomical telescopes, axial misalignments change the position of the Gaussian image plane, and they mainly introduce rotationally symmetric aberrations, such as defocus and spherical aberration [11]. Since axial misalignments in on-axis systems do not break the rotational symmetry of the system, their effects on the net aberration fields can easily be dealt with. Lateral misalignments mainly induce nonrotationally symmetric aberrations in this class of systems, such as astigmatism and coma. The effects of lateral misalignments in on-axis systems can be discussed using nodal aberration theory. Nodal aberration theory introduced by Shack and Thompson [12–17], with vector product serving as its mathematical tool, can analytically describe the aberration fields in the presence of lateral misalignments [18,19] or quantitatively compute misalignment parameters if wavefront measurements at several field points are available

[20–22]. In a word, the effects of axial misalignments and lateral misalignments in on-axis systems are relative simple and can effectively be described with the current aberration theories. These two kinds of effects bear little similarity to each other, and they can hardly couple with each other, making the alignment of on-axis systems comparatively simple and straightforward.

However, this is not the case for off-axis large astronomical telescopes. For off-axis telescopes, the rotational symmetry is broken and only plane symmetry is preserved due to pupil decentering. The effects of axial misalignments and lateral misalignments on the net aberration fields in off-axis astronomical telescopes are very different from those in on-axis telescopes. These effects are much harder to describe precisely. Besides, there exist strong similarities between the effects of the two kinds of misalignments, which poses a problem of separating them when they are coupled. On the other hand, the effects of axial misalignments are neglected in some of the current methods for the alignment of off-axis telescopes. Zhang *et al.* once proposed an analytic method to compute the lateral misalignments of off-axis telescopes [23]. They also proposed an analytic method to determine the lateral misalignment of the secondary mirror needed for compensating for those aberrations induced by other perturbations of the off-axis system [24]. However, the presence of axial misalignments can invalidate the computation methods proposed by Zhang. As will be shown in the following sections, axial misalignments can also introduce certain amounts of astigmatism and coma, as well as other types of aberrations. If we neglect the effects of axial misalignments and directly use the methods of Zhang *et al.* to compute the lateral misalignment parameters, all the aberration contribution of axial misalignments will be wrongly attributed to lateral misalignments, which can definitely introduce error in the computation results.

In our previous work, we presented a systematic discussion on the effects of lateral misalignments on the net aberration fields in off-axis two-mirror astronomical telescopes [25]. In that previous work, we made efforts to answer the following three questions: (1) how to analytically express the aberration fields of off-axis systems with lateral misalignments; (2) what new aberration field characteristics can be induced by lateral misalignments for off-axis systems compared to those for on-axis ones; (3) what valuable insights or theoretical guidance can be provided by the knowledge of the aberration fields induced by lateral misalignments in off-axis systems. This work can lead to a deep understanding of the aberration fields of off-axis astronomical telescopes induced by lateral misalignments. However, the effects of axial misalignments on the aberration field characteristics of off-axis astronomical telescopes have seldom been discussed in previous research. These effects are also quite different from those in on-axis telescopes. They still require further study for better understanding.

In this paper, we will discuss the aberration field characteristics of off-axis two-mirror astronomical telescopes induced by axial misalignments, especially for those nonrotationally symmetric aberrations. We first derive the expressions of several important nonrotationally symmetric aberrations under certain approximations. On this basis, we discuss the specific field characteristics of these aberrations. It will be shown that the field characteristics of these aberrations induced by axial misalignments bear strong similarities to those induced by lateral misalignments in the symmetry plane. This fact indicates that the effects of axial misalignments and lateral misalignments can couple together, and it is hard to separate them. On the other hand, we further reveal the inherent relationships between astigmatism and coma induced by axial misalignments, which are different from those induced by lateral misalignments. This fact presents the possibility of separating the effects of axial misalignments and lateral misalignments. This work can contribute to a deep understanding of the aberration field characteristics of off-axis two-mirror astronomical telescopes induced by axial misalignments. Most of this work can be extended to other off-axis astronomical telescopes with more freedom.

This paper is organized as follows. In Section 2, the expression for the net aberration contribution of axial misalignments is derived for off-axis two-mirror telescopes, which usually have a small field of view. Section 3 discusses the characteristics of the field dependence of several important nonrotationally symmetric aberrations (astigmatism, coma, and trefoil aberration) in off-axis two-mirror astronomical telescopes induced by axial misalignments. Then the inherent relationships between astigmatism and coma aberration induced by axial misalignments are further revealed and explicated in Section 4. In Section 5, we summarize and conclude the paper.

2. ABERRATION FUNCTION OF OFF-AXIS TWO-MIRROR ASTRONOMICAL TELESCOPES WITH AXIAL MISALIGNMENTS

We here first derive the aberration function for off-axis two-mirror astronomical telescopes induced by axial misalignments. To this end, we will apply a system-level pupil coordination transformation to the aberration function of an axially misaligned on-axis system. In the last part of this section, we present a general description of the effects of axial misalignments.

First consider the on-axis field point (field center) for an on-axis system in the nominal state. For an on-axis field point, the field-dependent aberrations are equivalent to zero, and we only need to consider spherical aberrations, including low-order and high-order ones. In this case, the aberration function can be expressed as

$$W^{(\text{On-axis})} = W_{040}(\vec{\rho}\cdot\vec{\rho})^2 + W_{060}(\vec{\rho}\cdot\vec{\rho})^3 + \ldots W_{0(2N)0}(\vec{\rho}\cdot\vec{\rho})^N, \tag{1}$$

where $W_{0(2N)0}$ represents the 2Nth-order wave spherical aberration coefficient. In the presence of axial misalignments, this expression can be rewritten as

$$W^{(\text{On-axis})} = W_{040}^{(AM)}(\vec{\rho}\cdot\vec{\rho})^2 + W_{060}^{(AM)}(\vec{\rho}\cdot\vec{\rho})^3 + \ldots W_{0(2N)0}^{(AM)}(\vec{\rho}\cdot\vec{\rho})^N, \tag{2}$$

where $W_{0(2N)0}^{(AM)}$ represents the 2Nth-order wave spherical aberration coefficient in the presence of axial misalignments, which can be further expressed as

$$W_{0(2N)0}^{(AM)} = W_{0(2N)0} + \Delta W_{0(2N)0}, \tag{3}$$

and here $\Delta W_{0(2N)0}$ represents the net contribution induced by axial misalignments.

The aberration function of an axially misaligned off-axis system can be obtained by applying a system-level pupil coordinate transformation [26] to the aberration function of an axially misaligned on-axis system. This pupil coordinate transformation is shown in Fig. 1.

In Fig. 1, $\vec{\rho}$ and $\vec{\rho}'$ denote the pupil vector for the on-axis system and the off-axis system, respectively. $\vec{s}$ represents the location of the off-axis pupil relative to the on-axis pupil. All of these three vectors are normalized by the half-aperture size of the on-axis pupil.

The aberration function for the on-axis field point of an off-axis system can then be expressed as

$$W^{(\text{off-axis})} = W_{040}^{(AM)}[(\vec{\rho}' + \vec{s}) \cdot (\vec{\rho}' + \vec{s})]^2 + W_{060}^{(AM)}[(\vec{\rho}' + \vec{s}) \cdot (\vec{\rho}' + \vec{s})]^3 + \ldots W_{0(2N)0}^{(AM)}[(\vec{\rho}' + \vec{s}) \cdot (\vec{\rho}' + \vec{s})]^N. \quad (4)$$

Combining Eqs. (3) and (4), we can obtain the net aberration contribution of axial misalignments for the on-axis field point of an off-axis system, which can be given by

$$\Delta W^{(\text{off-axis})} = \Delta W_{040}[(\vec{\rho}' + \vec{s}) \cdot (\vec{\rho}' + \vec{s})]^2 + \Delta W_{060}[(\vec{\rho}' + \vec{s}) \cdot (\vec{\rho}' + \vec{s})]^3 + \ldots \Delta W_{0(2N)0}[(\vec{\rho}' + \vec{s}) \cdot (\vec{\rho}' + \vec{s})]^N. \quad (5)$$

Then we continue to discuss the off-axis field point. Note that the net aberration contribution of axial misalignments on the spherical aberration coefficient of each surface is usually a first-order small quantity. Off-axis two-mirror astronomical telescopes usually have a small field of view. In this case, the net contribution of axial misalignments on the coefficients of those field-dependent aberrations in an on-axis system, such as coma and astigmatism, are usually high-order small quantities and therefore can be neglected. Here we take the net contribution of axial misalignments on the coma aberration coefficient as an example, which can be expressed as

$$\Delta W_{131} = \sum_{j=1} (\Delta W_{131j}^{(\text{sph})} + \Delta W_{131j}^{(\text{asph})}), \quad (6)$$

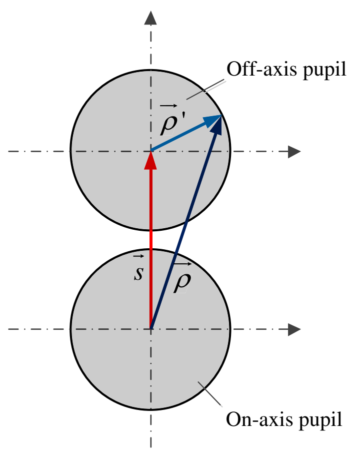

Fig. 1. Illustration of the system-level pupil coordinate transformation.

where

$$\Delta W_{131j}^{(\text{sph})} = \frac{\bar{i}_j}{i_j} \cdot \Delta W_{040j}^{(\text{sph})}, \qquad \Delta W_{131j}^{(\text{asph})} = \frac{\bar{y}_j}{y_j} \cdot W_{040j}^{(\text{asph})}, \quad (7)$$

where the superscripts (sph) and (asph) indicate that these aberration coefficients are for the spherical base sphere and the aspheric departure, respectively, $\bar{i}_j$ and i_j are the chief ray incident angle and marginal ray incident angle at surface j, respectively, and $\bar{y}_j$ and y_j are the chief ray height and marginal ray height at the surface j, respectively. In general, for the surfaces in large astronomical telescope systems with a relatively small field of view, especially for two-mirror astronomical telescopes, we can have $|\bar{i}_j| \ll |i_j|$, $|\bar{y}_j| \ll |y_j|$ (here $|\cdot|$ represents the absolute value operator). Therefore, both $\frac{\bar{i}_j}{i_j}$ and $\frac{\bar{y}_j}{y_j}$ are small quantities, i.e., $\Delta W_{131j}^{(\text{sph})}$ and $\Delta W_{131j}^{(\text{asph})}$ for each surface are second-order small quantities. Besides, it is very possible that $\Delta W_{131j}^{(\text{sph})}$ and $\Delta W_{131j}^{(\text{asph})}$ for different surfaces have different signs, and they can compensate for each other. In other words, the effects of axial misalignments (the level of which is usually very small) on the coma aberration coefficient can be neglected.

Since the effects of axial misalignments on the coma aberration coefficient (in the on-axis system) can be neglected, the effects of axial misalignments on those aberrations (in the on-axis system) with higher-order field dependence, such as astigmatism and trefoil, can also be neglected. Therefore, we can use Eq. (5) to express the effect of axial misalignments on the aberration fields of off-axis two-mirror telescopes.

Now we can present a general description of this effect. The axial misalignments can introduce small amounts of spherical aberrations in the on-axis system, and these aberrations can generate a series of nonrotationally symmetric aberrations with lower-order pupil dependence in the off-axis system through pupil coordinate transformation. As will be shown in the following sections, the magnitudes of these aberrations generated through pupil coordinate transformation are usually much bigger than the magnitudes of those axial-misalignments-induced spherical aberrations. We also can easily recognize that the effect of axial misalignments on the aberration fields of off-axis two-mirror telescopes is nearly field constant. For those off-axis three-mirror anastigmatic astronomical (TMA) telescopes with a much larger field of view, a field-dependent component can also be induced by axial misalignments. However, in general, the field-constant component is still the dominant one.

3. NONROTATIONALLY SYMMETRIC ABERRATIONS OF OFF-AXIS TWO-MIRROR ASTRONOMICAL TELESCOPES INDUCED BY AXIAL MISALIGNMENTS

This section discusses the aberration field characteristics of off-axis two-mirror telescopes induced by axial misalignments. The emphasis will be laid on several important nonrotationally symmetric aberrations. In on-axis telescopes, the nonrotationally symmetric aberrations induced by small axial misalignments can be neglected, while this is not the case for off-axis astronomical telescopes. As will be shown in this section, the nonrotationally symmetric aberrations are more sensitive to axial misalignments in off-axis telescopes, and they will exhibit a characteristic field dependence. The rotationally symmetric aberrations, such as defocus, are similar to those in on-axis telescopes, and so we no longer discuss them.

We will derive the specific expressions for the field dependence of several important nonrotationally symmetric aberrations with the vector product introduced by Shack under some approximations. The accuracy of the derived expressions will be demonstrated with real ray-trace data. The field characteristics of these aberrations are explicated. Meanwhile, we particularly point out the differences between the aberration field characteristics in off-axis telescopes and those in on-axis ones that are induced by axial misalignments. In addition, we also present the similarities and differences between the aberration field characteristics induced by axial misalignments and those induced by lateral misalignments in off-axis telescopes.

A. Astigmatism

Here we first discuss the astigmatic aberration field characteristics induced by axial misalignments. To this end, Eq. (5) can first be rewritten as

$$\Delta W = \sum_{n=2}^{N} \Delta W_{0(2n)0}[(\vec{\rho}\cdot\vec{\rho}) + 2(\vec{\rho}\cdot\vec{s}) + (\vec{s}\cdot\vec{s})]^n, \quad (8)$$

where the primes on the pupil coordinate and the superscript have been dropped. This expression can further be rewritten as

$$\Delta W = \sum_{n=2}^{N}\sum_{g=0}^{n}\sum_{h=0}^{n-g} \Delta W_{0(2n)0}\binom{n}{g}\binom{n-g}{h} \times 2^h(\vec{\rho}\cdot\vec{\rho})^g(\vec{s}\cdot\vec{\rho})^h(\vec{s}\cdot\vec{s})^{n-g-h}. \quad (9)$$

Here we suppose

$$\vec{s} = s\vec{i}, \quad (10)$$

where s is the modulus of $\vec{s}$, and $\vec{i}$ is a unit vector that has the same direction as $\vec{s}$ [27]. In this case, Eq. (9) can further be rewritten as

$$\Delta W = \sum_{n=2}^{N}\sum_{g=0}^{n}\sum_{h=0}^{n-g} \Delta W_{0(2n)0}\binom{n}{g}\binom{n-g}{h} \times 2^h s^{2n-2g-h}(\vec{i}\cdot\vec{\rho})^h(\vec{\rho}\cdot\vec{\rho})^g. \quad (11)$$

The astigmatic aberration term can only come from the term in Eq. (11) with $g = 0$ and $h = 2$. Using the following vector product identity:

$$2(\vec{i}\cdot\vec{\rho})^2 = (\vec{i}\cdot\vec{i})(\vec{\rho}\cdot\vec{\rho}) + \vec{i}^2\cdot\vec{\rho}^2, \quad (12)$$

we can derive the specific expression for net astigmatic aberration field induced by axial misalignments from Eq. (11), which can be given by

$$\Delta W_{AST} = K_{AST}\vec{i}^2\cdot\vec{\rho}^2, \quad (13)$$

where

$$K_{AST} = \sum_{n=2}^{N} 2\binom{n}{2} s^{2n-2}\Delta W_{0(2n)0}. \quad (14)$$

When only the fourth-order spherical aberration is considered here, i.e., $N = 2$, we can obtain

$$\Delta W_{AST} = 2s^2\Delta W_{040}\vec{i}^2\cdot\vec{\rho}^2. \quad (15)$$

When the sixth-order spherical aberration is further considered, i.e., $N = 3$, we have

$$\Delta W_{AST} = (2s^2\Delta W_{040} + 6s^4\Delta W_{060})\vec{i}^2\cdot\vec{\rho}^2. \quad (16)$$

We can infer the following two points from Eqs. (13)–(16):

(1) The direction of the field-constant astigmatism induced by axial misalignments is fixed. This is because the direction of astigmatism is determined by K_{AST} and $\vec{i}^2$. The direction of $\vec{i}^2$ is fixed ($\vec{i}$ points in the direction of the pupil decenter). The sign of K_{AST} can be positive or negative. In other words, there are only two possible directions for this field-constant astigmatism.

(2) The magnitude of this astigmatism is much larger than the change of those spherical aberrations ($\Delta W_{0(2n)0}$) of the on-axis parent system induced by axial misalignments. This is because to achieve an unobscured configuration, the magnitude of the relative pupil decenter is usually larger than two, i.e., $s > 2$ (taking NST as an example, $s = 2.3$ and s represents the ratio of pupil decenter to radius of the pupil). Referring to Eqs. (13)–(16), we can recognize that the pupil coordinate transformation has a magnifying effect, which further has a positive correlation with the order of pupil dependence for the spherical aberrations.

The astigmatic aberration field of the parent on-axis system of NST and NST without and with axial misalignments (the secondary mirror is axially misaligned by −0.2 mm) is shown in Figs. 2(a) and 2(b), respectively. The optical parameters and layout of the NST are shown in Appendix C of [25]. The field shown here is [−0.05°, 0.05°].

We can see from Fig. 2 that the astigmatism in NST is far more sensitive to axial misalignments than in its parent on-axis system. In its parent on-axis system, the effects of axial misalignment can hardly be recognized, while in NST, a large field-constant astigmatism will be induced, which lies perpendicular to the symmetry plane of NST. As will be shown in the following sections, if a positive axial misalignment is present, the direction of the astigmatism will lie along the symmetry plane of NST.

Here we also point out that this astigmatism is very similar to that induced by the lateral misalignments in the symmetry plane of the off-axis system (not the lateral misalignments perpendicular to the symmetry plane). This kind of lateral misalignment can also introduce a large field-constant astigmatism that lies along or perpendicular to the symmetry plane.

Here we begin to evaluate the accuracy of Eqs. (15) and (16) with the real ray-trace data. On one hand, we obtain the changes of the fringe Zernike coefficients (C_5/C_6) for three field points [(0°, 0°), (0.03°, 0.03°), and (0.03°, −0.03°)] induced by axial misalignments specified above from the optical simulation software (CODE V or Zemax). On the other hand, we calculate the changes of the fringe Zernike coefficients of these three field points with Eqs. (15) and (16), respectively. The values of ΔW_{040} and ΔW_{060} of the on-axis parent system needed in Eqs. (15) and (16) can be calculated with the traditional aberration theories [28] or directly obtained from optical simulation software (when the secondary is axially misaligned by −0.2 mm, $\Delta W_{040} = -0.0736\lambda$ and $\Delta W_{060} = 7.906\times10^{-4}\lambda$. Here $1\lambda = 500$ nm.). The results are shown in Table 1, where column A represents the results obtained from

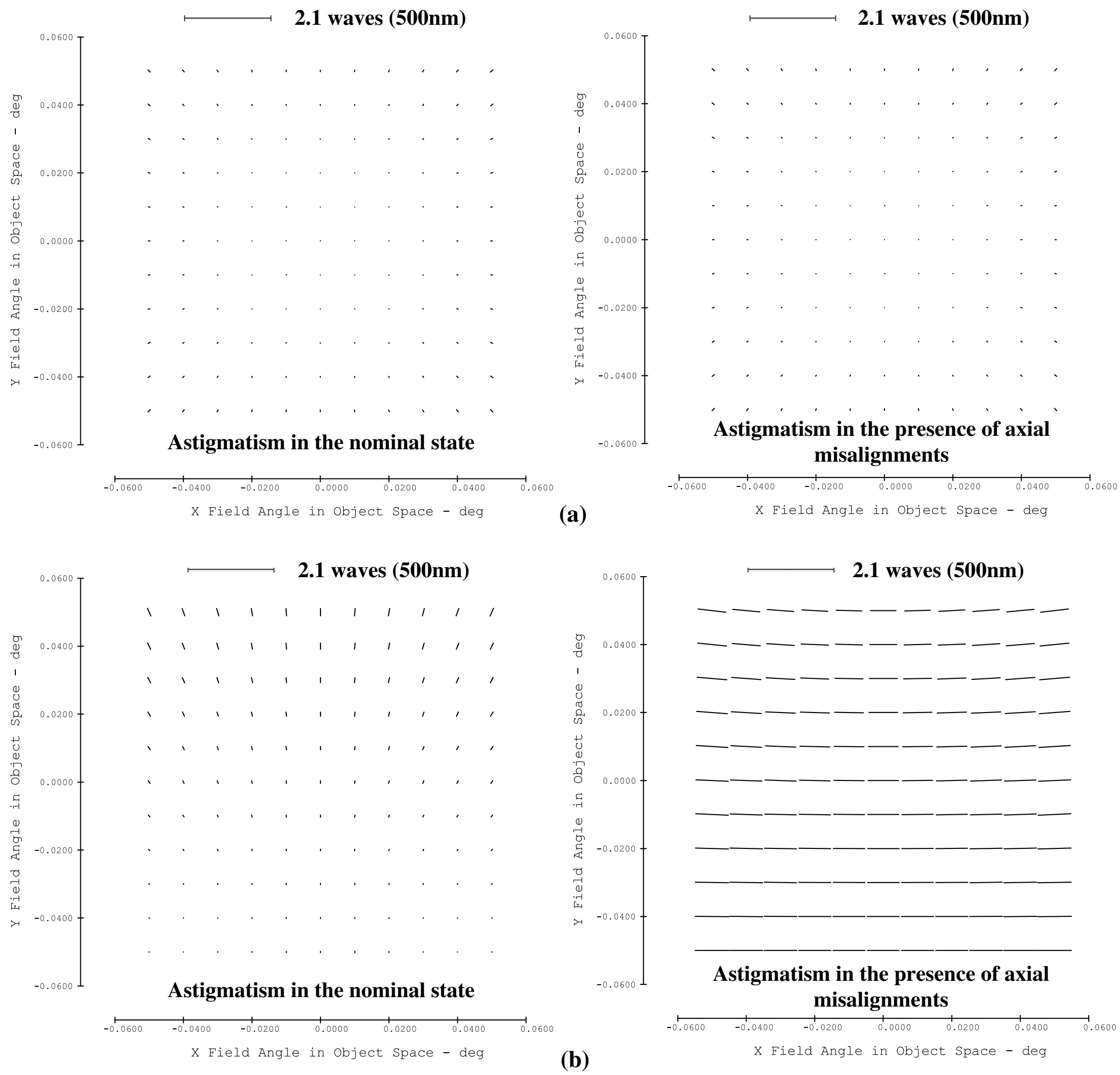

Fig. 2. Astigmatic aberration fields of the parent on-axis system of NST (a) and NST (b) without (left) and with (right) axial misalignments. On the one hand, we can see that the astigmatism in the on-axis parent system is very insensitive to axial misalignments. On the other hand, we can recognize that the astigmatic aberration field of the NST with axial misalignments can be seen as a combination of the astigmatism in the nominal state and a large field-constant astigmatism. In other words, a large field-constant astigmatism is induced by axial misalignments in NST.

Table 1. Verification for the Expressions Describing the Astigmatic Aberration Field in the Axially Misaligned NST

		A	B	C
(0°, 0°)	ΔC_5	0.6443	0.7787	0.6460
	ΔC_6	0.0000	0.0000	0.0000
(0.03°, 0.03°)	ΔC_5	0.6435	0.7787	0.6460
	ΔC_6	0.0010	0.0000	0.0000
(0.03°, −0.03°)	ΔC_5	0.6451	0.7787	0.6460
	ΔC_6	0.0010	0.0000	0.0000

optical simulation software, and column B and column C represent the results calculated with Eqs. (15) and (16), respectively.

We can see from Table 1 that Eq. (16) is more accurate than Eq. (15). While the change of the sixth-order spherical aberration ΔW_{060} in the on-axis parent system induced by axial misalignments is very small, this contribution can be magnified by $6s^4$ (for NST, $6s^4 = 168$) times when it is converted to astigmatism during pupil coordinate transformation. Therefore, Eq. (16) is more accurate.

However, we can still recognize a deviation between the results from optical simulation software and those calculated by Eq. (16). On the one hand, Eq. (16) does not consider the aberration contribution of those higher-order (>6) spherical aberrations. This is the main reason for the on-axis field point. On the other hand, we neglect the small field-dependent aberrations induced by axial misalignments.

While Eqs. (15) and (16) are not accurate enough, they can effectively describe the astigmatic aberration field induced by axial misalignments with a very concise expression. We can also clearly understand the mechanism for why astigmatism in an off-axis system is more sensitive to axial misalignments. Specifically, a small number of spherical aberrations will be induced by axial misalignments in the on-axis parent system. These spherical aberrations will generate astigmatism in the off-axis system during pupil coordinate transformation, the magnitude of which is far larger than the original spherical aberrations.

B. Coma

Here we continue to derive the expression of the coma aberration field from Eq. (11). Apparently, the coma aberration term can come from the aberration term in Eq. (11) with $g = 1$, $h = 1$. In addition, the coma aberration term can also come from the aberration term in Eq. (11) with $g = 0$, $h = 3$, which can be shown below:

$$8(\vec{\rho} \cdot \vec{i})^3 = 6(\vec{i} \cdot \vec{i})(\vec{i} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) + 2\vec{i}^3 \cdot \vec{\rho}^3. \quad (17)$$

Therefore, we can obtain the expression of the coma aberration field induced by axial misalignments, which is given by

$$\Delta W_{\text{Coma}} = K_{\text{Coma}}(\vec{i} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}), \quad (18)$$

where

$$K_{\text{Coma}} = \left[\sum_{n=2}^{N} 2\binom{n}{1}\binom{n-1}{1}\Delta W_{0(2n)0} + \sum_{n=3}^{N} 6\binom{n}{3}\Delta W_{0(2n)0}\right] s^{2n-3}. \quad (19)$$

When only the fourth-order spherical aberration is considered here, i.e., $N = 2$, we can obtain

$$\Delta W_{\text{Coma}} = 4s\Delta W_{040}(\vec{i} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}). \quad (20)$$

When the sixth-order spherical aberration is further considered, i.e., $N = 3$, we have

$$\Delta W_{\text{Coma}} = (4s\Delta W_{040} + 18s^3\Delta W_{060})(\vec{i} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}). \quad (21)$$

We can also see from Eqs. (18)–(21) that the direction of coma in off-axis two-mirror astronomical telescopes induced by axial misalignments is still fixed, which is determined by the direction of $\vec{i}$ and the sign of K_{Coma}. In other words, there are only two possible directions for this coma. On the other hand, the magnitude of this coma is also far larger than ΔW_{040} and ΔW_{060}.

The coma aberration fields of the parent on-axis system of NST without and with axial misalignments are shown in Fig. 3(a). The coma aberration fields of the NST without and with axial misalignments are shown in Fig. 3(b). We can see that in an on-axis system, coma is very insensitive to axial misalignments, while in NST, a relatively large coma is induced by axial misalignments. This is because the spherical aberration induced by axial misalignments in the on-axis parent system can generate a field-constant coma aberration, the magnitude of which is far larger than the original spherical aberrations. Besides, we can see that the direction of this coma lies in $+y$ direction (opposite to the direction of the pupil decenter for the NST used here). If the axial misalignment of the secondary is positive, the direction of the coma will lie opposite to the $+y$ direction. Note that the lateral misalignments in the symmetry plane of the off-axis system can also introduce a field-constant coma that lies along or opposite to the $+y$ direction.

Now we further validate the accuracy of the expressions [Eqs. (20) and (21)] describing coma aberration induced by axial misalignments. On the one hand, we obtain the changes of the fringe Zernike coefficients (C_7/C_8) for three field points [(0°, 0°), (0.03°, 0.03°), and (0.03°, −0.03°)] induced by axial misalignments from the optical simulation software. On the other hand, we calculate the changes of the fringe Zernike coefficients of these three field points with Eqs. (20) and (21), respectively. The results are shown in Table 2, where column A represents the results obtained from optical simulation software, and column B and column C represent the results calculated with Eqs. (20) and (21), respectively.

The results in Table 2 are similar to those in Table 1. We can see from Table 2 that Eq. (21) is more accurate than Eq. (20), because Eq. (21) further considers sixth-order spherical aberrations, the magnitude of which will be greatly magnified when it is converted to coma during pupil coordinate transformation. However, we can still recognize a deviation between the results from optical simulation software and those calculated by Eq. (21), for the contributions of the higher-order (>6) spherical aberrations are not considered in this expression. On the other hand, we can still consider that these two equations can effectively describe the field characteristics of coma aberration induced by axial misalignments with a very concise formation. They can also help us to understand why coma in off-axis astronomical telescopes is more sensitive to axial misalignments.

C. Trefoil Aberration

Here we further discuss another important nonrotationally symmetric aberration, i.e., trefoil aberration. In on-axis astronomical telescopes, this aberration is usually induced by trefoil figure errors. However, in off-axis astronomical telescopes, this is not the case. This part will show that in off-axis astronomical telescopes, trefoil aberration can also be induced by axial misalignments.

The trefoil aberration term can only come from the term in Eq. (11) with $g = 0$, $h = 3$. Combining Eqs. (11) and (17), the trefoil aberration induced by axial misalignments in off-axis astronomical telescopes can be given by

$$\Delta W_{\text{TRE}} = K_{\text{TRE}}(\vec{i}^3 \cdot \vec{\rho}^3), \quad (22)$$

where

$$K_{\text{TRE}} = \sum_{n=3}^{N} 2\Delta W_{0(2n)0}\binom{n}{3} s^{2n-3}. \quad (23)$$

When the sixth-order spherical aberration is considered, i.e., $N = 3$, we have

$$\Delta W_{\text{TRE}} = 2s^3\Delta W_{060}(\vec{i}^3 \cdot \vec{\rho}^3). \quad (24)$$

Note that trefoil aberration cannot be generated by the fourth-order spherical aberration through pupil coordinate transformation. We can see from Eqs. (22)–(24) that a field-constant trefoil aberration can be induced by axial misalignments. The directions of this aberration are also fixed, and there are only two possible directions for this aberration.

The trefoil aberration fields of the on-axis parent system and NST without and with axial misalignments are shown in Fig. 4. The results shown here are similar to those shown in Figs. 2 and 3. Specifically, hardly any trefoil aberration is induced by

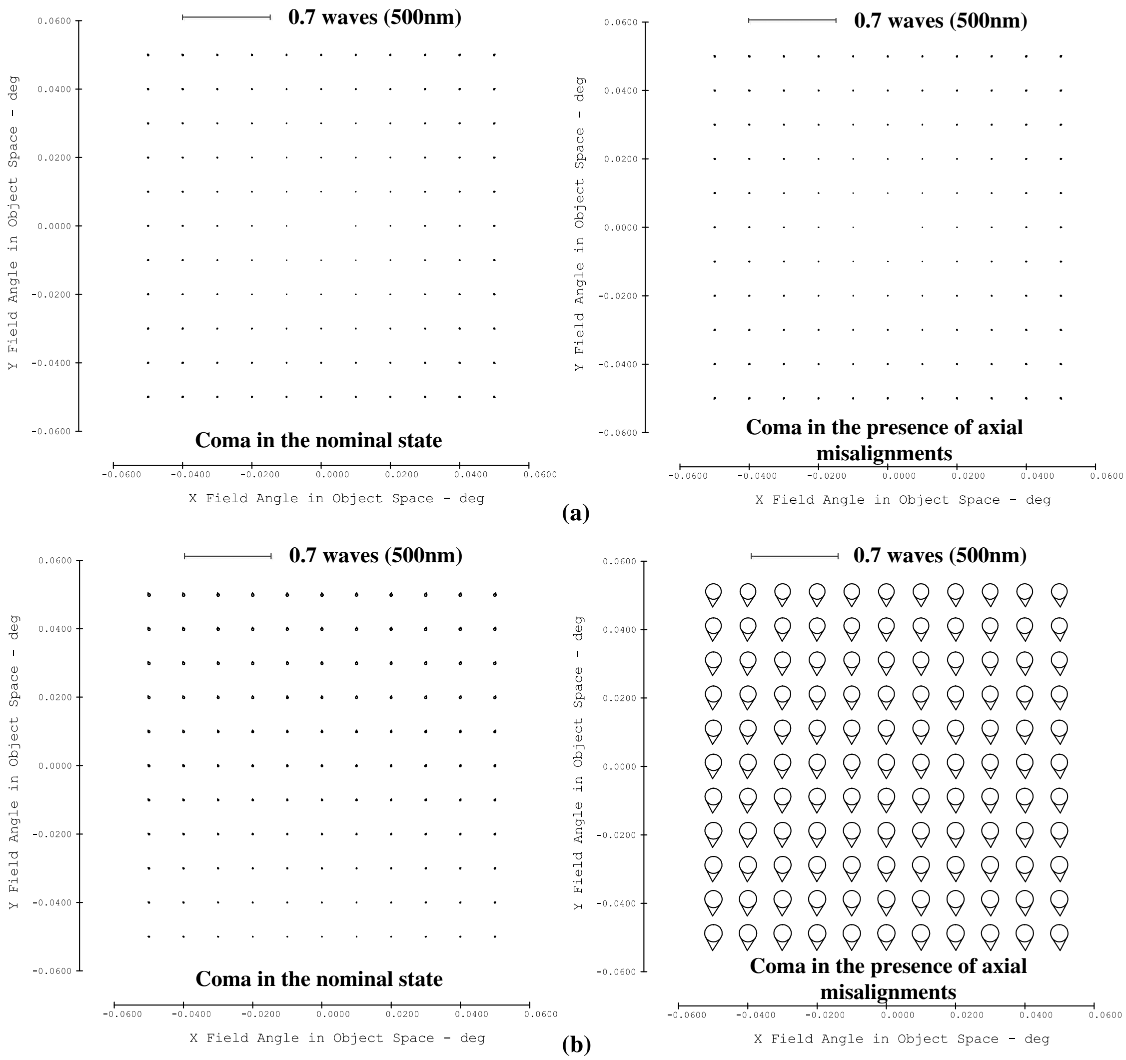

Fig. 3. Coma aberration fields of the parent on-axis system of NST (a) and NST (b) without (left) and with (right) axial misalignments. We can clearly see that hardly any coma is induced by axial misalignments in the on-axis parent system, while a field-constant coma is induced by axial misalignments in the NST.

Table 2. Verification for the Expressions Describing the Coma Aberration Field in the Axially Misaligned NST

		A	B	C
(0°, 0°)	ΔC_7	0.0000	0.0000	0.0000
	ΔC_8	0.1727	0.2257	0.1680
(0.03°, 0.03°)	ΔC_7	0.0000	0.0000	0.0000
	ΔC_8	0.1728	0.2257	0.1680
(0.03°, −0.03°)	ΔC_7	0.0000	0.0000	0.0000
	ΔC_8	0.1726	0.2257	0.1680

axial misalignments in the on-axis system, while a certain amount of field-constant trefoil aberration is induced by axial misalignments in NST. We can also recognize the major source of this trefoil aberration from Eqs. (23) and (24). While ΔW_{060} is usually very small, it can generate a comparatively large trefoil aberration due to the magnification effect of pupil coordinate transformation. Here we also point out that lateral misalignments in the symmetry plane can also introduce this kind of trefoil aberration.

The accuracy of Eqs. (23) and (24) is also demonstrated here with real ray-trace data. On the one hand, we obtain the changes of the fringe Zernike coefficients (C_{10}/C_{11}) induced by axial misalignments from the optical simulation software. On the other hand, we calculate these changes with Eqs. (23) and (24). The results are presented in Table 3, where column A represents the results obtained from optical simulation software and column B represents the results calculated with Eqs. (23) and (24).

On the one hand, we can see from Table 3 that Eqs. (23) and (24) can roughly represent the field characteristics of trefoil aberration. The actual trefoil aberration induced by axial misalignments is field constant, and the result of Eqs. (23) and (24) is also field constant. On the other hand, there still exists a deviation between the real ray-trace data and results calculated with Eqs. (23) and (24). To further improve accuracy, higher-order (>6) spherical aberration should be considered.

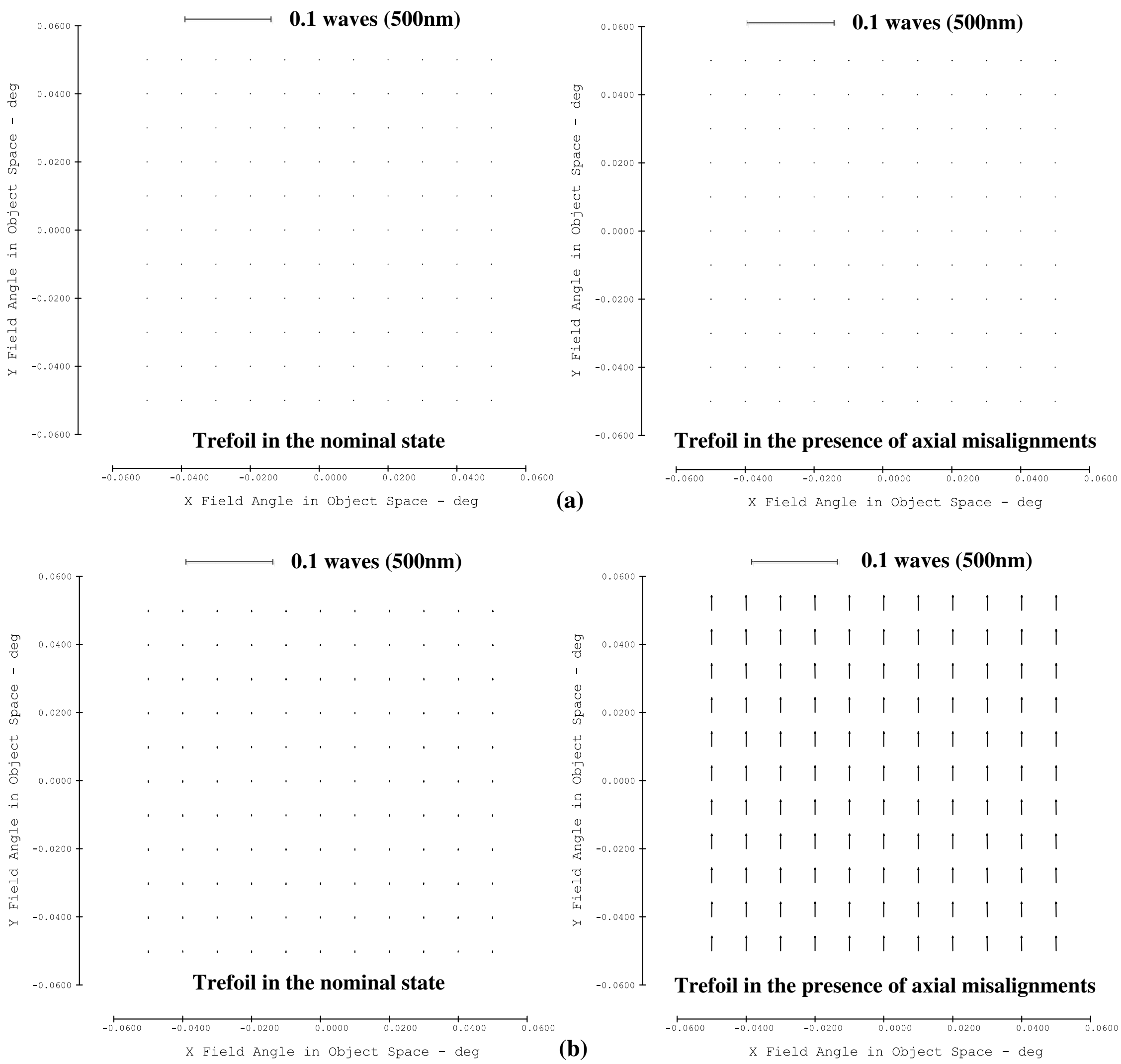

Fig. 4. Trefoil aberration fields of the parent on-axis system of NST (a) and NST (b) without (left) and with (right) axial misalignments. We can clearly see that hardly any trefoil aberration is induced by axial misalignments in the on-axis parent system, while a field-constant trefoil is induced by axial misalignments in the NST.

Table 3. Verification for the Expressions Describing the Trefoil Aberration Field in the Axially Misaligned NST

		A	B
(0°, 0°)	ΔC_{10}	0.0000	0.0000
	ΔC_{11}	0.0150	0.0192
(0.03°, 0.03°)	ΔC_{10}	0.0000	0.0000
	ΔC_{11}	0.0150	0.0192
(0.03°, −0.03°)	ΔC_{10}	0.0000	0.0000
	ΔC_{11}	0.0150	0.0192

4. INHERENT RELATIONSHIPS BETWEEN ASTIGMATISM AND COMA INDUCED BY AXIAL MISALIGNMENTS

In the section above, we discuss the specific field characteristics of several important nonrotationally aberrations induced by axial misalignments. Meanwhile, we point out that these aberration field characteristics bear strong similarities to those induced by the lateral misalignments in the symmetry plane of the off-axis system. Specifically, both axial misalignments and lateral misalignments can introduce some field-constant nonrotationally symmetric aberrations, mainly including astigmatism, coma, and trefoil aberration. Besides, the directions of these aberrations induced by axial misalignments are also the same as those induced by the lateral misalignments in the symmetry plane of the off-axis system (not the lateral misalignments perpendicular to the symmetry plane) [25]. Therefore, the effects of axial misalignments can couple with the effects of the lateral misalignments in the symmetry plane, and it is hard to separate them.

However, in this section we will show that the inherent relationships between (the magnitudes of) astigmatism and coma induced by axial misalignments are different from those induced by lateral misalignments. This fact presents the possibility of separating the effects of axial misalignments and lateral misalignments.

Here we first discuss the relationships between the directions of astigmatism and coma induced by axial misalignments. According to Eq. (13), the direction of astigmatism can be expressed as

$$\phi_{\mathrm{AST}} = \frac{1}{2}[2\xi(\vec{i}) + \psi(K_{\mathrm{AST}})], \quad \textbf{(25)}$$

where ϕ_{AST} represents the direction of the astigmatism induced by axial misalignments, $\xi(\vec{i})$ represents the azimuthal angle of the vector $\vec{i}$, which is measured from $+x$ axis, and $\psi(x)$ is a self-defined function, the value of which is determined by the sign of the constant x:

$$\psi(x) = \begin{cases} 0, & x > 0 \\ \pi, & x < 0 \end{cases}. \quad \textbf{(26)}$$

It makes no sense here when $x = 0$. The direction of astigmatism is in $[0, \pi)$, and the cycle period is π. If the result of Eq. (25) is not in the range of $[0, \pi)$, we should convert it into this range according to its periodicity.

According to Eq. (18), the direction of coma can be expressed as

$$\phi_{\mathrm{Coma}} = \xi(\vec{i}) + \psi(K_{\mathrm{Coma}}). \quad \textbf{(27)}$$

$\vec{i}$ lies in the symmetry plane of the off-axis system. Therefore, the direction of coma aberration in off-axis two-mirror astronomical telescopes induced by axial misalignments also lies in the symmetry plane of the system.

Besides, we should note that the sign of K_{AST} and K_{Coma} is determined by the sign of W_{040}, not the higher-order spherical aberrations. Therefore, we can obtain the relation between the direction of astigmatism and coma in off-axis two-mirror astronomical telescopes, which can be expressed as

$$\phi_{\mathrm{AST}} = \phi_{\mathrm{Coma}} - \frac{1}{2}\psi(\Delta W_{040}). \quad \textbf{(28)}$$

Note that the range for the direction of astigmatism is $[0, \pi)$, and the range for the direction of coma is $[0, 2\pi)$. When the result of Eq. (28) is out of the range of $[0, \pi)$, we still should convert it into $[0, \pi)$ according to its periodicity.

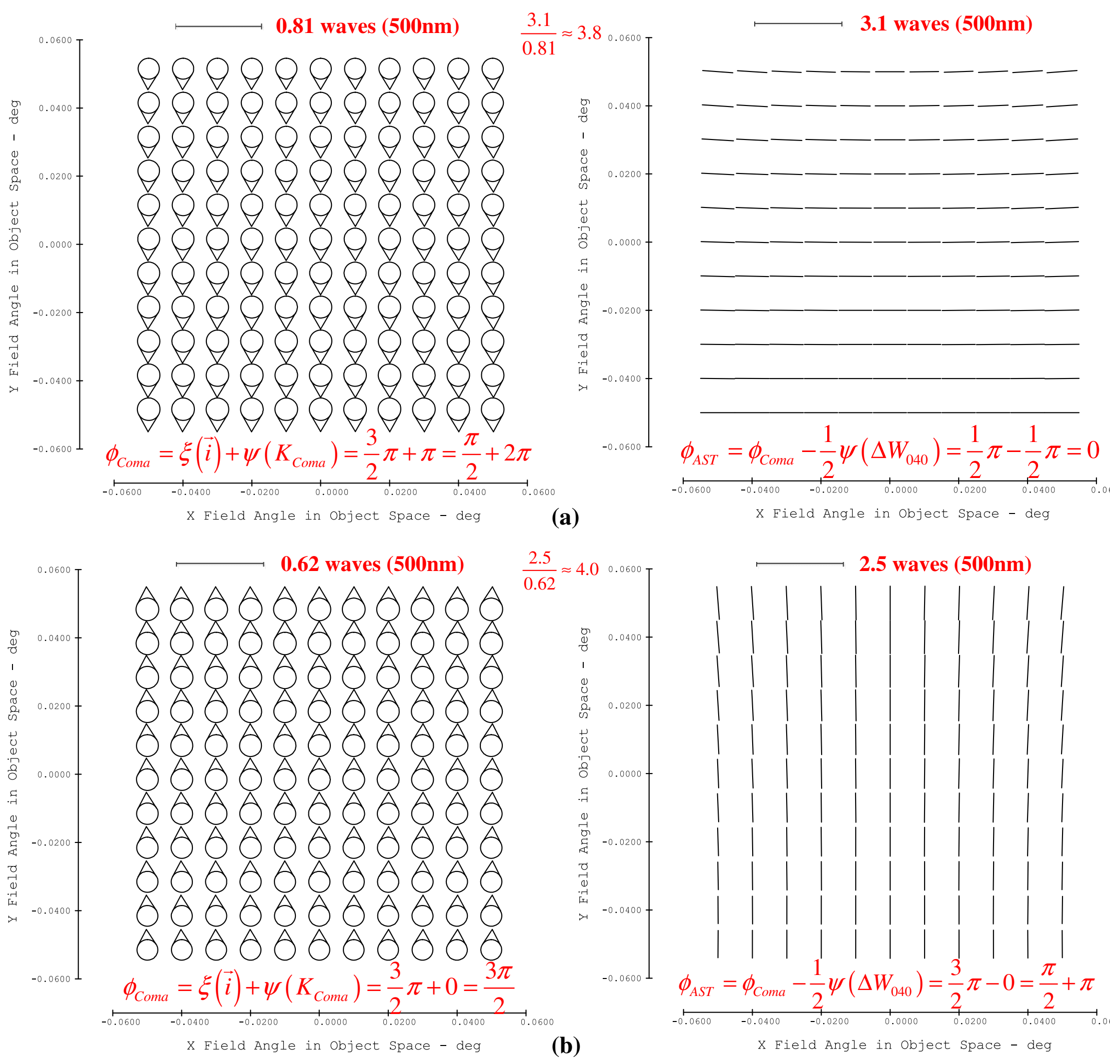

Fig. 5. Illustrations of the inherent relationships between the orientation and magnitude of coma and astigmatism in NST with axial misalignments. We can see that these relationships can roughly be represented by Eqs. (28) and (32), respectively. Here (a) and (b) are obtained in the presence of two different sets of axial misalignments, specified above.

Then we begin to reveal the relationship between the magnitudes of astigmatism and coma induced by axial misalignments. Referring to Eqs. (15), (16) and Eqs. (20), (21), we can obtain

$$\frac{K_{\mathrm{AST}}}{K_{\mathrm{Coma}}} \approx \frac{s}{2}. \tag{29}$$

On the other hand, in the optical alignment and testing process, aberrations are usually quantified based on the values of their fringe Zernike coefficients. In this case, the astigmatism and coma aberration can be expressed as

$$W_{\mathrm{AST}}^{(T)} = \begin{bmatrix} C_5 \\ C_6 \end{bmatrix} \cdot \begin{bmatrix} \rho^2 \cos(2\phi) \\ \rho^2 \sin(2\phi) \end{bmatrix} = \vec{C}_{5/6} \cdot \vec{\rho}^2, \tag{30}$$

$$W_{\mathrm{Coma}}^{(T)} = \begin{bmatrix} C_7 \\ C_8 \end{bmatrix} \cdot \begin{bmatrix} (3\rho^3 - 2\rho)\cos(\phi) \\ (3\rho^3 - 2\rho)\sin(\phi) \end{bmatrix} = 3\vec{C}_{7/8} \cdot \vec{\rho}(\vec{\rho} \cdot \vec{\rho}) - 2\vec{C}_{7/8} \cdot \vec{\rho}. \tag{31}$$

The superscripts (T) represent that here the astigmatism and coma are measured in the situation of optical testing. Referring to Eq. (29), we can infer that for the astigmatism and coma induced by axial misalignments,

$$\frac{|\vec{C}_{5/6}|}{3|\vec{C}_{7/8}|} = \frac{s}{2} \quad \text{or} \quad \frac{|\vec{C}_{5/6}|}{|\vec{C}_{7/8}|} = \frac{3s}{2}, \tag{32}$$

where $|\vec{C}_{5/6}|$ represents the magnitude of astigmatism and $|\vec{C}_{7/8}|$ represents the magnitude of coma. Equation (32) can represent the relationship between the astigmatism and coma induced by axial misalignments. We can see that the magnitude of the astigmatism induced by axial misalignments is about $\frac{3s}{2}$ times more than the magnitude of the coma. This is different from the relationship between astigmatism and coma induced by lateral misalignments (the former is $3s$ times more than the latter) [25]. This fact presents the possibility of separating the effects of lateral misalignments and axial misalignments.

To demonstrate the inherent relationships between the astigmatism and coma induced by axial misalignments for off-axis astronomical telescopes, we introduce two sets of axial misalignments to NST in the optical simulation software. The results are shown in Fig. 5. The axial misalignments of the secondary mirror are −0.3 mm for Fig. 5(a) ($\Delta W_{040} < 0$) and 0.2 mm for Fig. 5(b) ($\Delta W_{040} > 0$). Note that we suppose the direction of pupil decenter for NST is in $-y$ direction, i.e., $\xi(\vec{i}) = \frac{3}{2}\pi$ (referring to Appendix C of Ref. [25]). For NST, $\frac{3s}{2} = 3.45$. The results shown in this figure can validate the correctness of the derived relationships between astigmatism and coma induced by axial misalignments. While Eq. (32) is not accurate enough, we still consider that it can roughly represent the inherent relationship between the magnitude of astigmatism and coma in off-axis two-mirror telescopes induced by axial misalignments with a very concise and intuitive expression.

5. CONCLUSION

In unobscured off-axis astronomical telescopes with an offset pupil, the effects of axial misalignments are very different from those in on-axis ones. Specifically, a series of nonrotationally symmetric aberrations with characteristic field dependence will be induced by axial misalignments. This paper takes off-axis two-mirror astronomical telescopes as an example to discuss the field characteristics of several important nonrotationally symmetric aberrations (including astigmatism, coma, and trefoil aberration) induced by axial misalignments in off-axis astronomical telescopes. The aberration function is derived by applying a system-level pupil coordination transformation to the aberration function of an axially misaligned on-axis system under some proper approximations. The expressions of several important nonrotationally symmetric aberrations are obtained using the vector product identities. The accuracy of the proposed expressions is demonstrated with real ray-trace data. On this basis, the specific field characteristics of these aberrations are presented and explicated. The results show that the effects of axial misalignments bear strong similarities to the effects of the lateral misalignments in the symmetry plane of the off-axis system. On the other hand, the inherent relationships between astigmatism and coma induced by axial misalignments are further revealed, which are different from those induced by lateral misalignments. This fact presents the possibility of separating the effects of axial misalignments and lateral misalignments. Most of this work can be extended to the other off-axis astronomical telescopes with more freedom, such as off-axis TMA telescopes. For this class of off-axis telescopes with a relatively large field of view, the change of aberration coefficients [as shown in Eq. (7)] induced by axial misalignments may no longer be neglected. In this case, some field-dependent nonrotationally symmetric aberrations can also be induced by axial misalignments. However, in general, the field-constant aberration contributions are still the dominant components.

This work can not only lead to a more complete understanding of the effects of misalignments in off-axis astronomical telescopes, but also contribute to the development of reasonable and deterministic strategies for the alignment of off-axis astronomical telescopes.

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