Int J Theor Phys (2018) 57:279–289
https://doi.org/10.1007/s10773-017-3603-4

Quantum Entanglement of the Multiphoton Transition Jaynes-Cummings Model

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Received: 14 December 2016 / Accepted: 28 March 2017 / Published online: 29 November 2017

Abstract In this paper, we have studied the multiphoton transition Jaynes-Cummings model ($N = 1, 2, 3, 4, 5, 6$), and researched the effect of initial state superposition coefficient C_1, the initial photon number n, the transition photon number N, and the quantum discord δ on the quantum entanglement degrees, and given the quantum entanglement degrees curves with time evolution. We obtain some results, which should have important significance in the quantum computing and quantum information.

Keywords Jaynes-Cummings model · Multiphoton transition · Quantum entanglement degrees

1 Introduction

Quantum entanglement plays an important role in many quantum information and quantum computation tasks. And many well-known quantum systems poses quantum correlation that need to be characterized and realized for use in different applications. One of these very important quantum systems is Jaynes-Cummings model [1, 2]. The Jaynes-Cummings model, describing a harmonic oscillator coupled to a spin-1/2 system, underlies a wide variety of potential platforms for quantum computation, such as atoms in cavities [3], trapped ions [4], superconducting circuits [5, 6], and clouds of cold atoms [7]. One advantage of Jaynes-Cummings model is that it is exactly solvable model and we can investigate its entanglement properties analytically. One important application of Jaynes-Cummings model is

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in quantum computing for realization of quantum registers, namely cavity and circuit quantum electrodynamics. Gea-Banacloche and coworkers [8] have verified the existence of maximally entangled state in externally driven JCM. A large number of studies of such simple systems have been carried out for independent [9, 10] and also coupled Jaynes-Cummings models [11–14]. The many-body models being studied include quantum spin systems [15–17], models bearing novel topological properties [18, 19], models of electron phonon interaction, and systems in high energy physics [20–22].

In Jaynes-Cummings model with intensity-dependent coupling of a pair of two-level atoms the exact periodicity of the second-order squeezing oscillations is violated, whereas in a single atom one takes place the exact periodicity of the squeezing revivals. The squeezing revivals have also been observed in two-photon JaynesCCummings model of a pair of atoms. In this situation it is important to investigate the generation of higher-order squeezing of the quantized cavity fields in above-mentioned Jaynes-Cummings model.

The single photon and double photon Jaynes-Cummings model had been studied largely. In this paper, we have studied the multiphoton transition Jaynes-Cummings model ($N = 1, 2, 3, 4, 5, 6$), and researched the effect of initial state superposition coefficient C_1, the initial photon number n, the transition photon number N, and the quantum discord δ on the quantum entanglement degrees, and given the quantum entanglement degrees curves with time evolution. We find when the transition photon number N or the initial photon number n increases, the entanglement degrees oscillation get faster. When the initial state superposition coefficient $C_1 = 0$, the initial photon number $n = 0$ and the quantum discord $\delta^2 = 4g^2$, the entanglement degrees oscillation get slower, and keep the time of entanglement degrees 1 is longer than the quantum discord $\delta^2 = 0$. When the initial state superposition coefficient $C_1 = 0.76$, the initial photon number $n = 0$, the quantum discord $\delta^2 = 0$, the entanglement degrees $E \approx 1$ for $N = 1, 2, 3, 4, 5, 6$ When the quantum discord $\delta^2 = 4g^2$, the entanglement degrees variation range become larger for $N = 1, 2, 3$, but the entanglement degrees $E \approx 1$ for $N = 4, 5, 6$. These results have important significance in the quantum computing and quantum information.

2 The Multiphoton Jaynes-Cummings Model and Entanglement Degrees

Let us consider the N-photon Jaynes-Cummings model, the Hamiltonian is

$$H = \omega a^+ a + \frac{1}{2}\omega_0 \sigma_z + g(a^{+N}\sigma_- + a^N \sigma_+), \qquad (\hbar = 1) \tag{1}$$

where $\sigma_z =| a >< a | - | b >< b |, \sigma_+ =| a >< b |, \sigma_- =| b >< a |$.

The initial state is

$$| \psi(0) >= c_1 \mid a, n > +c_2 \mid b, n + N >, \tag{2}$$

where $|c_1|^2 + |c_2|^2 = 1$, the state $| \; b >$ is atom ground state, state $| \; a >$ is atom excited state, and the wave function at any time is

$$| \psi(t) >= c_1(t) \mid a, n > +c_2(t) \mid b, n + N >, \tag{3}$$

substituting (1) and (3) into Schrodinger equation

$$i\frac{\partial}{\partial t} \mid \psi(t) >= H \mid \psi(t) >, \tag{4}$$

we obtain

$$\begin{aligned}
&i\frac{\partial}{\partial t}\left[c_1(t) \mid a, n> +c_2(t) \mid b, n+N>\right] \\
&=\left[\omega a^+ a+\frac{1}{2}\omega_0\sigma_z+g(a^{+N}\sigma_-+a^N\sigma_+)\right]\left[c_1(t)\mid a,n> +c_2(t)\mid b,n+N>\right] \\
&=\left[\omega n c_1(t)+\frac{1}{2}\omega_0 c_1(t)+g c_2(t)\sqrt{(n+1)(n+2)\cdots(n+N)}\right]\mid n,a> \\
&+\left[\omega(n+N)c_2(t)-\frac{1}{2}\omega_0 c_2(t)+g c_1(t)\sqrt{(n+1)(n+2)\cdots(n+N)}\right]\mid n+N,b>, \quad (5)
\end{aligned}$$

comparing the both sides of (5), we have

$$i\frac{\partial}{\partial t}c_1(t)=\omega n c_1(t)+\frac{1}{2}\omega_0 c_1(t)+g c_2(t)\sqrt{(n+1)(n+2)\cdots(n+N)}, \quad (6)$$

$$i\frac{\partial}{\partial t}c_2(t)=\omega(n+N)c_2(t)-\frac{1}{2}\omega_0 c_2(t)+g c_1(t)\sqrt{(n+2)(n+1)\cdots(n+N)}, \quad (7)$$

the Laplace transform of functions $c_1(t)$ $c_2(t)$ are

$$L_1(p)=\mathcal{L}[c_1(t)], \qquad L_2(p)=\mathcal{L}[c_2(t)], \quad (8)$$

and Laplace transform formulas

$$\mathcal{L}[\dot{c_1}(t)]=pL_1(p)-c_1(0), \qquad \mathcal{L}[\dot{c_2}(t)]=pL_2(p)-c_2(0), \quad (9)$$

using Laplace transforms to the both sides of (6) and (7), we get

$$ipL_1(p)-ic_1(0)=\omega n L_1(P)+\frac{1}{2}\omega_0 L_1(p)+g\sqrt{(n+1)(n+2)\cdots(n+N)}L_2(p), \quad (10)$$

$$ipL_2(p)-ic_2(0)=\omega(n+N)L_2(P)+\frac{1}{2}\omega_0 L_2(p)+g\sqrt{(n+1)(n+2)\cdots(n+N)}L_1(p), \quad (11)$$

with $c_1(0)=c_1$ and $c_2(0)=c_2$.

$$L_1(p)=\frac{c_1}{p+i\omega n+\frac{i}{2}\omega_0}-i\frac{g\sqrt{(n+1)(n+2)\cdots(n+N)}}{p+i\omega n+\frac{i}{2}\omega_0}\left(\frac{D}{p+A}+\frac{E}{p+B}\right), \quad (12)$$

$$L_2(p)=\frac{D}{p+A}+\frac{E}{p+B}, \quad (13)$$

where

$$A=\frac{i\omega(2n+N)+i\sqrt{\delta^2+\omega_1^2}}{2}, \quad (14)$$

$$B=\frac{i\omega(2n+N)-i\sqrt{\delta^2+\omega_1^2}}{2}, \quad (15)$$

$$D=\frac{\left(\sqrt{\delta^2+\omega_1^2}-\delta\right)c_2+\omega_1 c_1}{2\sqrt{\delta^2+\omega_1^2}}, \quad (16)$$

$$E=\frac{\left(\sqrt{\delta^2+\omega_1^2}-\delta\right)c_2-\omega_1 c_1}{2\sqrt{\delta^2+\omega_1^2}}, \tag{17}$$

$$c_1(t)=e^{-\frac{i}{2}\omega(2n+N)t}\left[c_1\cos\left(\frac{\sqrt{\delta^2+\omega_1^2}}{2}t\right)-i\frac{c_1\delta+c_2\omega_1}{\sqrt{\delta^2+\omega_1^2}}\sin\left(\frac{\sqrt{\delta^2+\omega_1^2}}{2}t\right)\right], \tag{18}$$

$$c_2(t)=e^{-\frac{i}{2}\omega(2n+N)t}\left[c_2\cos\left(\frac{\sqrt{\delta^2+\omega_1^2}}{2}t\right)+i\frac{c_2\delta-c_1\omega_1}{\sqrt{\delta^2+\omega_1^2}}\sin\left(\frac{\sqrt{\delta^2+\omega_1^2}}{2}t\right)\right], \tag{19}$$

where $\delta=\omega_0-N\omega$, $\omega_1=2g\sqrt{(n+1)(n+2)\cdots(n+N)}$, ω_0 is the atomic transition frequency, and ω is the optical field frequency.

With the state (2), we can obtain the density operator of atom-photon system

$$\begin{aligned}\hat{\rho}(t)&=|\psi(t)><\psi(t)|\\&=|c_1(t)|^2|n,a><a,n|+c_1(t)c_2^*(t)|n,a><b,n+2|\\&+c_2(t)c_1^*(t)|n+2,b><a,n|+|c_2(t)|^2|n+2,b><b,n+2|,\end{aligned} \tag{20}$$

the reduce density operator of atom A is

$$\begin{aligned}\hat{\rho}_A(t)&=tr_{(a)}\hat{\rho}(t)\\&=<n|\hat{\rho}(t)|n>+<n+2|\hat{\rho}(t)|n+2>\\&=|c_1(t)|^2|a><a|+|c_2(t)|^2|b><b|,\end{aligned} \tag{21}$$

the matrix form of $\hat{\rho}_A(t)$ at basis vectors $|a>$ and $|b>$

$$\hat{\rho}_A(t)=\begin{pmatrix}<a|\hat{\rho}_A(t)|a> & <a|\hat{\rho}_A(t)|b>\\ <b|\hat{\rho}_A(t)|a> & <b|\hat{\rho}_A(t)|b>\end{pmatrix}=\begin{pmatrix}|c_1(t)|^2 & 0\\ 0 & |c_2(t)|^2\end{pmatrix}, \tag{22}$$

the quantum system entanglement degrees is

$$\begin{aligned}E(t)&=-tr\left[\hat{\rho}_A(t)\log_2\hat{\rho}_A(t)\right]\\&=-tr\left[\begin{pmatrix}|c_1(t)|^2 & 0\\ 0 & |c_2(t)|^2\end{pmatrix}\cdot\begin{pmatrix}|\log_2 c_1(t)|^2 & 0\\ 0 & \log_2|c_2(t)|^2\end{pmatrix}\right]\\&=-(|c_1(t)|^2\log_2 c_1(t)|^2+|c_2(t)|^2\log_2|c_2(t)|^2).\end{aligned} \tag{23}$$

3 Numerical Result

In this section, we shall calculate the quantum entanglement degrees with (18), (19) and (23). From the Figs. 1, 2, 3, 4, 5, 6, 7, 8 and 9, we have given the curves of the atom and light field entanglement degrees, which are periodic oscillation with time evolution, the

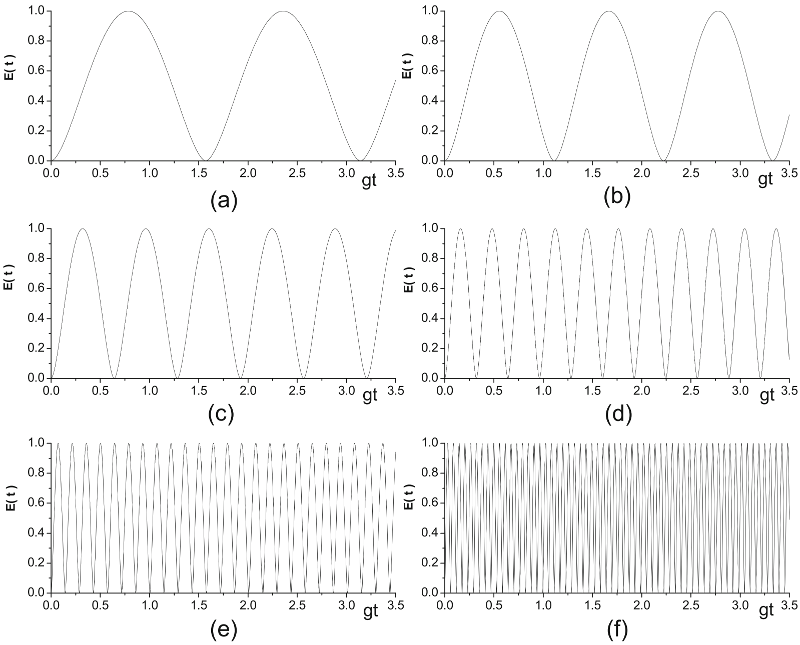

Fig. 1 The curves of the atom and light field entanglement degrees with time evolution, the initial state superposition coefficient $C_1 = 0$, the initial photon number $n = 0$, the quantum discord $\delta^2 = 0$

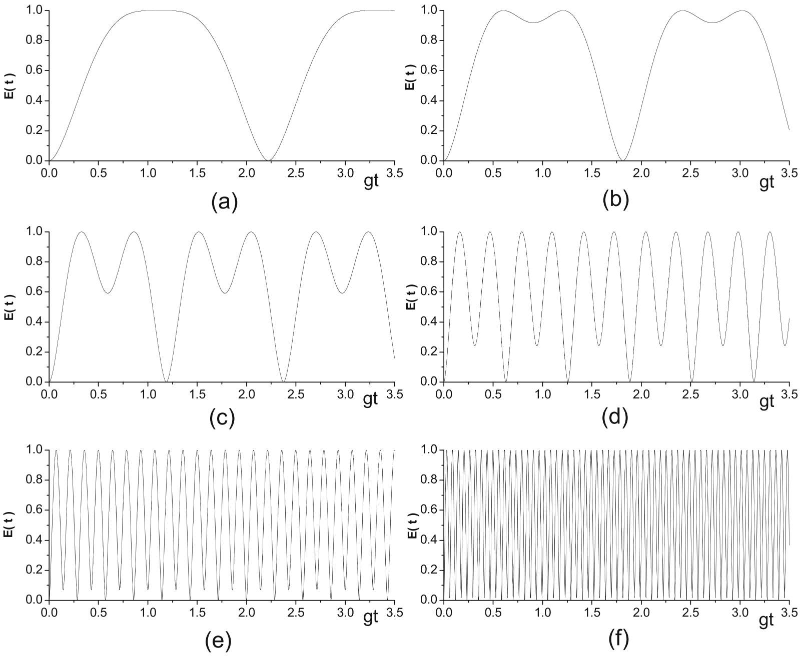

Fig. 2 The curves of the atom and light field entanglement degrees with time evolution, the initial state superposition coefficient $C_1 = 0$, the initial photon number $n = 0$, the quantum discord $\delta^2 = 4g^2$

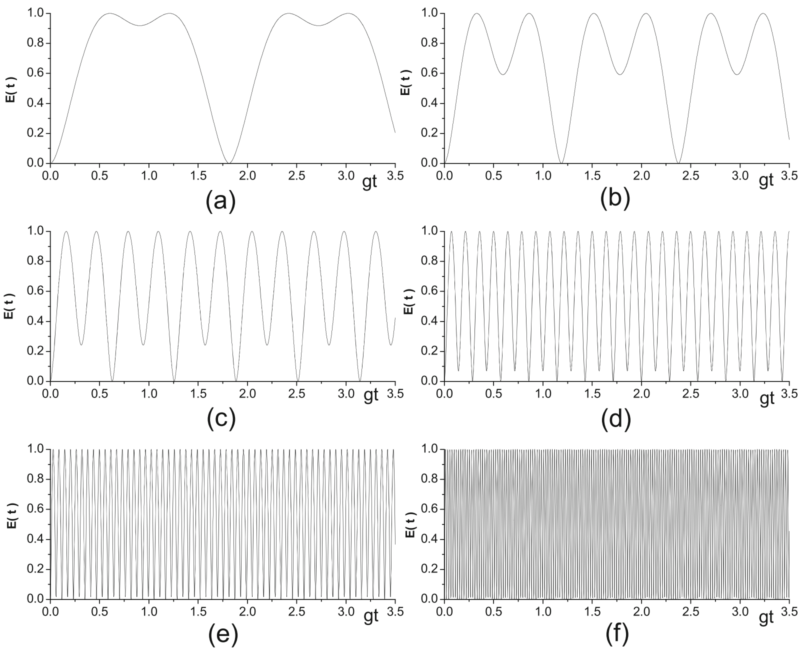

Fig. 3 The curves of the atom and light field entanglement degrees with time evolution, the initial state superposition coefficient $C_1 = 0$, the initial photon number $n = 1$, the quantum discord $\delta^2 = 4g^2$

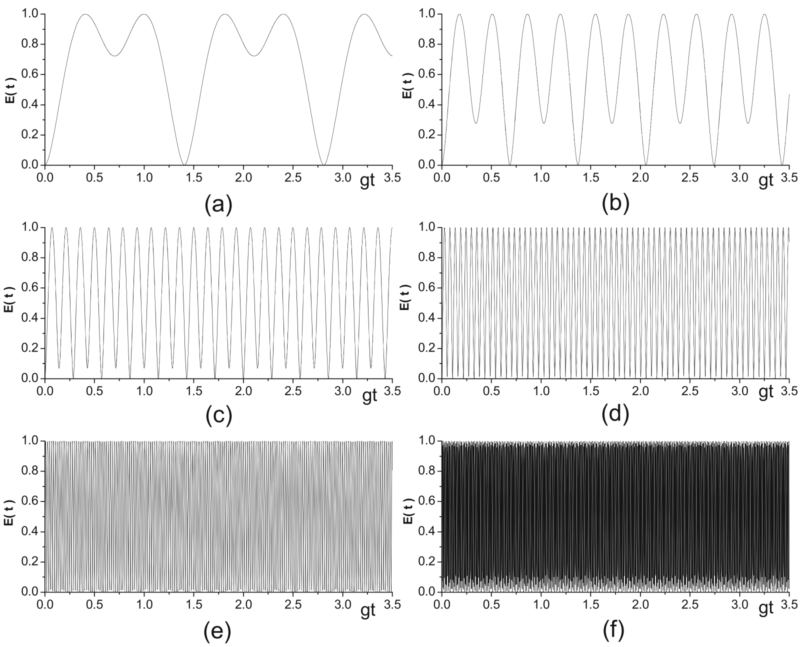

Fig. 4 The curves of the atom and light field entanglement degrees with time evolution, the initial state superposition coefficient $C_1 = 0$, the initial photon number $n = 3$, the quantum discord $\delta^2 = 4g^2$

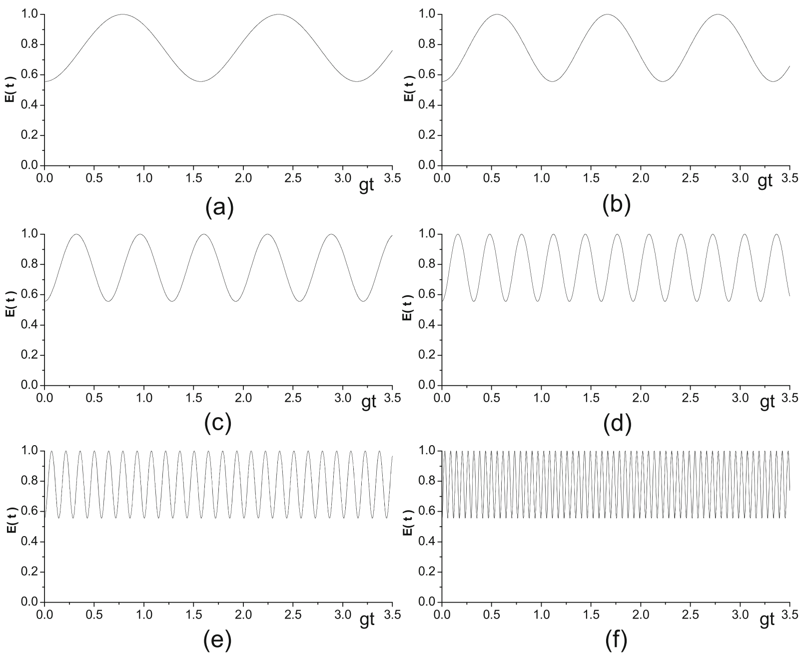

Fig. 5 The curves of the atom and light field entanglement degrees with time evolution, the initial state superposition coefficient $C_1 = 0.36$, the initial photon number $n = 0$, the quantum discord $\delta^2 = 0$

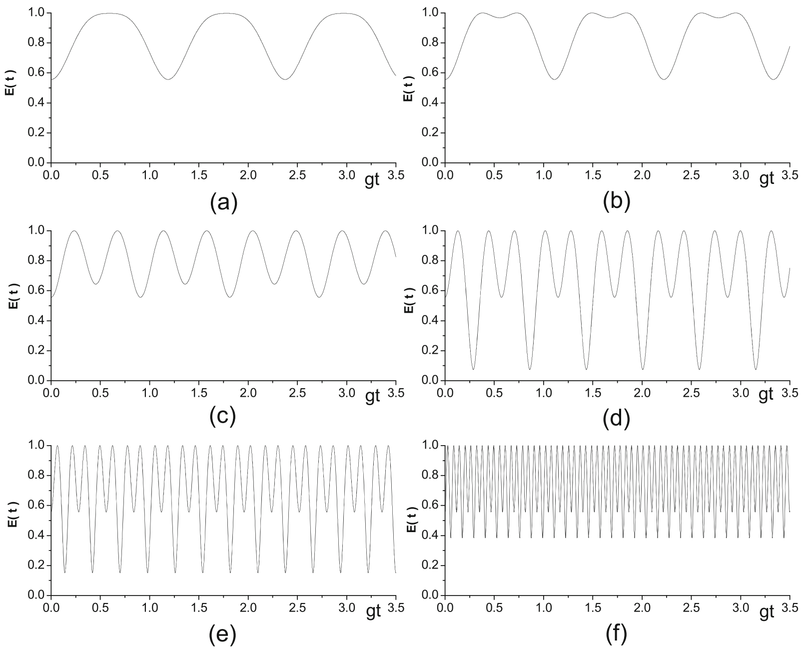

Fig. 6 The curves of the atom and light field entanglement degrees with time evolution, the initial state superposition coefficient $C_1 = 0.36$, the initial photon number $n = 0$, the quantum discord $\delta^2 = 24g^2$

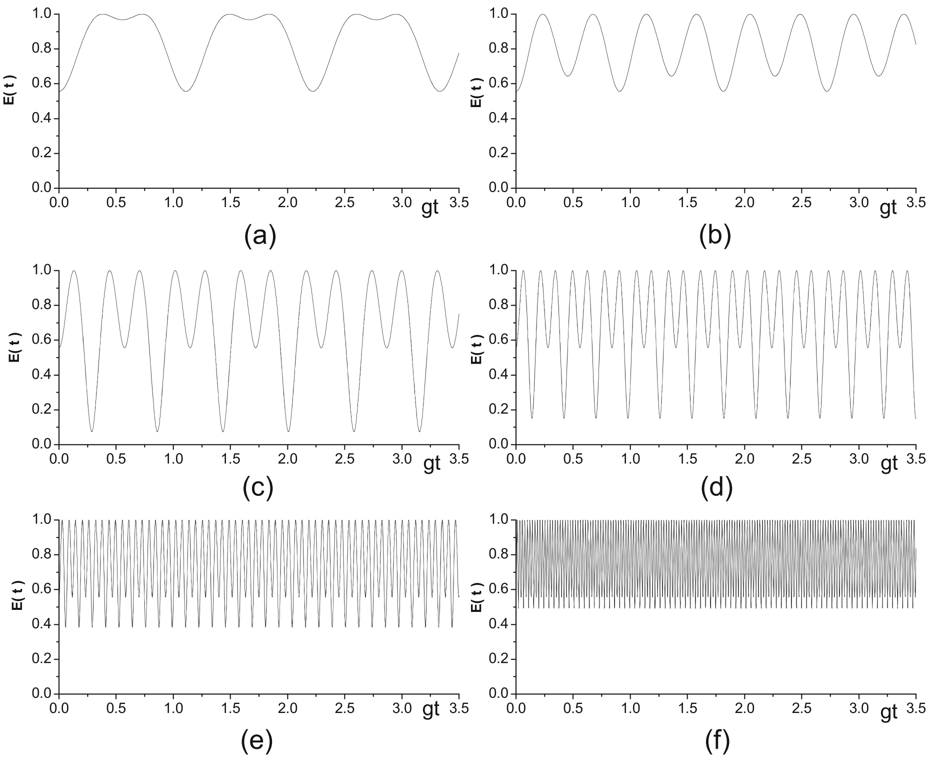

Fig. 7 The curves of the atom and light field entanglement degrees with time evolution, the initial state superposition coefficient $C_1 = 0.36$, the initial photon number $n = 1$, the quantum discord $\delta^2 = 24g^2$

entanglement degree E is in the range of $0-1$. From Figs. 1–4, the initial state superposition coefficient $C_1 = 0$, i.e., the atom and light field entanglement degree is 0 at initial state. With the time evolution, the atom and light field should be in the entangled state. In Fig. 1a–f, the initial photon number $n = 0$, the quantum discord $\delta^2 = 0$, the transition photon numbers N are 1, 2, 3, 4, 5 and 6, respectively. In Fig. 1a-f, the entanglement degree $0 \leq E \leq 1$. When the numbers of photons N increases, the entanglement degrees oscillation get faster, i.e., the period becomes small. When $N = 1, 2$ and 3, the evolution curves of entanglement degrees change slowly with time t, and keep the time of entanglement degrees 1 longer. When $N > 3$, the entanglement degree oscillate quickly, and keep the time of entanglement degree 1 short. When $N = 6$, the entanglement degree oscillate very quickly, and keep the time of entanglement degree 1 very short. In Fig. 2, the quantum discord $\delta^2 = 4g^2$, and other parameters are the same as Fig. 1. Comparing Fig. 2 with Fig. 1, When the quantum discord $\delta^2 = 4g^2$, the entanglement degrees oscillation get slower, and keep the time of entanglement 1 is longer than the quantum discord $\delta^2 = 0$. In Figs. 3 and 4, the initial photon numbers $n = 1$ and 3, respectively, and other parameters are the same as Fig. 2. Comparing Fig. 2 with Figs. 3 and 4, when the initial photon numbers increase, the entanglement degrees oscillation get faster.

From Figs. 5, 6 and 7, the initial state superposition coefficient $C_1 = 0.36$, i.e., the atom and light field are in the entangled state at initial state. In Fig. 5a–f, the initial photon number $n = 0$, the quantum discord $\delta^2 = 0$, the transition photon numbers N are 1, 2, 3, 4, 5 and

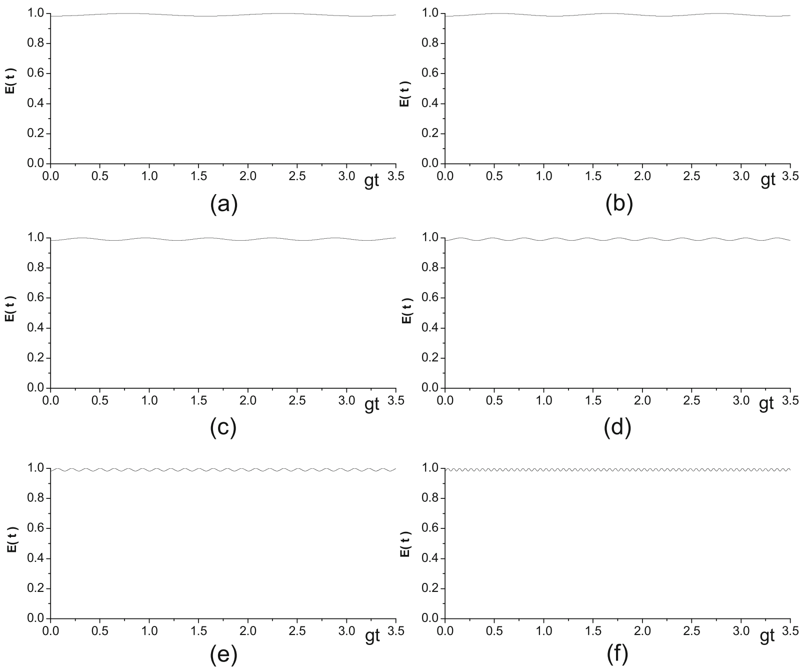

Fig. 8 The curves of the atom and light field entanglement degrees with time evolution, the initial state superposition coefficient $C_1 = 0.76$, the initial photon number $n = 0$, the quantum discord $\delta^2 = 0$

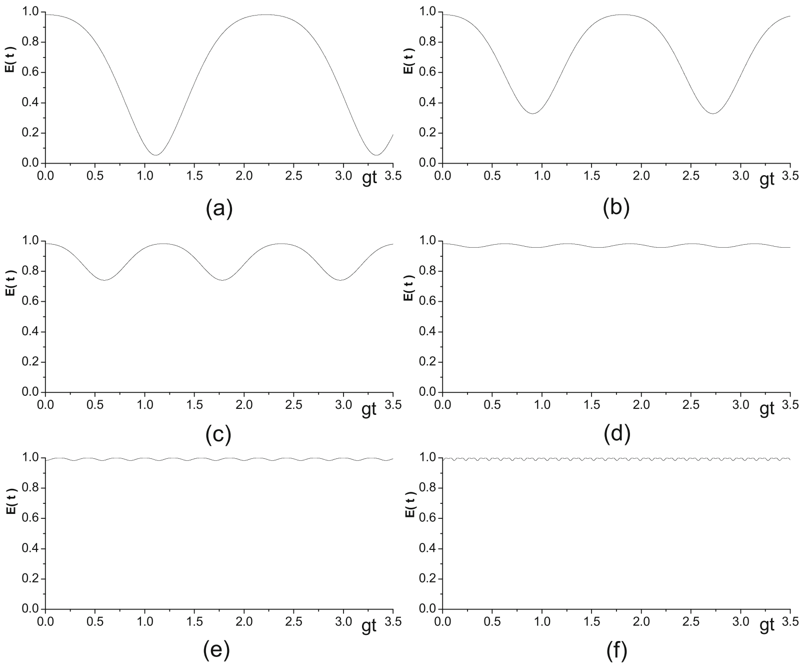

Fig. 9 The curves of the atom and light field entanglement degrees with time evolution, the initial state superposition coefficient $C_1 = 0.76$, the initial photon number $n = 0$, the quantum discord $\delta^2 = 4g^2$

6, respectively. In Fig. 5a–f, the entanglement degree $E > 0$. When the number of photons N increases, the entanglement degrees oscillation get faster. In Fig. 6, the quantum discord $\delta^2 = 24g^2$, and other parameters are the same as Fig. 5. Comparing Figs. 6 with Fig. 5, we find when the quantum discord increases, the entanglement degree variation range increase, and keep the time of entanglement 1 is longer. In Fig. 7, the initial photon number $n = 1$, and other parameters are the same as Fig. 6. Comparing Fig. 7 with Fig. 6, we find when the initial photon number increases, the entanglement degrees oscillation get faster.

From Figs. 8 and 9, the initial state superposition coefficient $C_1 = 0.76$, i.e., the initial state of atom and light field is close to maximum entanglement. In Fig. 8a–f, the initial photon number $n = 0$, the quantum discord $\delta^2 = 0$, the transition photon numbers N are $1, 2, 3, 4, 5$ and 6, respectively. In Fig. 8a–f, we find when the numbers of photons N increases, the entanglement degrees oscillation get faster, and their entanglement degrees $E \approx 1$. In Fig. 9, the quantum discord $\delta^2 = 4g^2$, and other parameters are the same as Fig. 8. Comparing Fig. 9 with Fig. 8, we find when the quantum discord increases, the entanglement degrees oscillation get faster. When $N = 1, 2, 3$, the entanglement degrees variation range are larger, when $N = 4, 5, 6$, their entanglement degrees $E \approx 1$.

4 Conclusion

In this paper, we have studied the effect of initial state superposition coefficient C_1, the initial photon number n, the transition photon number N, and the quantum discord δ on the quantum entanglement degrees with the Jaynes-Cummings model, and given the quantum entanglement degrees curves with time evolution. We find when the transition photon number N or the initial photon number n increases, the entanglement degrees oscillation get faster. When the initial state superposition coefficient $C_1 = 0$, the initial photon number $n = 0$ and the quantum discord $\delta^2 = 4g^2$, the entanglement degrees oscillation get slower, and keep the time of entanglement degrees 1 is longer than the quantum discord $\delta^2 = 0$. When the initial state superposition coefficient $C_1 = 0.76$, the initial photon number $n = 0$, the quantum discord $\delta^2 = 0$, the entanglement degrees $E \approx 1$ for $N = 1, 2, 3, 4, 5, 6$ When the quantum discord $\delta^2 = 4g^2$, the entanglement degrees variation range become larger for $N = 1, 2, 3$, but the entanglement degrees $E \approx 1$ for $N = 4, 5, 6$. These results have important significance in the quantum computing and quantum information.

Acknowledgments This work is supported by Scientific and Technological Development Foundation of Jilin Province, Grant Number: 20130101031JC.

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