# Evaluation of different calibration equations for NTC thermistor applied to high-precision temperature measurement 

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#### Abstract

Using a negative temperature coefficient (NTC) thermistor for high-precision temperature measurement can give a resolution and accuracy as low as 5 mK . The performance of an NTC thermistor is affected markedly by its calibration equation. Two series of high precision calibration for NTC thermistors in a precision water bath by means of comparison method were presented. Nine approximate calibration equations for the re-sistance-temperature characteristics of the MF501 NTC thermistor are evaluated within a temperature range of $278.15-328.15 \mathrm{~K}$. It is confirmed that the fitting quality is influenced greatly by the number of coefficients used in the calibration equation, and that the Hoge-2 equation is the best calibration equation for the MF501 NTC thermistor for high-precision temperature measurements. The combined standard uncertainty of the thermistor calibration system is estimated as 4.31 mK . The calibration procedure and evaluation method proposed can be used for calibration of any types of NTC thermistors.


## 1. Introduction

Negative temperature coefficient (NTC) thermistors and platinum resistance thermometers are both widely used in spacecraft as temperature sensors. Compared with the latter, NTC thermistors are smaller, faster, more reliable, and less sensitive to mechanical shock or vibration. As the performance of space optical remote sensors improves, so their temperature-control requirements become stricter [1-3]. For example, in some large space telescopes, the temperature precision for the primary mirror during the working period needs to be better than 50 mK over a relatively narrow temperature range over $290.15-295.15 \mathrm{~K}$. As a consequence, a high-precision temperature sensor is required to monitor tiny changes in the temperature of the primary mirror. Because the measurement uncertainty of the temperature sensor is only one aspect of the target uncertainty (i.e., 50 mK ), the former must be lower ( $\sim 5 \mathrm{mK}$ ) to allow for other uncertainties in the measurement chain. The high-precision NTC thermistors used in space optical remote sensors are MF501 NTC thermistors, the measurement accuracy of which is $\pm 0.3 \mathrm{~K}$, namely 60 times larger than that required. In addition, because of the highly nonlinear characteristics of NTC thermistors, their performance for temperature measurement is heavily dependent on the choice of calibration equations. Therefore, the challenge of high-precision calibration must be met to achieve the requirements of high-precision temperature
measurement.
Chung and Oh [4] proposed a residual compensation method for the calibration equation of NTC thermistors in a calibration temperature range of $288.15-308.15 \mathrm{~K}$. Standard fitting errors ( $1 \sigma$ ) in temperature were compared to both the Basic equation and the Steinhart-Hart equation, and the residual compensation method was found to give a more exact calibration results. However, that method requires five parameters to be determined, making its data processing very complex. In a different study, two series of high-precision calibration experiments were carried out for a so-called super-stable NTC thermistor in the temperature range of $273.15-333.15 \mathrm{~K}$ at intervals of 5 K by means of the comparison method [5]. The variables affecting the calibration uncertainty of the thermistor were discussed in detail. The results in that study showed that the interpolation error was largely influenced by the number of parameters in the interpolation equations, and that the Steinhart-Hart equation performed poorly. A fourth-order polynomial in terms of the resistance-ratio model was recommended as a more suitable model. Chen [6] selected seven calibration equations to assess the fitting agreement of the resistance-temperature relationship of four types of NTC thermistor under a relatively large calibration temperature range of $273.15-343.15 \mathrm{~K}$. Several statistics were used to compare the performance of those equations, and the results indicated that the Basic equation and the Steinhart-Hart equation were inadequate calibration equations for all NTC thermistors; the Hoge-3 equation was the

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Fig. 1. Thermistor calibration system.
best among the seven calibration equations. Ilić et al. [7] proposed two approximation curves (a three-parameter AC1 equation and a twoparameter AC2 equation) for the resistance-temperature relationship of thermistors. Those two curves were compared with the Steinhart-Hart equation by using tabular resistance data for different NTC resistors; differences from the tabular data were mostly better than $\pm 10 \mathrm{mK}$ for the temperature range of $273.15-343.15 \mathrm{~K}$. However, the form of the two equations was quite complicated. Alexander and MacQuarrie [8] proposed a general method for calibrating the resistance-temperature curve of NTC thermistors to obtain consistent, high-quality data. Six temperature steps were imposed during the calibration procedure, of which data from three steps were used to calculate the coefficients of the Steinhart-Hart equation; the others were used to verify the fitting results. An accuracy of $\pm 0.156 \mathrm{~K}$ over $268.15-308.15 \mathrm{~K}$ was achieved after calibration through the Steinhart-Hart equation. Nevertheless, that is still far from the accuracy required for fine temperature measurement or control. Bennett [9] discussed the resistance-temperature relationship of several types of thermistor at 10 temperature points in the range of $273.15-303.15 \mathrm{~K}$. Those results showed that a thermistor resistance-temperature accuracy of better than 1 mK could be obtained with a formula with four parameters, and that a squared term did little to improve the fitting agreement compared to the Steinhart-Hart equation.

The aforementioned previous research was focused mainly on the resistance-temperature characteristics of NTC thermistors whose nominal resistance was roughly $10 \mathrm{k} \Omega$ at $25^{\circ} \mathrm{C}$. In contrast, the MF501 NTC thermistor has a nominal resistance of roughly $5 \mathrm{k} \Omega$, a tolerance of less than $\pm 1 \%$, and tends to be used in a vacuum. Hence, a suitable calibration equation must be determined for the MF501 NTC thermistor. In the present paper, we investigate nine approximate calibration equations for the resistance-temperature characteristics of the MF501 NTC thermistor within a temperature range of $278.15-328.15 \mathrm{~K}$. In Section 2, we present the experimental setup of the calibration system, the detailed calibration procedure, and we outline the theoretical basis for evaluating calibration equations and calculating calibration uncertainties. In Section 3, we present the performance of each of the nine calibration equations, and we estimate the uncertainties of the thermistor calibration system. Finally, we draw our conclusions in Section Section 4.

## 2. Materials and methods

### 2.1. Thermistor calibration system

In this study, we prepared seven MF501 NTC thermistors, each with a nominal resistance at $25{ }^{\circ} \mathrm{C}\left(R_{25}\right)$ of around $5 \mathrm{k} \Omega$ and a nominal beta value of 4100 K . The dissipation constant of each thermistor was about $2.0-3.0 \mathrm{~mW} / \mathrm{K}$, and the measurement accuracy was $\pm 0.3 \mathrm{~K}$ as provided by the manufacturer. To minimize the impact of ambient temperature fluctuations on the thermistor calibration system, we built a thermostatic room in which the temperature was kept at $293.15 \pm 5 \mathrm{~K}$ and high-precision thermistor calibration was executed. Furthermore, we used a high-precision temperature-controlled water bath (Hart Scientific, Model 7012) to maintain a uniform calibration temperature; its specifications were as follows: temperature range $=263.15-383.15 \mathrm{~K}$; stability $= \pm 0.8 \mathrm{mK}$ at 298.15 K (water); uniformity $= \pm 2 \mathrm{mK}$ at 298.15 K (water).

Because of the high precision required of the resistance of each MF501 NTC thermistor, we measured the resistances of our thermistors using a Fluke Super-QAD Precision Temperature Scanner Model 1586A (Fluke 1586A) configured with an external DAQ-STAQ multiplexer. The Fluke 1586A was calibrated with standard resistors at the Jilin (a province of China) Institute of Metrology, and an uncertainty of $5 \times 10^{-5}(k=2)$ was achieved over a $10-\mathrm{k} \Omega$ range. The QAD software developed by the Fluke company was convenient for automatic data acquisition with a personal computer connected to the Fluke 1586A via a cable (Fig. 1). The resistance and temperature of the thermistor were read out continuously with a 10 -s sampling period.

A calibrated standard platinum resistance thermometer (Hart Standard Platinum Resistance Thermometer Model 5628, SPRT 5628) was used for temperature sensing during the calibration. The properties of the SPRT 5628 were as follows: short-term stability $= \pm 2 \mathrm{mK}$ ( $k=2$ ); calibration uncertainty $= \pm 4 \mathrm{mK}(k=2)$ at 273.15 K . The four-wire technique was used to eliminate wire resistances when connecting the SPRT 5628 to the DAQ-STAQ multiplexer. The resistance of the SPRT 5628 was converted automatically into temperature with the aforementioned QAD software. The measurement current was 1 mA for the SPRT 5628 and $10 \mu \mathrm{~A}$ for the thermistors.

To maintain a uniform measurement temperature and avoid unwanted side effects due to placing the thermistor directly into an electrically conductive liquid (i.e., water), we inserted each thermistor into its own hermetically sealed glass tube that was 400 mm in length
and 7 mm in diameter. The thermistor was located at the bottom of the glass tube to minimize the thermal resistance between the two. Fig. 1 shows the entire thermistor calibration system.

We determined the resistance-temperature characteristics of the MF501 NTC thermistor based on calibration by means of the comparison method, which is widely used for temperature sensors [4,5,10]. The calibration temperature range was $278.15-328.15 \mathrm{~K}$, and 11 measurement points were preselected at $5.0-\mathrm{K}$ intervals. The calibration procedure was as follows.
(a) Both the glass tubes with the MF501 NTC thermistors hermetically sealed and the SPRT 5628 were carefully attached to the hold plate with several grippers. Each pair of glass tubes was inserted to the same depth (roughly 300 mm below the top lips of water) in the bath liquid.
(b) Set the desired temperature for the water bath and run the QAD software to monitor the real-time resistance and temperature of the thermistor. As advised by the manufacturer, to minimize the uncertainty, it is advisable to start at the highest temperature and progress down to the lowest temperature [11].
(c) After temperature stabilization of the water bath for more than half an hour, both the temperature of the SPRT 5628 and the resistance of the MF501 NTC thermistor were logged.
(d) Repeat steps (b) and (c) for the next measurement point until all measurement points have been calibrated.

### 2.2. Thermistor calibration equations

The temperature measurement accuracy of the NTC thermistors is determined directly by the calibration equation. In this section, nine approximate thermistor calibration equations are presented to describe the resistance-temperature characteristic of the NTC thermistors.

## (a) Basic equation

The Basic equation, a two-parameter exponential equation expressed as Eq. (1), is the most popular equation and is widely used to describe the relationship between thermistor resistance and temperature [12,13]:
$R_{T}=R_{T_{0}} e^{\beta\left(\frac{1}{T}-\frac{1}{T_{0}}\right)}$,
where $R_{T}$ is the thermistor resistance at temperature $T$ in Kelvin and $R_{T_{0}}$ is the resistance of the thermistor at reference temperature $T_{0}$, usually $298.15 \mathrm{~K}\left(25^{\circ} \mathrm{C}\right)$. The parameter $\beta$ is a characteristic value of the thermistor material, with typical values in the range $2000-6000 \mathrm{~K}^{-1}$.

Because the raw data obtained from the thermistors were always the resistance, Eq. (1) can be expressed as follows:
$\frac{1}{T}=A+B \ln R_{T}$,
where $A$ and $B$ are fitting coefficients.
Several other calibration equations were selected as discussed by Hoge [14] and Chen [6].
(b) Hoge-1 equation
$\frac{1}{T}=A_{0}+A_{1} \ln R_{T}+A_{2}\left(\ln R_{T}\right)^{2}$
(c) Hoge-2 equation
$\frac{1}{T}=A_{0}+A_{1} \ln R_{T}+A_{2}\left(\ln R_{T}\right)^{2}+A_{3}\left(\ln R_{T}\right)^{3}$
(d) Hoge-3 equation
$\frac{1}{T}=A_{0}+A_{1} \ln R_{T}+A_{2}\left(\ln R_{T}\right)^{2}+A_{3}\left(\ln R_{T}\right)^{3}+A_{4}\left(\ln R_{T}\right)^{4}$
(e) Hoge-4 equation
$\frac{1}{T}=A_{0}+A_{1} \ln R_{T}+A_{2}\left(\ln R_{T}\right)^{2}+A_{5} / \ln R_{T}$,
where $A_{0}, A_{1}, A_{2}, A_{3}, A_{4}$, and $A_{5}$ are constants.
(f) Hoge-5 equation
$\frac{1}{T}=\frac{C_{1}+C_{2} \ln R_{T}}{1+C_{3} \ln R_{T}}$,
where $C_{1}, C_{2}$, and $C_{3}$ are constants.
(g) Steinhart-Hart equation [15]
$\frac{1}{T}=A_{0}+A_{1} \ln R_{T}+A_{3}\left(\ln R_{T}\right)^{3}$,
where $A_{0}, A_{1}$, and $A_{3}$ are the Steinhart-Hart coefficients for the specified thermistor.
(h) Second-order equation

The calibration equation recommended by the manufacturer is expressed by Eq. (9). In this paper, we refer to Eq. (9) as the second-order equation:
$\ln R_{T}=a+\frac{b}{T}+\frac{c}{T^{2}}$,
where $a, b$, and $c$ are the parameters to be determined for the MF501 NTC thermistor. The inverse relationship of $R$ to $T$ for Eq. (9) can be expressed as
$\frac{1}{T}=\frac{-b+\sqrt{b^{2}-4 c\left(a-\ln R_{T}\right)}}{2 c}$.
For high-precision temperature measurement, it is assumed that the $1 / T$ term is a polynomial in $\ln R_{T}$ or vice versa [16]:
$\frac{1}{T}=\sum_{i=0}^{m} a_{i}\left(\ln R_{T}\right)^{i}$.
In this paper, we refer to Eq. (11) with $m=5$ as the fifth-order equation.
(i) Fifth-order equation
$\frac{1}{T}=a_{0}+a_{1} \ln R_{T}+a_{2}\left(\ln R_{T}\right)^{2}+a_{3}\left(\ln R_{T}\right)^{3}+a_{4}\left(\ln R_{T}\right)^{4}+a_{5}\left(\ln R_{T}\right)^{5}$,
where $a_{0}, a_{1}, a_{2}, a_{3}, a_{4}$, and $a_{5}$ are constants.

### 2.3. Criteria for evaluation

To evaluate the fitting agreement of the resistance-temperature characteristic of the MF501 NTC thermistor with each of the nine approximate calibration equations, several criteria were used in this paper [6,17].

The fitting residual of temperature is
$\Delta T_{i}=T_{i}^{*}-T_{i}$,
where $\Delta T_{i}$ is the fitting residual of the calibration equation, $T_{i}$ is the temperature measured at each calibration point, and $T_{i}^{*}$ is the temperature calculated from the calibration equation. The specified temperature point is denoted with subscript $i$. The maximum and minimum values of $\Delta T_{i}$ are expressed as $\Delta T_{\max }$ and $\Delta T_{\min }$, respectively.

The absolute value of the average fitting residual is defined as
follows:
$|\Delta T|_{\text {avg }}=\frac{\sum\left|\Delta T_{i}\right|}{n}$,
where $\left|\Delta T_{i}\right|$ is the absolute value of $\Delta T_{i}$ and $n$ is the number of calibration points.

An effective evaluation of the underlying probability distribution of the results is the experimental standard deviation $\Delta T_{s t d}$, which is given by
$\Delta T_{s t d}=\sqrt{\frac{\sum_{i=1}^{n}\left(\Delta T_{i}-\overline{\Delta T_{i}}\right)^{2}}{n-1}}$,
where $\overline{\Delta T_{i}}$ is the average of the fitting residuals of the calibration equation and $n$ is the number of data. Typically, the smaller the values of $\Delta T_{\text {std }}$ and $|\Delta T|_{\text {avg }}$, the better the calibration equation represents the resistance-temperature characteristic of the thermistor.

### 2.4. Methods for evaluating uncertainties

Evaluation of the thermistor calibration uncertainties mainly follows the guidelines described in ISO GUM [18]. Linear interpolation was used to establish the uncertainties between calibration points with the help of manuals. To evaluate the uncertainty associated with the measurement error in the thermistor resistance, we rearrange Eq. (1) to give
$\beta=\frac{d \ln R}{d(1 / T)}=-\frac{T^{2}}{R} \frac{d R}{d T}$.
Thus, the uncertainty due to the resistance measurement at each calibration point can be calculated by Eq. (17), where the subscript $i$ denotes the specified temperature point:
$u_{T_{i}}=\frac{T_{i}^{2}}{\beta} \frac{u_{R_{i}}}{R_{i}}$.
The self-heating effects of the calibration thermistor cannot be ignored when high-precision temperature measurement is required. The self-heating effects can be estimated by
$\Delta T_{T_{i}}=\frac{I^{2} R\left(T_{i}\right)}{\delta}$,
where $I$ is the sensing current passing through the calibration thermistor, and $\delta$ is the dissipation constant of the thermistor with typical values in the range $0.5-20.0 \mathrm{~mW} / \mathrm{K}$ in the stirred oil [5,12].

The estimated standard deviation of the calibration result is given as the combined standard uncertainty. It is obtained by combining the individual standard uncertainties, whether they arise from Type-A or Type-B evaluation:
$u_{c}=\sqrt{\sum s_{i}^{2}+\sum u_{j}^{2}}$,
where $u_{c}$ is the combined standard uncertainty, $s_{i}$ is the Type-A standard uncertainty, and $u_{j}$ is the Type-B standard uncertainty.

## 3. Results and discussion

### 3.1. Calibration results

The resistance-temperature measurement data for the seven MF501 NTC thermistors are listed in Table 1. Note that $T$ is the average bath temperature and $R$ is the average thermistor resistance after stabilization of the water bath for over half an hour. Two series of calibrations were performed with the same experimental setup; in Table 1, the upper part is referenced as experiment 1 (March 2017) and the lower part is referenced as experiment 2 (April 2017).

Based on the measurement data in Table 1, the calibration results
and characteristic of the MF501 NTC thermistors over a temperature range of $278.15-328.15 \mathrm{~K}$ with $5.0-\mathrm{K}$ temperature intervals are presented in Fig. 2.

Using the measurement data presented in Table 1, the coefficients of the nine selected calibration equations for the seven MF501 NTC thermistors were obtained from least-squares fits. Fig. 3 shows a typical least-squares fit to the resistance-temperature measurement data (experiment 1) for thermistor No. 3 according to Eq. (9). The coefficients of the other calibration equations for thermistor No. 3 are listed in Table 2.

### 3.2. Evaluation of calibration equations

The fitting residual plots of the nine selected calibration equations for MF501 NTC thermistor No. 3 (experiment 1) are shown in Fig. 4. In the plots, the Steinhart-Hart, second-order, and fifth-order equations are expressed as "S\&H," "2nd," and "5th," respectively.

The residual results presented in Fig. 4 indicate that the Basic equation is the worst representation of the resistance-temperature characteristic of the MF501 NTC thermistor. However, the residual plot of the Basic equation shows a strong symmetrical residual pattern for the distribution of fitting errors. The maximum temperature difference from the fitting residuals of the Basic equation is no more than 60 mK . In Fig. 4, the residual plots of the Hoge-1, Hoge-5, Steinhart-Hart, and second-order equations show little difference; the Steinhart-Hart equation performs only marginally better compared to the other three equations. A similar phenomenon is also found for the residual plots of the Hoge-2, Hoge-3, and fifth-order equations, which indicates that an approximate accuracy can be achieved by means of these three calibration equations. Besides, based on Fig. 4, the maximum temperature difference from the fitting residual of the Hoge-4 equation is less than 0.9 mK .

According to the criteria defined in Section 2, Table 3 provides some numerical parameters for the nine approximate calibration equations for the seven MF501 NTC thermistors. Note that each criterion listed in the Table 3 is based on the average of the seven MF501 NTC thermistors. From Table 3, the basic equation has the largest values of the evaluated criteria among the nine selected calibration equations. The values of $\Delta T_{\max }, \Delta T_{\min },|\Delta T|_{\text {avg }}$, and $\Delta T_{s t d}$ for the Basic equation are $54.89 \mathrm{mK},-35.77 \mathrm{mK}, 27.25 \mathrm{mK}$, and 32.57 mK , respectively. As a result, although the Basic equation has a very simple form, it is inadequate as a calibration equation for high-precision temperature measurement with MF501 NTC thermistors.

The $\Delta T_{\text {std }}$ values listed in Table 3 for the Hoge-1, Hoge-5, Stein-hart-Hart, and second-order equations are $6.69 \mathrm{mK}, 6.63 \mathrm{mK}, 5.93 \mathrm{mK}$, and 6.56 mK , respectively. The $|\Delta T|_{\text {avg }}$ values for those four calibration equations are $5.61 \mathrm{mK}, 5.56 \mathrm{mK}, 4.96 \mathrm{mK}$, and 5.51 mK , respectively. This indicates that the second-order equation as recommended by the manufacturer gives a less accurate representation of the characteristic of MF501 NTC thermistors compared with the Steinhart-Hart equation. Furthermore, the fitting agreement of the resistance-temperature characteristics of the MF501 NTC thermistors among the Hoge-1, Hoge5 , and second-order equations shows little difference. A similar result pertains for the Hoge-2, Hoge-3, and fifth-order equations. The $\Delta T_{\text {std }}$ value for the Steinhart-Hart equation is 25 times larger than that of the Hoge-2 equation, which is a complete third-order equation. Also, the Steinhart-Hart equation performed awkwardly when compared with the Hoge-4 equation. As a result, we discourage the use of the Stein-hart-Hart equation for high-precision temperature measurement. The Hoge-1, Hoge-5, Steinhart-Hart, and second-order equations are adequate calibration equations for the MF501 NTC thermistor for common temperature measurement.

Usually, the fitting error due to interpolation decreases with polynomial order. The $\Delta T_{s t d}$ values of the Basic, Hoge-1, Hoge-2, Hoge-3, and fifth-order equations are $32.57 \mathrm{mK}, 6.69 \mathrm{mK}, 0.23 \mathrm{mK}, 0.22 \mathrm{mK}$, and 0.21 mK , respectively. Given that the fitting quality does not

Table 1
Measurement data of resistance and temperature for the MF501 NTC thermistors.

| T/K | $R / \Omega$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. 1 | No. 2 | No. 3 | No. 4 | No. 5 | No. 6 | No. 7 |
| 278.2574 | 13164.11 | 13190.12 | 13080.40 | 13129.21 | 13110.11 | 13189.72 | 13204.24 |
| 283.3417 | 10162.63 | 10182.11 | 10095.95 | 10136.69 | 10121.23 | 10184.30 | 10192.92 |
| 288.2827 | 7966.33 | 7981.11 | 7912.63 | 7946.84 | 7934.14 | 7984.82 | 7989.61 |
| 293.1597 | 6311.24 | 6322.62 | 6267.79 | 6296.60 | 6286.05 | 6327.18 | 6329.45 |
| 298.0455 | 5034.14 | 5042.90 | 4998.79 | 5023.13 | 5014.35 | 5047.91 | 5048.47 |
| 302.9663 | 4037.07 | 4043.75 | 4008.14 | 4028.75 | 4021.45 | 4048.95 | 4048.37 |
| 307.9471 | 3251.18 | 3256.26 | 3227.44 | 3244.93 | 3238.84 | 3261.47 | 3260.13 |
| 312.9821 | 2629.81 | 2633.66 | 2610.29 | 2625.16 | 2620.08 | 2638.75 | 2636.96 |
| 318.0535 | 2138.22 | 2141.11 | 2122.13 | 2134.78 | 2130.53 | 2145.99 | 2143.94 |
| 323.1317 | 1749.18 | 1751.34 | 1735.87 | 1746.68 | 1743.13 | 1755.99 | 1753.83 |
| 328.1941 | 1440.67 | 1442.25 | 1429.59 | 1438.87 | 1435.91 | 1446.65 | 1444.47 |
| 278.2566 | 13164.48 | 13190.44 | 13080.69 | 13129.55 | 13110.47 | 13190.09 | 13204.71 |
| 283.3415 | 10162.71 | 10182.23 | 10096.08 | 10136.80 | 10121.34 | 10184.40 | 10193.12 |
| 288.2823 | 7966.34 | 7981.16 | 7912.70 | 7946.88 | 7934.19 | 7984.86 | 7989.71 |
| 293.1601 | 6311.13 | 6322.55 | 6267.71 | 6296.52 | 6285.97 | 6327.09 | 6329.40 |
| 298.0437 | 5034.53 | 5043.30 | 4999.17 | 5023.52 | 5014.74 | 5048.28 | 5048.85 |
| 302.9653 | 4037.32 | 4044.00 | 4008.38 | 4029.00 | 4021.69 | 4049.17 | 4048.60 |
| 307.9465 | 3251.37 | 3256.44 | 3227.60 | 3245.10 | 3239.01 | 3261.60 | 3260.30 |
| 312.9824 | 2629.86 | 2633.71 | 2610.37 | 2625.21 | 2620.13 | 2638.75 | 2637.00 |
| 318.0546 | 2138.11 | 2141.01 | 2122.02 | 2134.68 | 2130.45 | 2145.86 | 2143.86 |
| 323.1331 | 1749.13 | 1751.29 | 1735.82 | 1746.63 | 1743.08 | 1755.93 | 1753.78 |
| 328.1950 | 1440.61 | 1442.21 | 1429.57 | 1438.83 | 1435.87 | 1446.58 | 1444.43 |



Fig. 2. $R-T$ measurement data of MF501 NTC thermistor No. 3 over 278.15-328.15 K.


Fig. 3. Least-squares fit of Eq. (9) to thermistor measurement data.
improve greatly as the calibration equation changes from third to fourth (or fifth) order, and the fact that the Hoge-2 equation has a simpler form, we deem the Hoge-2 equation (third-order) to be the best calibration equation for the MF501 NTC thermistor for high-precision temperature measurement.

### 3.3. Evaluation of calibration uncertainties

Several calibration uncertainties must be evaluated to determine the total uncertainty of the MF501 NTC thermistor calibration system. Given the calibration procedure outlined in Section 2, the total measurement uncertainty can be divided into two main parts: the uncertainties in temperature measurement, and the uncertainties in resistance measurement.

The readout accuracies for the SPRT temperature and the thermistor resistance were calculated based on the Fluke 1586A manuals and the calibration results of the Jilin Institute of Metrology, respectively. Eq. (17) was used to convert resistance uncertainty to temperature uncertainty. Measurement noise is an uncertainty due to noise or instability in the measurement readings. The uncertainty due to noise in the readings was calculated by dividing the standard deviation of the number of readings (of which there was roughly 200) by the square root of the same. To account for self-heating effects, a dissipation constant of $2.0 \mathrm{~mW} / \mathrm{K}$ was used in the calculations. All other uncertainties were determined according to the specifications provided by the manufacturer.

Table 4 lists the uncertainty budgets of the MF501 NTC thermistor calibration system in the temperature range of $278.15-328.15 \mathrm{~K}$. The maximum combined standard uncertainties of the calibration system with respect to the Steinhart-Hart, second-order, and Hoge-2 equations are $7.33 \mathrm{mK}, 7.85 \mathrm{mK}$, and 4.31 mK , respectively. Considering that most uncertainties were chosen to represent the worst case, and that the measurement noise of the reference SPRT and calibration thermistor for each measurement varied only slightly, we state an uncertainty of no more than 5 mK for the Hoge-2 equation.

## 4. Conclusions

In this study, nine approximate calibration equations were selected to evaluate the fitting agreement of the resistance-temperature characteristics of seven MF501 NTC thermistors within a temperature range of $278.15-328.15 \mathrm{~K}$. The MF501 NTC thermistor had a nominal resistance of roughly $5 \mathrm{k} \Omega$ at a temperature of 298.15 K , and a measurement accuracy of $\pm 0.3 \mathrm{~K}$. The parameters of the nine approximate calibration equations were evaluated by means of least-squares fits.

The results of this study indicate that the Basic equation performs

Table 2
Coefficients of the selected calibration equations for MF501 NTC thermistor No.3. Note that the coefficients listed below are based on the measurement data of experiment 1 .

| Calibration equation | Coefficient | Value | Calibration equation | Coefficient | Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Basic eqn. | A | $1.2527737 \mathrm{E}-03$ | Hoge-5 eqn. | $C_{1}$ | $1.3057717 \mathrm{E}-03$ |
|  | B | $2.4689828 \mathrm{E}-04$ |  | $C_{2}$ | $2.3025616 \mathrm{E}-04$ |
| Hoge-1 eqn. | $A_{0}$ | $1.3071339 \mathrm{E}-03$ | Steinhart-Hart eqn. | $C_{3}$ | $-3.0927204 \mathrm{E}-03$ |
|  | $A_{1}$ | $2.3380151 \mathrm{E}-04$ |  | $A_{0}$ | $1.2892287 \mathrm{E}-03$ |
|  | $A_{2}$ | $7.8332888 \mathrm{E}-07$ |  | $A_{1}$ | $2.4030186 \mathrm{E}-04$ |
| Hoge-2 eqn. | $A_{0}$ | $1.1514978 \mathrm{E}-03$ | Second-order eqn. | $A_{3}$ | $3.1333020 \mathrm{E}-08$ |
|  | $A_{1}$ | $2.9006090 \mathrm{E}-04$ |  | $a$ | $-5.6450553 \mathrm{E}+00$ |
|  | $A_{2}$ | -5.9671318E-06 |  | $b$ | $4.3954696 \mathrm{E}+03$ |
|  | $A_{3}$ | $2.6886975 \mathrm{E}-07$ |  | c | $-5.2036790 \mathrm{E}+04$ |
| Hoge-3 eqn. | $A_{0}$ | $1.1554887 \mathrm{E}-03$ | Fifth-order eqn. | $A_{0}$ | $1.1708917 \mathrm{E}-03$ |
|  | $A_{1}$ | $2.8813670 \mathrm{E}-04$ |  | $A_{1}$ | $2.7884968 \mathrm{E}-04$ |
|  | $A_{2}$ | $-5.6202529 \mathrm{E}-06$ |  | $A_{2}$ | $-3.3854807 \mathrm{E}-06$ |
|  | $A_{3}$ | $2.4115921 \mathrm{E}-07$ |  | $A_{3}$ | $-2.7120942 \mathrm{E}-08$ |
|  | $A_{4}$ | $8.2770221 \mathrm{E}-10$ |  | $A_{4}$ | $1.6895089 \mathrm{E}-08$ |
| Hoge-4 eqn. | $A_{0}$ | $1.7721058 \mathrm{E}-03$ |  | $A_{5}$ | -3.8405941E-10 |
|  | $A_{1}$ | $1.7791526 \mathrm{E}-04$ |  |  |  |
|  | $A_{2}$ | $3.0130351 \mathrm{E}-06$ |  |  |  |
|  | $A_{5}$ | $-1.2841107 \mathrm{E}-03$ |  |  |  |



Fig. 4. Residuals plots of the nine calibration equations for MF501 NTC thermistor No. 3.

Table 3
Evaluated parameters of the nine approximate calibration equations for the seven MF501 NTC thermistors (March 2017).

| Calibration equation | Criteria/(mK) |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $\Delta T_{\max }$ | $\Delta T_{\min }$ | $\|\Delta T\|_{\text {avg }}$ | $\Delta T_{\text {std }}$ |
| Basic | 54.89 | -35.77 | 27.25 | 32.57 |
| Hoge-1 | 8.38 | -11.15 | 5.61 | 6.69 |
| Hoge-2 | 0.41 | -0.27 | 0.18 | 0.23 |
| Hoge-3 | 0.43 | -0.27 | 0.16 | 0.22 |
| Hoge-4 | 0.69 | -0.78 | 0.48 | 0.56 |
| Hoge-5 | 8.32 | -11.09 | 5.56 | 6.63 |
| Steinhart-Hart | 7.53 | -10.08 | 4.96 | 5.93 |
| Second-order | 8.24 | -10.93 | 5.51 | 6.56 |
| Fifth-order | 0.47 | -0.24 | 0.16 | 0.21 |

poorly in representing the resistance-temperature characteristic of the MF501 NTC thermistor. The Steinhart-Hart equation performed better than the normal two-term equation (Hoge-1) only sometimes. Hence, the use of the Steinhart-Hart equation for high-precision temperature measurement is discouraged. The Hoge-2 equation was the best calibration equation of the nine equations for the MF501 NTC thermistor for high-precision temperature measurement. The average experimental standard deviation of the Hoge-2 equation was estimated as 0.23 mK . The calibration procedure and evaluation method proposed can be used for calibration of any types of NTC thermistors.

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Table 4
Uncertainty budgets for the thermistor calibration system ( $k=1$ ).

| Code | Uncertainty contribution | Type | 278.15 K (mK) | 303.15 K (mK) | 328.15 K (mK) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $u_{1}$ | Reference SPRT calibration | B | 2.03 | 2.18 | 2.33 |
| $u_{2}$ | Reference SPRT short-term stability | B | 1.00 | 1.00 | 1.00 |
| $u_{3}$ | Bath temperature non-uniformity | B | 1.00 | 1.00 | 1.00 |
| $u_{4}$ | Bath temperature stability | B | 0.40 | 0.40 | 0.40 |
| $u_{5}$ | Bath temperature drift | B | 0.10 | 0.10 | 0.10 |
| $u_{6}$ | Reference SPRT readout | B | 2.56 | 2.85 | 3.14 |
| $u_{7}$ | Calibration thermistor readout | B | 0.47 | 0.56 | 0.66 |
| $u_{8}$ | Self-heating of thermistor | B | 0.65 | 0.20 | 0.07 |
| $s_{1}$ | Interpolation error | A |  |  |  |
|  | Steinhart-Hart equation |  | 5.93 | 5.93 | 5.93 |
|  | Second-order equation |  | 6.56 | 6.56 | 6.56 |
|  | Hoge-2 equation |  | 0.23 | 0.23 | 0.23 |
| $s_{2}$ | Measurement noise of reference SPRT | A | 0.67 | 0.74 | 0.76 |
| $s_{3}$ | Measurement noise of calibration thermistor | A | 0.13 | 0.17 | 0.24 |
| $u_{c}$ | Total standard uncertainty ( $k=1$ ) | - |  |  |  |
|  | Steinhart-Hart equation |  | 7.01 | 7.15 | 7.33 |
|  | Second-order equation |  | 7.55 | 7.68 | 7.85 |
|  | Hoge-2 equation |  | 3.74 | 4.00 | 4.31 |

2015173).

## References

[1] T.S. Topham, G.E. Bingham, H. Latvakoski, et al., Observational study: microgravity testing of a phase-change reference on the International Space Station, NPJ Microgravity 1 (2015) 15009.
[2] H. Latvikoski, G.E. Bingham, T.S. Topham, et al., Phase change cells and the verification of gallium as a thermal calibration reference in space, SPIE Opt. Eng. Appl. (2015) 96080Q.
[3] P.J. Gero, J.K. Taylor, F.A. Best, et al., On-orbit absolute blackbody emissivity determination using the heated halo method, Metrologia 49 (2012) S1.
[4] J.P. Chung, S.W. Oh, A residual compensation method for the calibration equation of negative temperature coefficient thermistors, Thermochim. Acta 616 (2015) 27-32.
[5] S. Rudtsch, C.v. Rohden, Calibration and self-validation of thermistors for highprecision temperature measurements, Measurement 76 (2015) 1-6.
[6] C.C. Chen, Evaluation of resistance-temperature calibration equations for NTC thermistors, Measurement 42 (2009) 1103-1111.
[7] D. Ilić, J. Butorac, L. Ferković, Temperature measurements by means of NTC resistors and a two-parameter approximation curve, Measurement 41 (2008) 294-299.
[8] M.D. Alexander, K.T.B. MacQuarrie, Toward a standard thermistor calibration
method and data correction spreadsheets, Groundwater Monit. Remediation 25 (4) (2005) 75-81.
[9] A.S. Bennett, The calibration of thermistors over the temperature range $0-30{ }^{\circ} \mathrm{C}$, Deep Sea Res. 19 (1971) 157-163.
[10] B.S. Elkins, M. Huang, J.I. Frankel, Higher-time derivative of in-depth temperature sensors for aerospace heat transfer, Int. J. Therm. Sci. 52 (2012) 31-39.
[11] 7012 Calibration Bath User Manual (English). < http://download.flukecal.com/ pub/literature/7012_ugeng0000.pdf > .
[12] D.R. White, K. Hill, D. del Campo, C. Garcia Izquierdo, Guide on Secondary Thermometry - Thermistor Thermometry, BIPM, Consultative Committee for Thermometry (CCT), 2014.
[13] D.R. White, In pursuit of a fit-for-purpose uncertainty guide, Metrologia 53 (2016) S107-124.
[14] H.J. Hoge, Useful procedure in least squares, and tests of some equations for thermistors, Rev. Sci. Instrum. 59 (1988) 975-979.
[15] J.S. Steinhart, S.R. Hart, Calibration curves for thermistors, Deep Sea Res. 15 (3) (1968) 497-503.
[16] B.R. Kimball, B. DeCristofano, G. Caldarella, Thermistor Calibration Procedure for Simulated Skin Sensors, Army Natick Soldier Research Development And Engineering Center, 1993.
[17] Y. Cong, Z. Wang-chao, S. Bin, et al., Study on NTC thermistor characteristic curve fitting methods, Int. Conf. Comput. Sci. Network Technol. (2012) 2209-2213.
[18] G.U.M. Iso, Guide to the expression of uncertainty, Measurement (1993).


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