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Research on System Coherence Evolution of Different Environmental Models

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Abstract In this paper, we have studied the evolution curve of two-level atomic system that the initial state is excited state. At the different of environmental reservoir models, which include the single Lorentzian, ideal photon band-gap, double Lorentzian and square Lorentzian reservoir, we researched the influence of these environmental reservoir models on the evolution of energy level population. At static no modulation, comparing the four environmental models, the atomic energy level population oscillation of square Lorentzian reservoir model is fastest, and the atomic system decoherence is slowest. Under dynamic modulation, comparing the photon band-gap model with the single Lorentzian reservoir model, no matter what form of dynamic modulation, the time of atoms decay to the ground state is longer for the photonic band-gap model. These conclusions make the idea of using the environmental change to modulate the coherent evolution of atomic system become true.

Keywords Dynamic reservoir · Quantum control · Excited atom · Spontaneous emission

1 Introduction

In quantum world, the most fascinating feature is the quantum coherence of microscopic world, it makes us see a great application prospect. However, the reality quantum system is not isolated, it is inevitable to interact with the environment, by the impact of environment appears irreversible quantum decoherence phenomenon. So, how to effectively suppress

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the decoherence of quantum system become an important problem in quantum information science. The research [1–8] on spontaneous emission has attracted lots of interesting for a long time. Now the knowledge about spontaneous emission works up deeply, photoelectron and quantum information rapidly develop, and the big progress in the technology is made to prepare the environment controlling the photons, such as all kind of high-Quality cavities [9–11], photonic crystals and so on. All those can help us to control and change spontaneous emission, and provide us theoretical foundations and experimental suggestions. Broadly speaking, the main approaches of control spontaneous emission are: quantum measurement [12], quantum interference [13–15], and other methods [16–20]. To a certain extent, the spontaneous emission process can reflect the decay of atomic system.

In Ref. [21], it is pointed out that the evolution process of a two-level atom quantum state is manipulated with a dynamic dissipative environment. In this paper, we have studied the evolution curve of two-level atomic system that the initial state is excited state. At the different of environmental reservoir models [22–25], which include the single Lorentzian, ideal photon band-gap, double Lorentzian and square Lorentzian reservoir, we researched the influence of these environmental reservoir models on the evolution of energy level population. At static no modulation, comparing the four environmental models, the atomic energy level population oscillation of square Lorentzian reservoir model is fastest, the atomic system decoherence is slowest. The time of atomic attenuation to ground state is shortest for the single Lorentzian reservoir model. Under dynamic modulation, by comparing the photon band-gap model with the single Lorentzian reservoir model, we can find that no matter what form of dynamic modulation, the time of atoms decay to the ground state is longer, the energy dissipation to the cavity and the atomic system decoherence is slower for the photonic band-gap model. Due to the periodic modulation, the atoms are affected by different environments, which make the idea of using the environmental change to modulate the coherent evolution of the atomic system become true.

2 State Evolution of a Two-Level Atom in Photonic Crystal Heat Reservoir

Let us consider a two-level atom in the cavity, it initial state in the excited state. At the dipole approximation, the system Hamiltonian is

$$H = \hbar\omega_1 \mid 1 >< 1 \mid + \sum_k \hbar\omega_k a_k^+ a_k + i\hbar \sum_k g_k (a_k^+ \mid 0 >< 1 \mid - a_k \mid 1 >< 0 \mid), \quad (1)$$

where $\mid 0 >$ is atom ground state, $\mid 1 >$ is atom excited state, ω_1 is the atomic resonance transition frequency, ω_k is the radiation photon frequency, a_k^+ is the creation operator for kth mode with frequency ω_k, a_k is the annihilation operator for kth mode with frequency ω_k, and k represents both the momentum and polarization of the vacuum mode.

where g_k is the coupling constant between the kth mode and the atomic transitions.

The wave function of the system at any time t is

$$\mid \psi(t) >= A(t)e^{-i\omega_1 t} \mid 1, 0 > + \sum_k B_k(t)e^{-i\omega_k t} \mid 0, 1_k >, \quad (2)$$

where $\mid 1, 0 >$ shows atom in the excited state $\mid 1 >$ with no photon, $\mid 0, 1_k >$ shows atom in ground state $\mid 0 >$ with a k pattern photon.

With the initial conditions, $|\ A(0)\ |^2 = 1$, $|\ B_k(0)\ | = 0$. When the radiation field frequency is continuous distribution, the summation of (1) and (2) should be became integral, i. e., $\sum_k \rightarrow \int \rho(\omega, t) d\omega$, where $\rho(\omega, t)$ is photon state density, ω is photon frequency.

The (1) and (2) become

$$H = \hbar\omega_1 \mid 1 >< 1 \mid + \int \hbar\omega a_\omega^+ a_\omega \rho(\omega, t) d\omega + i\hbar \int g_\omega (a_\omega^+ \mid 0 >< 1 \mid - a_\omega \mid 1 >< 0 \mid) \rho(\omega, t) d\omega, \tag{3}$$

$$\mid \psi(t) >= A(t) e^{-i\omega_1 t} \mid 1, 0 > + \int B_\omega(t) e^{-i\omega t} \rho(\omega, t) d\omega \mid 0, 1_k > . \tag{4}$$

The evolution state of $\mid \psi(t) >$ satisfies the Schrödinger equation ($\hbar = 1$)

$$i \frac{\partial}{\partial t} \mid \psi(t) >= H \mid \psi(t) >, \tag{5}$$

substituting (3) and (4) into Schrödinger (5), we have

$$\begin{aligned} & i \frac{\partial}{\partial t} [A(t) e^{-i\omega_1 t} \mid 1, 0 > + \int B_\omega(t) e^{-i\omega t} \rho(\omega, t) d\omega \mid 0, 1_\omega >] \\ & = [\omega_1 \mid 1 >< 1 \mid + \int \omega a_\omega^+ a_\omega \rho(\omega, t) d\omega + i \int g_\omega (a_\omega^+ \mid 0 >< 1 \mid - a_\omega \mid 1 >< 0 \mid) \rho(\omega, t) d\omega] \\ & \cdot [A(t) e^{-i\omega_1 t} \mid 1, 0 > + \int B_\omega(t) e^{-i\omega t} \rho(\omega, t) d\omega \mid 0, 1_\omega >], \end{aligned} \tag{6}$$

comparing the both sides coefficient of state $\mid 1, 0 >$ and $\mid 0, 1_\omega >$, we get

$$i \frac{\partial}{\partial t} A(t) e^{-i\omega_1 t} = \omega_1 A(t) e^{-i\omega_1 t} - i \int g_\omega B_\omega(t) e^{-i\omega t} \rho(\omega, t) d\omega, \tag{7}$$

$$i \frac{\partial}{\partial t} [\int B_\omega(t) e^{-i\omega t} \rho(\omega, t) d\omega] = i \int g_\omega A(t) e^{-i\omega_1 t} \rho(\omega, t) d\omega + \int \omega B_\omega(t) e^{-i\omega t} \rho(\omega, t) d\omega, \tag{8}$$

Simplifying (7) and (8), we obtain the dynamics equations of system evolution

$$A'(t) = - \int g_\omega B_\omega(t) e^{-i(\omega - \omega_1) t} \rho(\omega, t) d\omega, \tag{9}$$

$$B'_\omega(t) = g_\omega A(t) e^{-i(\omega_1 - \omega) t} - B_\omega(t) \frac{\rho'(\omega, t)}{\rho(\omega, t)}, \tag{10}$$

with $\rho'(\omega, t) = \frac{d\rho(\omega, t)}{dt}$, $g_\omega = 1$.

3 Numerical Result

In a general cavity, the main photon distribution densities are the Lorentzian distribution, which are the single Lorentzian, ideal photon band-gap, double Lorentzian and square Lorentzian reservoir. For the single Lorentzian photon reservoir, the photon density is:

$$\rho(\omega) = \frac{1}{\pi} \frac{\gamma}{(\omega - \omega_c)^2 + \gamma^2}, \tag{11}$$

where ω_c is the resonant frequency of cavity, γ is the half width of single Lorentzian reservoir.

For the ideal photonic band-gap model, the state density function form is subtracted by two Lorentzian photon reservoir with the same center frequency. It is as follows:

$$\rho(\omega) = \frac{W_1\gamma_1}{(\omega-\omega_c)^2+(\frac{\gamma_1}{2})^2} - \frac{W_2\gamma_2}{(\omega-\omega_c)^2+(\frac{\gamma_2}{2})^2}, \tag{12}$$

where W_1, W_2 are the proportion of two Lorentzian reservoir, and $W_1 - W_2 = 1$. γ_1, γ_2 are the half width of two Lorentzian reservoirs, in order to guarantee the positive qualitative, require $\gamma_1 > \gamma_2$.

For the double Lorentzian model, the state density function form is simply summed up by two Lorentzian photon reservoirs with the same center frequency. It is:

$$\rho(\omega) = \frac{W_1\gamma_1}{(\omega-\omega_c)^2+(\frac{\gamma_1}{2})^2} + \frac{W_2\gamma_2}{(\omega-\omega_c)^2+(\frac{\gamma_2}{2})^2}, \tag{13}$$

where W_1, W_2 need to satisfy the relation $W_1 + W_2 = 1$.

For the square Lorentzian model, the state density function form can be written as:

$$\rho(\omega) = \frac{\frac{\gamma^3}{2}}{[(\omega-\omega_c)^2+(\frac{\gamma}{2})^2]^2}, \tag{14}$$

In Fig. 1, we give the evolution curve of energy level population in a static non-modulated single Lorentzian reservoir, which is calculated by the (11). The parameters are: the atomic resonance transition frequency $\omega_1 = 100\beta$ (β is the unitless relative amount), the resonant frequency of cavity $\omega_c = 100\beta$, and the half width $\gamma = 1$. In Fig. 2, we give the evolution curve of energy level population in a static non-modulated ideal photonic band-gap model reservoir, which is calculated by the (12). The parameters are: $\omega_1 = 100\beta$, $\omega_c = 100\beta$, the half width $\gamma_1 = 1$, $\gamma_2 = 0.8$, and the proportion $W_1 = 1.3$, $W_2 = 0.3$. In Fig. 3, we give the evolution curve of energy level population in a static non-modulated double Lorentzian reservoir, which is calculated by the (13). The parameters are: $\gamma_1 = 1$, $\gamma_2 = 0.8$, $W_1 = 1.3$, $W_2 = 0.3$, $\omega_1 = 100\beta$, and $\omega_c = 100\beta$. In Fig. 4, we give the evolution curve of energy level population in a static non-modulated square Lorentzian reservoir, which is calculated by the (14). The parameters are: $\gamma = 1$, $\omega_1 = 100\beta$, and $\omega_c = 100\beta$.

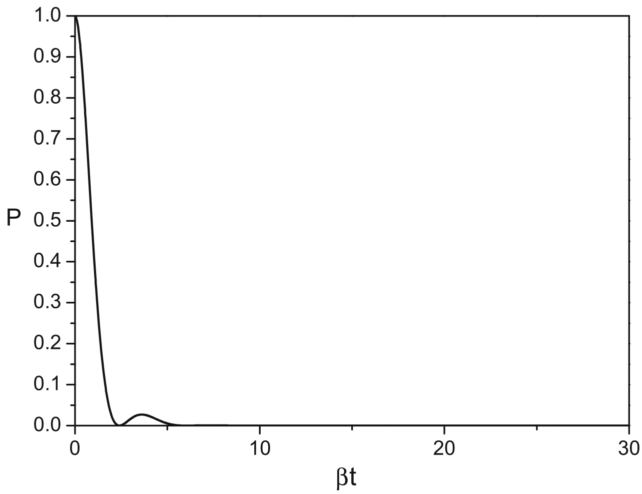

Fig. 1 The evolution curve of energy level population in a static non-modulated single Lorentzian reservoir

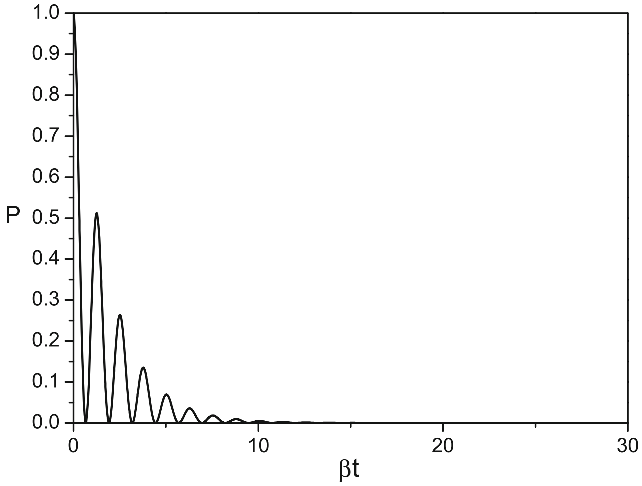

Fig. 2 The evolution curve of energy level population in a static non-modulated ideal photonic band-gap model

In static no modulation, we compare the four environmental models, i.e., single Lorentzian, ideal photon band-gap, double Lorentzian and square Lorentzian on the evolution of excited state atoms. From Figs. 1, 2, 3 and 4, we find the energy level population oscillation of photon band-gap model is faster than the single Lorentzian model reservoir, the atomic system decoherence and the energy dissipation to the outside become slower. The square Lorentzian model reservoir is compared with the other three kinds of environmental reservoirs, the atomic energy level population oscillation is the fastest, the atomic system decoherence and energy dissipation to the cavity are the slowest. In a single Lorentzian model reservoir, the time of atomic attenuation to ground state is the shorter.

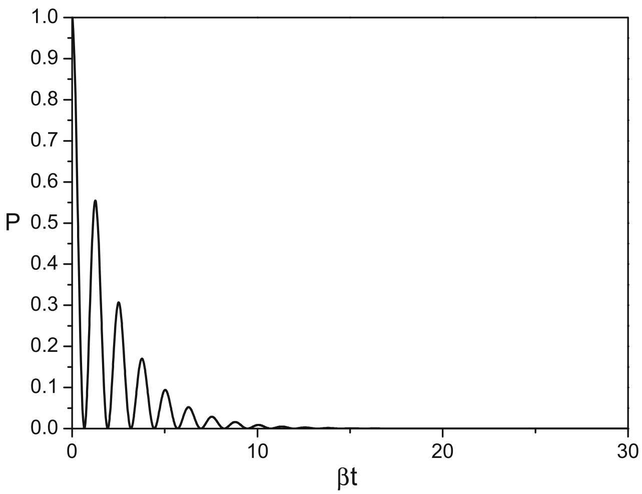

Fig. 3 The evolution curve of energy level population in a static non-modulated double Lorentzian reservoir

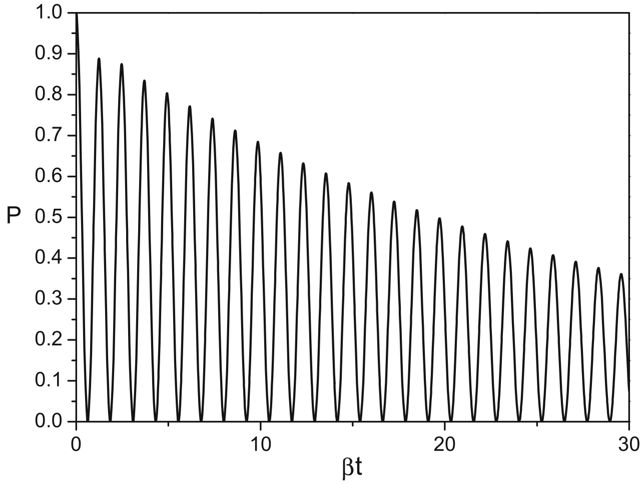

Fig. 4 The evolution curve of energy level population in a static non-modulated square Lorentzian reservoir

In the following, we shall study the coherent evolution of atomic system in dynamically environment reservoir, and compare the photon band-gap model with the single Lorentzian model reservoir. In Fig. 5, the center resonant frequency $\omega_c(t)$ of reservoir is modulated by continuous rectangular pulse. The Fig. 5a and b are corresponding the single Lorentzian model and the photonic band-gap model. In the non-modulation period, the center resonant frequency is $\omega_c(t) = 100\beta$. In the modulation period, the center resonant frequency is $\omega_c(t) = 102\beta$. The other parameters are: $\gamma = 1$, $\omega_1 = 100\beta$, $\gamma_1 = 1$, $\gamma_2 = 0.8$, $W_1 = 1.3$

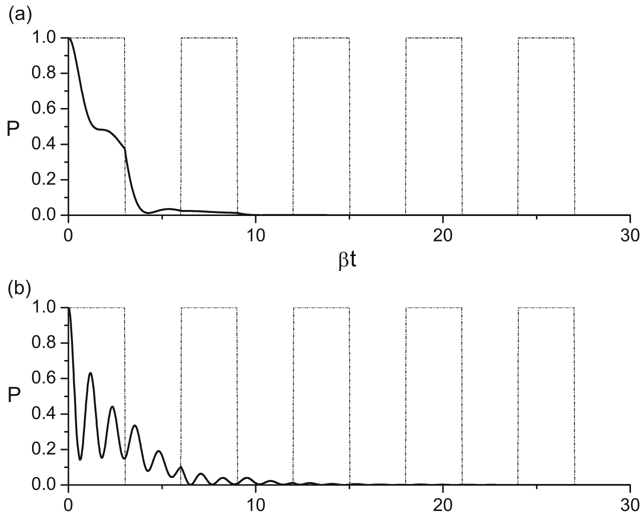

Fig. 5 The evolution curve of atomic energy level population with the center frequency by continuous rectangular pulse modulation (black solid line), the rectangular dotted line is the modulation pulse, **a** the single Lorentzian model, **b** the photonic band-gap model

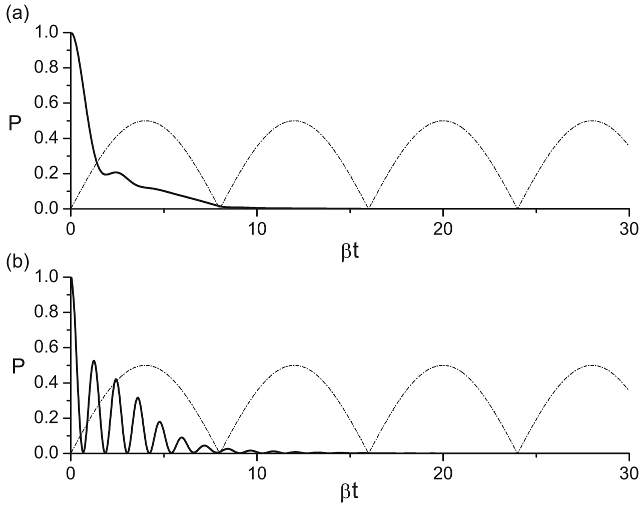

Fig. 6 The evolution curve of atomic energy level population with the center frequency by slowly continuous periodic pulse modulation (black solid line), the dotted line is the modulation pulse, **a** the single Lorentzian model, **b** the photonic band-gap model

and $W_2 = 0.3$. The black solid line shows the evolution curve of atomic energy level population with the center frequency by continuous rectangular pulse modulation. The rectangular dotted line is the modulation pulse.

In Fig. 6, the center resonant frequency $\omega_c(t)$ of reservoir is modulated by slowly continuous periodic pulse, the Fig. 6a and b are corresponding the single Lorentzian model and the photonic band-gap model. We consider the center frequency $\omega_c(t)$ is $100\beta + 2\beta \times |\sin(\frac{\pi}{8}t)|$,

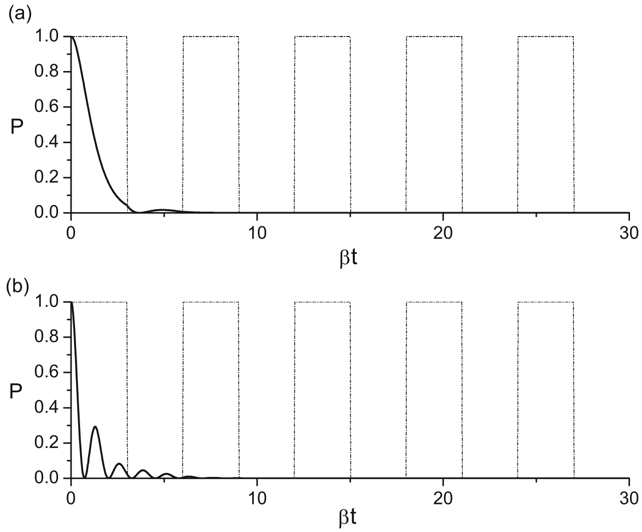

Fig. 7 The evolution curve of atomic energy level population with the half width by rectangular continuous pulse modulation (black solid line), the rectangular dotted line is the modulation pulse, **a** the single Lorentzian model, **b** the photonic band-gap model

the other parameters are $\gamma = 1$, $\omega_1 = 100\beta$, $\gamma_1 = 1$, $\gamma_2 = 0.8$, $W_1 = 1.3$ and $W_2 = 0.3$. The black solid line shows the evolution curve of atomic energy level population with the center frequency by slowly continuous periodic pulse modulation. The dotted line is the modulation pulse, the change range of $\omega_c(t)$ is $100\beta - 102\beta$.

In Fig. 7, the half width of reservoir is modulated by continuous rectangular pulse, the Fig. 7a and b are corresponding the single Lorentzian model and the photonic band-gap model. In single Lorentzian model reservoir, the half width of reservoir is $\gamma = 1$ in the non-modulation period and $\gamma = 2$ in the modulation period. In photon band-gap model reservoir, the half width of reservoir are $\gamma_1 = 1$ and $\gamma_1 = 0.8$ in the non-modulation period, $\gamma_1 = 2$ and $\gamma_1 = 1.8$ in the modulation period, the other parameters are $\omega_1 = 100\beta$, $\omega_c(t) = 100\beta$, $W_1 = 1.3$ and $W_2 = 0.3$. The black solid line shows the evolution curve of atomic energy level population with the half width by rectangular continuous pulse modulation. The rectangular dotted line is the modulation pulse.

In Fig. 8, the half width of reservoir is modulated by slowly continuous periodic pulse, the Fig. 8a and b are corresponding the single Lorentzian model and the photonic band-gap model. In single Lorentzian model reservoir, the half width is: $\gamma(t) = 1 + |\sin(\frac{\pi}{8}t)|$. In photon band-gap model reservoir, the half width are $\gamma_1(t) = 1 + |\sin(\frac{\pi}{8}t)|$ and $\gamma_2(t) = 0.8 + |\sin(\frac{\pi}{8}t)|$, the other parameters are $\omega_1 = 100\beta$, $\omega_c(t) = 100\beta$, $W_1 = 1.3$ and $W_2 = 0.3$. The black solid line shows the evolution curve of atomic energy level population with the half width by slowly continuous periodic pulse modulation. The dotted line is the modulation pulse.

By comparing the photon band-gap model with the single Lorentzian model reservoir, we can find that no matter what form of dynamic modulation, the time of atoms decay to the ground state is longer, the energy dissipation to the cavity and the atomic system decoherence become slower for the photonic band-gap model. Due to the periodic modulation, the atoms are affected by different environments, which make the idea of using the environmental change to modulate the coherent evolution of the atomic system become true.

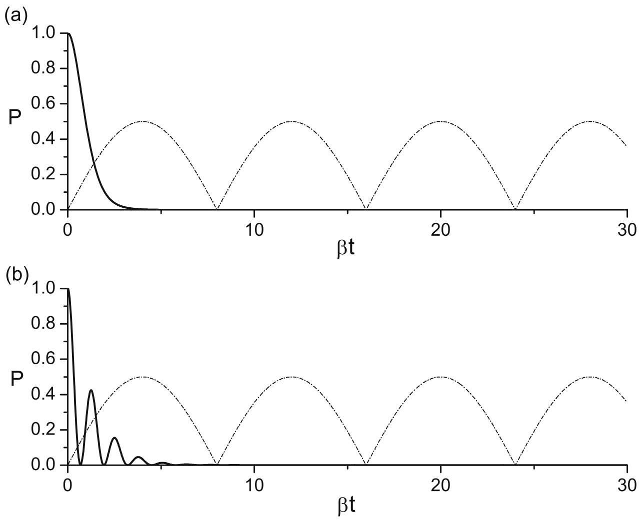

Fig. 8 The evolution curve of atomic energy level population with the half width by slowly continuous periodic pulse modulation (black solid line), the dotted line is the modulation pulse, **a** the single Lorentzian model, **b** the photonic band-gap model

4 Conclusion

In this paper, we have studied the evolution curve of two-level atomic system that the initial state is excited state. At the different of environmental reservoir models, which include the single Lorentzian, ideal photon band-gap, double Lorentzian and square Lorentzian reservoir, we researched the influence of these environmental reservoir models on the evolution of energy level population. At static no modulation, comparing the four environmental models, the atomic energy level population oscillation of square Lorentzian reservoir model is fastest, the atomic system decoherence is slowest. The time of atomic attenuation to ground state is shortest for the single Lorentzian reservoir model. The frequency of reciprocating energy exchange between atoms and cavities are determined by the coupling of atoms and electromagnetic modes. When the cavity and the atom exist a strong coupling, the photon of spontaneous radiation have the chance to be reabsorbed by atoms, the atoms are excited to return to the excited state and repeat the process, there is a kind of Rabi oscillation behavior in the decay process. Under dynamic modulation, by comparing the photon band-gap model with the single Lorentzian reservoir model, we can find that no matter what form of dynamic modulation, the time of atoms decay to the ground state is longer, the energy dissipation to the cavity and the atomic system decoherence is slower for the photonic band-gap model. Due to the periodic modulation, the atoms are affected by different environments, which make the idea of using the environmental change to modulate the coherent evolution of the atomic system become true.

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