Optics Communications 387 (2017) 281–289

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Novel theory for propagation of tilted Gaussian beam through aligned optical system

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ARTICLE INFO

Keywords:
Tilted Gaussian beams
Virtual optical systems
Decentered Gaussian beams
Optical field distributions
ZEMAX simulation

ABSTRACT

A novel theory for tilted beam propagation is established in this paper. By setting the propagation direction of the tilted beam as the new optical axis, we establish a virtual optical system that is aligned with the new optical axis. Within the first order approximation of the tilt and off-axis, the propagation of the tilted beam is studied in the virtual system instead of the actual system. To achieve more accurate optical field distributions of tilted Gaussian beams, a complete diffraction integral for a misaligned optical system is derived by using the matrix theory with angular momentums. The theory demonstrates that a tilted TEM_{00} Gaussian beam passing through an aligned optical element transforms into a decentered Gaussian beam along the propagation direction. The deviations between the peak intensity axis of the decentered Gaussian beam and the new optical axis have linear relationships with the misalignments in the virtual system. ZEMAX simulation of a tilted beam through a thick lens exposed to air shows that the errors between the simulation results and theoretical calculations of the position deviations are less than 2‰ when the misalignments ε_x, ε_y, ε_x', ε_y' are in the range of [−0.5, 0.5] mm and [−0.5, 0.5]°.

1. Introduction

In a laser system, the beam propagation direction should coincide with the system axis, such that after the beam passes through an optical system, the optical field distribution and the laser parameters follow certain accurate and concise rules. However, the laser can tilt because of misalignment of the resonant cavity and the mechanical and thermal disturbances in the assembly and propagation processes. Examples of the direct application of tilted beams in practice have been reported [1–4]. The direct use of the Collins formula cannot produce the desired results, owing to the tilt and off-axis between the propagation direction of the tilted beam and the system axis.

In the main methods and theories of tilted Gaussian beams, Goodman [5] applied Fourier optics to describe the tilt and off-axis of the beam; the phase-tilt factor [6–8] was utilized to characterize the effect of tilted beams; Hadad [9,10] employed the parabolic wave equation to depict the optical field distribution and propagation; tensor optics can deal with generalized beans including tilted Gaussian beams [11,12]; the propagation characteristics of three-dimensional beams can obtained by Wigner distribution function in space domain [13]; and in references [4,14], the method of coordinate transformation was used to determine the optical field distributions of tilted Gaussian beams. Nonetheless, these theories have complex forms or limited precision.

In this study, the propagation direction of the tilted beam is set as the new optical axis. We establish a virtual optical system that is aligned with the new optical axis. The actual optical element that is aligned with original optical axis can be treated as the misaligned virtual element in the virtual system, which maintains the transformation properties of the tilted beam. Thus, we can study the propagation of the tilted beam in the virtual system. To obtain a more accurate optical field distribution in the virtual system, a complete diffraction integral for a misaligned optical system is derived by using matrix theory with angular momentums for the virtual system is misaligned. This method of the virtual system combines the easy transformation and concise form of coaxial systems and the accuracy of the Collins formula, resulting in advantages such as easy operation and high precision.

In Section 2, we establish the virtual element and the virtual optical system on the basis of the description of the tilted beam and the theory of misaligned optical systems. The misalignments and other system parameters are also calculated. In Section 3, the Collins formula for misaligned optical systems is derived by applying the ray transfer matrix with angular momentums. In Section 4, the theory developed in

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http://dx.doi.org/10.1016/j.optcom.2016.11.076
Received 14 September 2016; Received in revised form 14 November 2016; Accepted 28 November 2016
Available online 01 December 2016

the preceding two sections is used to calculate and analyze the optical field distribution of a tilted TEM_{00} Gaussian beam passing through an optical system. The validity of the theory is verified by performing ZEMAX simulation in Section 5. The results are summarized in Section 6.

2. Establishing virtual optical system and calculating misalignments

The tilted beam is not aligned with the axis of the optical system, which makes it difficult to study the propagation of the tilted beam along the system axis by using the Collins formula. Thus, a virtual coaxial optical system is established along the beam propagation direction so that we can study the tilted beam in the virtual system. To distinguish them from the virtual system and the virtual element that we propose later, the original system and the original element are denoted as the actual system and the actual element.

2.1. Description and coordinate system for tilted beam

The actual optical element has symmetry of revolution around the system axis. Thus, the actual element is an aligned element, and the actual system is an aligned system. The propagation in the actual system is shown in Fig. 1.

The coordinates of point o' are (a_0, b_0) in the coordinate system xoy. The coordinates of point o_{1m}' are (a_1, b_1) in the coordinate system $x_1o_1y_1$. The angles between the z axis and the projections of the z' axis in planes xoz and yoz are θ_x, θ_y, and the angle between the z' axis and the z axis is θ_z. In the paraxial approximation, Formula (1) is satisfied as follows:

$$\theta_x \approx \tan\theta_x = \frac{a_1 - a_0}{u_0}, \quad \theta_y \approx \tan\theta_y = \frac{b_1 - b_0}{u_0},$$
$$\theta_z \approx \tan\theta_z = \pm\sqrt{\tan\theta_x^2 + \tan\theta_y^2}$$
$$\approx \pm\sqrt{\theta_x^2 + \theta_y^2}. \tag{1}$$

Here, the value of θ_z will be negative only when the value of $\theta_x$is negative, which indicates the positive direction of θ_z.

In the actual system, the refractive indices on the two sides of the element are n_1 and n_2; the distance from the input plane to the exit plane is l; the distance between the source plane and the input plane is u_0; the distance between the exit and the receiver plane is v_0; and the ray transfer matrix of the actual element is$\underrightarrow{\mathbf{M}}$. The arrow under the matrix symbol indicates propagation from left to right.

A ray traveling through an optical system can be completely defined by the position parameter $\boldsymbol{q}$ and the "momentum" $\boldsymbol{p}$[15,16]. In a Cartesian system of coordinate axes$\boldsymbol{i}, \boldsymbol{j}$, with origin o on the z axis, $\boldsymbol{q}$ and $\boldsymbol{p}$ can be defined as

$$\mathbf{q} = x\mathbf{i} + y\mathbf{j}, \quad p = \frac{n(x, y, z)}{\sqrt{1 + \left(\frac{dx}{dz}\right)^2 + \left(\frac{dy}{dz}\right)^2}}\left(\frac{dx}{dz}\boldsymbol{i} + \frac{dy}{dz}\boldsymbol{j}\right). \tag{2}$$

In this equation, $n(x, y, z)$ is the refractive index of the medium. In the paraxial approximation, the components of $\boldsymbol{p}$ along the x and y axes, namely, p_x and p_y, can be expressed as

$$\begin{cases} p_x = n\frac{dx}{dz} = n\theta_x \\ p_y = n\frac{dy}{dz} = n\theta_y \end{cases}. \tag{3}$$

For the axisymmetric element in Fig. 1, the parameters $(x_1, p_{x1})^{\mathrm{T}}$, $(y_1, p_{y1})^{\mathrm{T}}$ of the incoming ray at the input plane and those $(x_2, p_{x2})^{\mathrm{T}}$, $(y_2, p_{y2})^{\mathrm{T}}$ of the outgoing ray at the exit plane satisfy

$$\begin{pmatrix} x_2 & y_2 \\ p_{x2} & p_{y2} \end{pmatrix} = \underrightarrow{\mathbf{M}}\begin{pmatrix} x_1 & y_1 \\ p_{x1} & p_{y1} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 & y_1 \\ p_{x1} & p_{y1} \end{pmatrix}, \tag{4}$$

$$Det(\underrightarrow{\mathbf{M}}) = ad - bc = 1. \tag{5}$$

To study the propagation of the tilted beam in the virtual system, a coordinate system for the tilted beam needs to be constructed according to the transverse intensity distribution of the tilted beam along the z' axis. If point o' is taken as the origin, we can establish a reference coordinate system $o'x_s'y_s'z'$, which can be easily obtained by a coordinate transformation of the coordinate system $oxyz$. The tilted beam coordinate system $o'x'y'z'$ can then be obtained by a counter-clockwise rotation of the reference coordinate system $o'x_s'y_s'z'$ by $\zeta(-\pi/2 \le \zeta \le \pi/2)$ around the z' axis.

As shown in Fig. 1, in the coordinate transformation of the reference coordinate system $o'x_s'y_s'z'$ relative to the original coordinate system $oxyz$, the Euler angles, namely, the precession angle, the nutation angle and the spin angle are α_y, $\theta_z$and 0, and the rotating shafts are the z, y_s' and z' axes, respectively. α_y is defined as follows:

$$\alpha_y = arc\tan\left(\frac{\tan\theta_y}{\tan\theta_x}\right) = arc\tan\left(\frac{\theta_y}{\theta_x}\right). \tag{6}$$

Here, $\alpha_y = \pm\pi/2$ corresponds to the cases where θ_y/θ_x infinitely approaches $\pm\infty$, respectively.

The spin angle ζ indicates that the beam may rotate about the z' axis besides the tilt and off-axis relative to the z axis, and the coordinate system $o'x_s'y_s'z'$ is a special case of $o'x'y'z'$ when ζ becomes 0. Thus, the tilted beam coordinate system $o'x'y'z'$ is applicable for tilted beams of many kinds, including TEM_{00} Gaussian beams and high-order beams. For beams whose transverse distribution has symmetry of revolution around the z' axis (e.g., TEM_{00} Gaussian beams), the reference coordinate system $o'x_s'y_s'z'$ is more convenient than other systems.

The preceding analysis suggests that the beam tilt can be completely described by five parameters, namely, the offset coordinates (a_0, b_0), beam angles θ_x, θ_y in the x and y axes, and the spin angle ζ around the beam propagation axis.

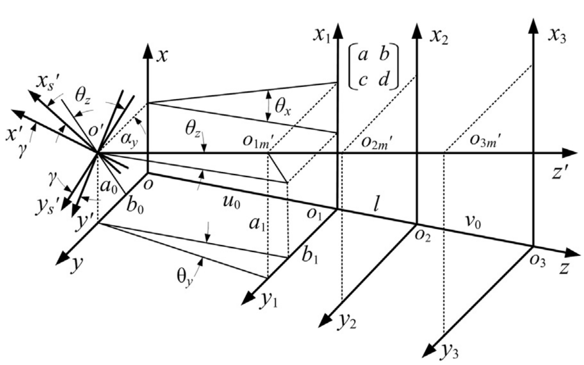


Fig. 1. Schematic of propagation in the actual optical system. The z axis is the axis of the actual system. The z' axis is the propagation axis of the tilted beam. Point o' is the beam waist center. xoy, $x_1o_1y_1$, $x_2o_2y_2$, and $x_3o_3y_3$ are, respectively, the coordinate systems in the source plane, input plane, exit plane, and receiver plane of the actual system. These planes intersect with the z' axis at points o', o_{1m}', o_{2m}' and o_{3m}', respectively. ζ is the spin angle of the tilted beam coordinate system $o'x'y'z'$ relative to the coordinate system $oxyz$..

2.2. Establishment of virtual element and virtual optical system

To maintain the same transformation properties of the tilted beam in the virtual system and the actual system, the element on which the tilted beam actually impinges in the virtual system must coincide with the actual element.

By setting the beam propagation direction as the new optical axis, a virtual element aligned with the new optical axis can be constructed along the z' axis. Minor decentering and tilting relative to the new optical axis are applied to the virtual element. The moved virtual element on which the tilted beam actually impinges in the virtual system can then coincide with the actual element. Thus, the actual element can be treated as the misaligned virtual element in the virtual

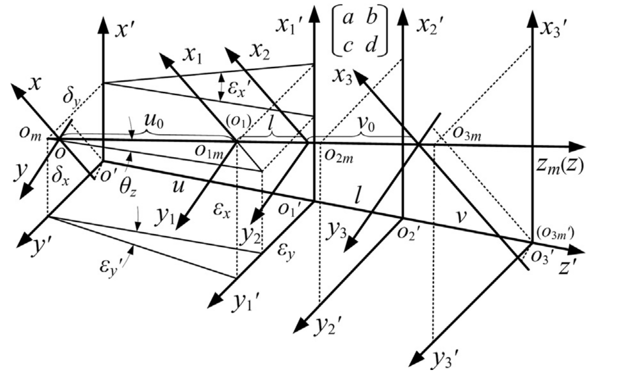


Fig. 2. Schematic of beam propagation in the virtual optical system. $x'o'y'$, $x_1'o'y_1'$, $x_2'o'y_2'$ and $x_3'o'y_3$, respectively, are the coordinate systems of the source plane, input plane, exit plane, and receiver plane of the virtual system. These planes intersect with the z axis at points o_m, o_{1m}, o_{2m}, and o_{3m}, respectively. The linear misalignments ε_x and ε_y are the coordinates of point o_{1m} in the coordinate system $x_1'o'y_1'$. The angular misalignments ε_x' and ε_y' are the angles between the z' axis and the projections of the z axis in planes $x_1'o_1'z'$ and $y_1'o_1'z'$. The positive directions of the initial offset distances δ_x and $\delta_y$are opposite to those of the x' and y' axes, respectively.

system, and the virtual system is then considered a misaligned optical system.

We first fix the position of the input plane of the virtual element along the z' axis, and then the entire virtual system can be completed along the z' axis.

The beam propagation in the virtual system shown in Fig. 2 can help us position the virtual element and the virtual system. The input plane of the misaligned virtual element is that of the actual element, and their respective centers of o_{1m} and o_1 are at the same point. Therefore, we have a vertical plane of the z' axis through point o_1 and its intersection o_1' with the z' axis. The vertical plane is the input plane of the virtual element, and point o_1' is the corresponding center. The output plane of the virtual element and its center o_2' can be obtained by translating the input plane and its center o_1' by a distance l along the z' axis. We have vertical planes of the z' axis respectively over the beam waist center o' and point o_{3m}'. These two planes are the source plane and the receiver plane of the virtual system with respective centers o' and o_3'. Obviously, o_3' and $o_{3\ m}'$ are the same point.

The light path in the virtual system remains unchanged compared with that in the actual system. Meanwhile, the beam propagation direction overlaps with the virtual system axis, the actual optical axis is converted to the misaligned optical axis, the actual element is converted to the misaligned virtual element, and the beam tilt can be translated to the two-dimensional misalignment of the virtual element. Consequently, in the small scope of the tilt and off-axis, the propagation of the beam's oblique incidence on the actual element in the actual system is converted to the propagation of the beam's coaxial incidence on the misaligned virtual element in the virtual system.

As is shown in Fig. 3, in the virtual system the bean travels through a plane parallel layer of thickness u in a uniform medium of n_1 from plane RP_0 to plane RP_1, passes through the virtual element with the ray transfer matrix $\underrightarrow{\mathbf{M}}$ from plane RP_1 to plane RP_2, and ultimately reaches plane PR_3 through a plane parallel layer of thickness v in a uniform medium of n_2. Accordingly, the ray transfer matrix of the virtual system can be expressed as

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \tag{7}$$

where

$$A = \frac{n_2 a + cv}{n_2},\ B = \frac{n_2 au + n_1 n_2 b + cuv + n_1 dv}{n_1 n_2},\ C = c,\ D = \frac{cu + n_1 d}{n_1} \tag{8}$$

with

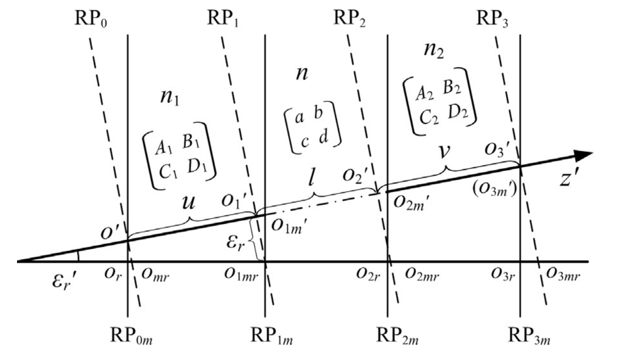


Fig. 3. Schematic of the virtual optical system. The subscript r represents the meridional or sagittal direction. RP_0, RP_1, RP_2, and RP_3 are the source plane, input and exit planes of the virtual element, and receiver plane of the virtual optical system, respectively. RP_{0m}, RP_{1m}, RP_{2m}, and RP_{3m} are, respectively, the source plane, input plane, exit plane, and the receiver plane of the actual system. ε_r and ε_r' are the corresponding misalignments. n is the refractive index of the element. Line $o_r o_{3mr}$ is the projection of the z axis in plane $x'o'z'$ or $y'o'z'$.

$$\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} 1 & u/n_1 \\ 0 & 1 \end{pmatrix},\ \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} = \begin{pmatrix} 1 & v/n_2 \\ 0 & 1 \end{pmatrix} \tag{9}$$

which are the ray transfer matrices of the plane parallel layers on both sides of the virtual element.

2.3. Calculation of misalignments and other system parameters

In view of the spin angle ζ, the coordinate axes (the y' and y_1' axes) of coordinate systems $x'o'y'$ and $x_1'o_1'y_1'$ are deviated from the reference planes (RP_{1m} and RP_{2m}) of the actual system, so that the direct calculation of related parameters is tedious and complicated. Therefore, we first determine the parameters in the reference coordinate system $x_{1s}'o_1'y_{1s}'$, followed by the relations between the parameters in the reference coordinate system $x_{1s}'o_1'y_{1s}'$ and those in the tilted beam coordinate system $x_1'o_1'y_1'$.

As shown in Fig. 4, the tilted beam coordinate system $x_1'o_1'y_1'$ can be obtained by rotating the reference coordinate system $x_{1s}'o_1'y_{1s}$ by ζ around the z' axis. The misalignments ε_x, ε_x', ε_y, ε_y' in the coordinate system $x_1'o_1'y_1'$ and ε_{xs}, ε_{xs}', ε_{ys}, ε_{ys}' in the coordinate system $x_{1s}'o_1'y_{1s}$ have the following relationship:

$$\begin{aligned} &\varepsilon_x = \varepsilon_{xs}\cos\zeta + \varepsilon_{ys}\sin\zeta,\ \varepsilon_x' = \varepsilon_{xs}'\cos\zeta + \varepsilon_{ys}'\sin\zeta \\ &\varepsilon_y' = -\varepsilon_{xs}'\sin\zeta + \varepsilon_{ys}'\cos\zeta,\ \varepsilon_y = -\varepsilon_{xs}\sin\zeta + \varepsilon_{ys}\cos\zeta. \end{aligned} \tag{10}$$

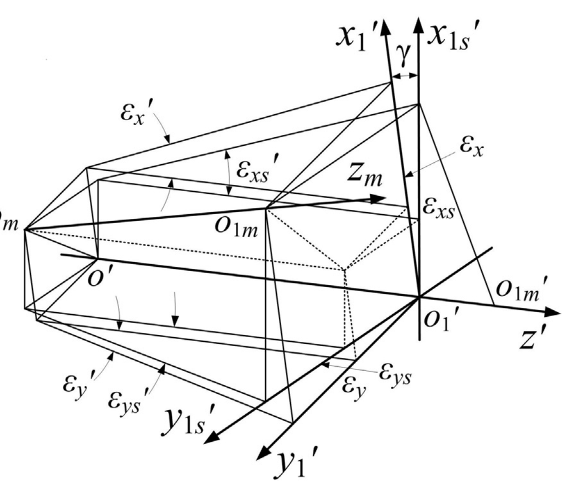


Fig. 4. Conversion of misalignments in the tilted beam coordinate system $x_1'o_1'y_1'$ and the reference coordinate system $x_{1s}'o_1'y_{1s}$. $x_{1s}'o_1'y_{1s}'$ is the reference coordinate system for the input plane of the virtual element. ε_{xs}, ε_{xs}' are the linear and angular misalignments along the x_{1s}axis, and ε_{ys}, ε_{ys}' are those along the y_{1s}axis.

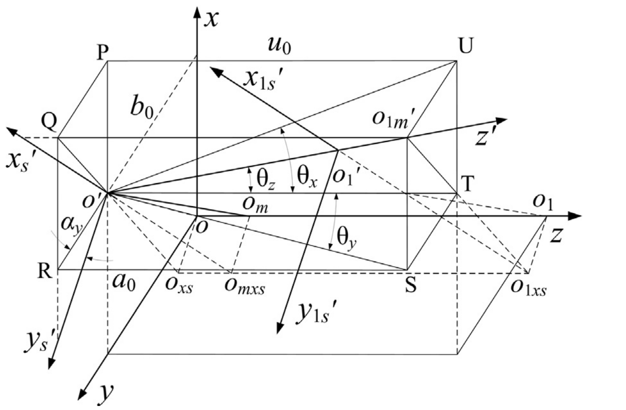


Fig. 5. Schematic of parameters in the reference coordinate system $x_{1s}'o_1'y_{1s}$. o'PQR-o_{1m}'STU is a cuboid with sides parallel to the coordinate axes of $oxyz$. Line $o_{xs}o_{1xs}$ is the projection of line oo_1 in plane $x_s'o'z'$. The initial offset in the x_s' and y_s' axes can be expressed as $\delta_{xs}= o_{mxs}o'$ and $\delta_{ys}= o_m o_{mxs}$, respectively. The misalignments can be expressed as $\varepsilon_{xs}= o_1'o_{1xs}$, $\varepsilon_{ys}= o_{1xs}o_1$, $\varepsilon_{xs}'=-\theta_z$ and $\varepsilon_{ys}'=0$.

When the tilted beam rotates around the z' axis, planes RP_0, RP_1, RP_2, and RP_3 of the virtual system, as well as their corresponding centers, remain fixed; only the coordinate systems rotate around the z' axis. Consequently, parameters u and v in the virtual system remain the same.

As shown in Fig. 5, the z axis can be proved to be parallel to plane $x_s'o'z'$. Thus, line oo_1 is parallel to its projection in plane $x_s'o'z'$. The y_s' axis is proved in plane xoy. We obtain parallels of the y_s' axis through points o and o_1, and their intersections o_{xs} and $o_{1xs}$with the extended lines of lines Qo' and o_{1m}'T, respectively. Lines oo_{xs} and o_1o_{1xs} are proved in planes xoy and $x_1o_1y_1$, respectively. Line $o_{1xs}o_{xs}$ can then be proved to be the projection of line oo_1 in plane $x_s'o'z'$. The reverse extension line of the x_s' axis and line $o_{xs}o_{1xs}$ intersect at point o_{mxs}. Line $o_{mxs}o_m$ is parallel to line oo_{xs} through point o_{mxs}. Point o_m, which can be proved to be the intersection of plane $x_s'o'y_s'$ and the z axis, can then be established. We obtain the parallel line $o_{1xs}o_1'$ of the x_s' axis through point o_{1xs} and its intersection o_1' with the z' axis. Line $o_1'o_{1xs}$ is extended, and the parallel line of the y_s' axis is drawn through point o_1'. In this way, the reference coordinate system $x_{1s}'o_1'y_{1s}'$ for the input plane of the virtual element can be obtained.

The formula for plane RP_0 in the coordinate system $oxyz$ and the z-axis intercept ($c_z=oo_m=o_{xs}o_{mxs}$) are

$$\tan\theta_x x + \tan\theta_y y + z - (a_0\tan\theta_x + b_0\tan\theta_y) = 0, \tag{11}$$

$$c_z = a_0\tan\theta_x + b_0\tan\theta_y. \tag{12}$$

The positive directions of the initial offset distances δ_{xs} and $\delta_{ys}$are opposite to those of the x_s' and y_s' axes, respectively. Based on Fig. 5,

$$\delta_{xs} = \frac{c_z}{\sin\theta_z} = \frac{a_0\tan\theta_x + b_0\tan\theta_y}{\sin\theta_z} = \frac{a_0\theta_x + b_0\theta_y}{\theta_z},$$
$$\delta_{ys} = \frac{b_0\tan\theta_x - a_0\tan\theta_y}{\tan\theta_z} = \frac{b_0\theta_x - a_0\theta_y}{\theta_z} \tag{13}$$

Given that line $o_{xs}o_{3mx}$ is the projection of the z axis in plane $x_s'o'z'$, and it is parallel to the z axis, in Fig. 3, we have $o_r o_{1mr}=u_0$, $o_{1mr}o_{2r}=l$, $o_{2r}o_{3r}= v_0$. u and v can then be calculated in Fig. 3 as follows:

$$u = u_0\cos\theta_z - \delta_{xs}\sin\theta_z\cos\theta_z = (u_0 - a_0\theta_x - b_0\theta_y)(1 - \frac{\theta_z^2}{2}), \tag{14}$$

$$v = \frac{u_0 + l}{\cos\theta_z} - (u + l) + \frac{v_0}{\cos\theta_z}$$
$$= u_0\theta_z^2 + l\frac{\theta_z^2}{2} + (a_0\theta_x + b_0\theta_y)(1 - \frac{\theta_z^2}{2}) + v_0(1 + \frac{\theta_z^2}{2}). \tag{15}$$

Given that line oo_1 is parallel to its projection in plane $x_s'o'z'$, ε_{ys}' is 0. The positive direction of ε_{xs}' and that of θ_z are opposite. According to Fig. 5 and Fig. 2,

$$\varepsilon_{xs} = -u\tan\theta_z - \delta_{xs} = -u_0\theta_z - (\frac{a_0\theta_x + b_0\theta_y}{\theta_z})(1 - \theta_z^2),$$
$$\varepsilon_{ys} = -\delta_{ys} = \frac{a_0\theta_y - b_0\theta_x}{\theta_z},\ \ \varepsilon_{xs}' = -\theta_z,\ \ \varepsilon_{ys}' = 0. \tag{16}$$

By substituting these quantities into Formula (10), ε_x, ε_x', ε_y, and ε_y' can be acquired. Thus, all the parameters ε_x, ε_x', ε_y, ε_y', u, and v required for the virtual system are obtained.

3. Diffraction integral for misaligned optical system with angular momentums

Given that the virtual system is a misaligned optical system, a suitable diffraction integral for a misaligned optical system is needed to calculate the optical field distribution in the receiver plane of the virtual system.

The diffraction integral of a misaligned optical system in [19,20] was deduced with the theories proposed by Arnaud [17] and Kogelnik [18], and the refractive indexes n_1, n_2 on the two sides of the element are simplified to n_1=n_2=1, which means the diffraction integral can only be used to obtain calculations of limited precision for element exposed to air.

In this section, we apply the matrix optics theories proposed by Gerrard [15] and Dragt [16] with angular momentums. A more complete and precise derivation process is presented. Thus, the diffraction integral formula of a misaligned optical system generated in this study can be applied to scenario wherein n_2 and n_1 are not limited to 1, and produce a more accurate result.

In the virtual system in Fig. 3, the subscript r represents either the horizontal or vertical direction. The paraxial ray can be specified by the position parameter r and the angular parameter p_r. r_{1v} and p_{r1v} are the parameters at the input plane RP_1 of the virtual element, whereas r_{2v} and p_{r2v} are those at the exit plane RP_2 of the virtual element. We can introduce parameters r_{1m} and p_{r1m} at RP_{1m} as well as r_{2m} and p_{r2m} at RP_{2m} of the misaligned virtual element, which signifies the actual element. In the first order approximation, we have

$$\begin{pmatrix} r_{2m} \\ p_{r2m} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} r_{1m} \\ p_{r1m} \end{pmatrix}, \tag{17}$$

$$r_{1m} = r_{v1} - \varepsilon_r,\ \ p_{r1m} = p_{r1v} - n_1\varepsilon_r',\ \ r_{2m} = r_{2v} - \varepsilon_r - l\varepsilon_r',\ \ p_{r2m} = p_{r2v} - n_2\varepsilon_r'. \tag{18}$$

Formula (18) indicates the scope the tilt and off-axis of the tilted beam. By denoting $\boldsymbol{r}$=(r, p_r)$^{\mathrm{T}}$ and combining Formulas (17) and (18), we can express the parameters in the following matrix form:

$$\boldsymbol{r}_{2v} = \underset{\longrightarrow}{\mathbf{M}}\boldsymbol{r}_{1v} + \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} \varepsilon \\ \varepsilon' \end{pmatrix}, \tag{19}$$

where α, β, γ and δ are the elements of the misalignment matrix, that are given by

$$\alpha = 1 - a,\ \ \beta = l - n_1 b,\ \ \gamma = -c,\ \ \delta = n_2 - n_1 d. \tag{20}$$

To satisfy the requirements of matrix multiplication, Formula (19) can be rewritten in the following augmented matrix form:

$$\boldsymbol{r}_{2v}^{(4)} = \underset{\longrightarrow}{\mathbf{M}^{(4)}}\boldsymbol{r}_{1v}^{(4)} \tag{21}$$

where

$$\boldsymbol{r}^{(4)} = \begin{pmatrix} r \\ p_r \\ 1 \\ 1 \end{pmatrix},\ \ \ \underset{\longrightarrow}{\mathbf{M}^{(4)}} = \begin{pmatrix} a & b & \alpha\varepsilon_r & \beta\varepsilon'_r \\ c & d & \gamma\varepsilon_r & \delta\varepsilon'_r \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{22}$$

r_1 and p_{r1} are the parameters at the source plane RP_0 of the virtual element, and r_2 and p_{r2} are those at the receiver plane RP_3 of the virtual element. The matrix for a plane parallel layer with thickness l in a uniform medium with a refractive index n becomes

$$\underset{\longrightarrow l}{\mathbf{M}}^{(4)}(l, n) = \begin{pmatrix} 1 & l/n & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{23}$$

From RP_0 to RP_3, the ray transfer matrix $\underset{\longrightarrow}{\mathbf{M}'}^{(4)}$of the virtual system can be obtained by relating parameters u, v, n_1, and n_2.

$$\boldsymbol{r}_2^{(4)} = \underset{\longrightarrow}{\mathbf{M}'}^{(4)}\boldsymbol{r}_1^{(4)} \tag{24}$$

$$\underset{\longrightarrow}{\mathbf{M}'}^{(4)} = \underset{\longrightarrow l}{\mathbf{M}}^{(4)}(v, n_2)\underset{\longrightarrow}{\mathbf{M}}^{(4)}\underset{\longrightarrow l}{\mathbf{M}}^{(4)}(u, n_1) \tag{25}$$

Therefore,

$$\underset{\longrightarrow}{\mathbf{M}'}^{(4)} = \begin{pmatrix} A & B & \alpha_T\varepsilon_r & \beta_T\varepsilon_r' \\ C & D & \gamma_T\varepsilon_r & \delta_T\varepsilon_r' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{26}$$

$$\alpha_T = \alpha + \frac{\gamma v}{n_2},\ \ \beta_T = \beta + \frac{\delta v}{n_2},\ \ \gamma_T = \gamma,\ \ \delta_T = \delta \tag{27}$$

where A, B, C, D are given by Formula (8), and α_T, β_T, γ_T, δ_T are the elements of misalignment matrix for the total system.

According to reference [19] the eikonal function of a misaligned system can be written as

$$L = L_0 + \varphi r_1 + \psi r_2 + \frac{1}{2}\Phi r_1^2 - \Omega r_1 r_2 + \frac{1}{2}\Psi r_2^2, \tag{28}$$

where L_0 is the optical path length from RP_0 to RP_3 along the z' axis. The coefficients of the quadratic terms depend only on the properties of the virtual system, and the linear terms account for the variations caused by misalignment.

According to the formula in references [16,21],

$$\frac{\partial L}{\partial r_1} = -r_1',\ \ \frac{\partial L}{\partial r_2} = r_2'. \tag{29}$$

The slope of a ray in a medium can be evaluated as the radial gradient of the eikonal divided by the refractive index of the medium [19]. According to the definition of "momentum" $\boldsymbol{p}$, r' corresponds to the component p_r of the angular momentum $\boldsymbol{p}$, that is,

$$\frac{\partial L}{\partial r_1} = -p_{r1},\ \ \frac{\partial L}{\partial r_2} = p_{r2}. \tag{30}$$

Substituting Formula (28) into the Formula (30), we obtain

$$-p_{r1} = \varphi + \Phi r_1 - \Omega r_2,\ \ p_{r2} = \psi + \Psi r_2 - \Omega r_1,$$

Hence,

$$r_2 = \frac{1}{\Omega}(\Phi r_1 + p_{r1} + \varphi),\ \ p_{r2} = (\frac{\Phi\Psi}{\Omega} - \Omega)r_1 + \frac{\Psi}{\Omega}p_{r1} + \frac{\psi\varphi}{\Omega} + \psi. \tag{31}$$

By comparing with $(r_2, p_{r2})^{\mathrm{T}}$ derived by Formula (24), namely,

$$r_2 = Ar_1 + Bp_{r1} + \alpha_T\varepsilon_r + \beta_T\varepsilon_r',\ \ p_{r2} = Cr_1 + Dp_{r1} + \gamma_T\varepsilon_r + \delta_T\varepsilon_r'. \tag{32}$$

the coefficients of the eikonal function for the virtual system can be obtained as follows:

$$\Phi = \frac{A}{B},\ \ \Omega = \frac{1}{B},\ \ \Psi = \frac{D}{B},\ \ \varphi = \frac{(\alpha_T\varepsilon_r + \beta_T\varepsilon_r')}{B},$$
$$\psi = \frac{[(B\gamma_T - D\alpha_T)\varepsilon_r + (B\delta_T - D\beta_T)\varepsilon_r']}{B}. \tag{33}$$

Therefore, the eikonal function for the virtual system can be given by

$$L = L_0 + \frac{1}{2B}[Ar_1^2 - 2r_1r_2 + Dr_2^2 + 2(\alpha_T\varepsilon + \beta_T\varepsilon')r_1 +$$
$$2\{(B\gamma_T - D\alpha_T)\varepsilon + (B\delta_T - D\beta_T)\varepsilon'\}]. \tag{34}$$

According to References [21,22], the complete diffraction integral for a misaligned optical system can be expressed as

$$E_2(x_2, y_2) = \frac{i}{\lambda_0 B}\exp(-ikL_0)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}E_1(x_1, y_1)\exp\left\{-\frac{1}{2B}[A(x_1^2 + y_1^2) - 2(x_1x_2 + y_1y_2) + D(x_2^2 + y_2^2) + ex_1 + fy_1 + gx_2 + hy_2]\right\}dx_1dy_1, \tag{35}$$

with

$$e = 2(\alpha_T\varepsilon_x + \beta_T\varepsilon_x'),$$
$$f = 2(\alpha_T\varepsilon_y + \beta_T\varepsilon_y'),\ g = 2[(B\gamma_T - D\alpha_T)\varepsilon_x + (B\delta_T - D\beta_T)\varepsilon_x'],\ h = 2[(B\gamma_T - D\alpha_T)\varepsilon_y + (B\delta_T - D\beta_T)\varepsilon_y'], \tag{36}$$

where λ_0 is the wavelength in vacuum, x_1, y_1 are coordinates in the coordinate system $x'o'y'$ for plane RP_0, and x_2, y_2 are coordinates in the coordinate system $x_3'o_3'y_3'$ for plane RP_3. A, B, C, D are given by Formula (8), and α_T, β_T, γ_T, δ_T are given by Formula (27).

When a beam passes through a tilted element, the foci of the meridional and sagittal planes are separated, resulting in so-called astigmatism [23]. In addition, the matrices in the meridional and sagittal planes are different [24]. Given that the misalignments in the virtual element are small, we can disregard the aberrations in the meridian and sagittal planes in this paper as in references [19,20]. Accordingly, matrices of the meridional and sagittal planes remain unchanged.

4. Optical field distribution of tilted TEM_{00} Gaussian beam in actual optical system

A tilted TEM_{00} Gaussian beam is set to impinge onto the actual optical system. We can study the propagation of the tilted beam in the virtual system, and calculate the optical field distribution at the receiver plane of the virtual system. The optical field distribution of the tilted beam in the actual system can then be obtained.

As shown in Fig. 1, point o' is the waist center of the tilted TEM_{00} Gaussian beam, the propagation axis is the z' axis, and (a_0, b_0), θ_x, θ_y, u_0, v_0 are parameters for the tilted beam. ε_x, ε_x', ε_y, ε_y', u and v have been discussed in Section 2. According to the conclusions about the spin angle ζ in Section 2.1, we can use the generic coordinate system $o'x'y'z'$ to study the propagation of the tilted TEM_{00} Gaussian beam. The spin angle ζ does not affect the final result. For the tilted TEM_{00} Gaussian beam, ζ is 0. Given that high-order Gaussian beams can be analyzed by embedding a fundamental mode [25], the method and conclusions for tilted TEM_{00} Gaussian beams are also applicable for high-order ones.

The optical field distribution of the tilted TEM_{00} Gaussian beam at the source plane RP_0 of the virtual system is as follows:

$$E(x_1, y_1, z_1) = A_0\left(\frac{iZ_R}{q_1}\right)\exp\left[-ik(\frac{x_1^2 + y_1^2}{2q_1}) - ikz_1\right], \tag{37}$$

$$Z_R = \frac{\pi\omega_0^2}{\lambda}, \tag{38}$$

$$q_1 = z_1 + iZ_R, \tag{39}$$

where x_1, y_1, z_1 are coordinates in the coordinate system $o'x'y'z'$ of plane RP_0; A_0 is the amplitude constant; ω_0 is the beam waist radius; Z_R is the Rayleigh length; q_1 is the complex curvature radius. In this case, z_1=0.

By substituting Formula (37) into Formula (35), the optical field distribution of the tilted beam at the receiver plane RP_3 of the virtual system can be obtained as follows: .

$$E_2(x_2, y_2) = \frac{iA_0 Z_R}{Aq_1 + B}\exp(-ikL_0 + z_1)\exp\left[\frac{-ik}{2q_2}(x_2 - \frac{e}{2})^2 - \frac{ik}{2B}(\mathrm{De} + g)x_2 + \frac{ikDe^2}{8B}\right] \times \exp\left[\frac{-ik}{2q_2}(y_2 - \frac{f}{2})^2 - \frac{ik}{2B}(\mathrm{Df} + h)x_2 + \frac{ikDf^2}{8B}\right], \tag{40}$$

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}, \tag{41}$$

$$L_0 = n_1 u + n_2 v + nl, \tag{42}$$

where x_2,y_2 are coordinates in the coordinate system $x_3'o_3'y_3'$ of plane RP$_3$; q_2 is the complex curvature radius of the beam along the z' axis after the virtual element; L_0 is the optical path length along the z' axis; n is the refractive index of the element.

According to references [26–28], the beam described in Formula (40) is a decentered Gaussian beam along the z' axis. Thus the tilted beam is transformed into a decentered Gaussian beam along the propagation direction after passing through the misaligned virtual element (i.e., the actual element).

The peak intensity of the decentered Gaussian beam lies on a straight line called the peak intensity axis. The peak intensity axis is regarded as the z'' axis. The position deviations $x_d(z')$ and $y_d(z')$ at RP$_3$ are the coordinates where the z'' axis intersects with plane RP$_3$ in the coordinate system $x_3'o_3'y_3'$. The angular momentum deviations $\varepsilon_x(z')$ and $\varepsilon_y(z')$ at RP$_3$ are the components of the angular momentum(the product of the angle and the refractive index n_2) between the z'' axis and the z' axis along the $x_3'$and y_3' axes. $x_d(z')$, $y_d(z')$, $\varepsilon_x(z')$,$\varepsilon_y(z')$ can be given by

$$x_d(z') = \frac{e}{2} = \alpha_T\varepsilon_x + \beta_T\varepsilon_x',$$
$$\varepsilon_x(z') = \frac{De + g}{2B} = \gamma_T\varepsilon_x + \delta_T\varepsilon_x' y_d(z') = \frac{f}{2} = \alpha_T\varepsilon_y + \beta_T\varepsilon_y',$$
$$\varepsilon_y(z') = \frac{Df + h}{2B} = \gamma_T\varepsilon_y + \delta_T\varepsilon_y'. \tag{43}$$

Substituting Formulas (20) and (27) into Formula (43) yields the following results:

$$\varepsilon_x(z') = n_2\varepsilon_x' - (c\varepsilon_x + dn_1\varepsilon_x'),\ x_d(z') = (\varepsilon_x + l\varepsilon_x') - (a\varepsilon_x + bn_1\varepsilon'_x) + v\frac{\varepsilon_x(z')}{n_2} = x_{d0}(z') + v\frac{\varepsilon_x(z')}{n_2},\ \varepsilon_y(z') = n_2\varepsilon_y' - (c\varepsilon_y + dn_1\varepsilon_y'),\ y_d(z') = (\varepsilon_y + l\varepsilon_y') - (a\varepsilon_y + bn_1\varepsilon_y') + v\frac{\varepsilon_y(z')}{n_2} = y_{d0}(z') + v\frac{\varepsilon_y(z')}{n_2}. \tag{44}$$

where x_{dO} (z'), y_{dO} (z') are the position deviations of the z'' axis at the exit plane RP$_2$ of the virtual system.

The deviations at RP$_3$ have linear relationships with the misalignments. The angular momentum deviations do not vary with the propagation distance v after the virtual element. Thus $x_{dO}(z')$, $y_{dO}(z')$, $\varepsilon_x(z')$, $\varepsilon_y(z')$ can be regarded as the deviations at the exit plane RP$_2$ of the virtual system.

The first terms of the deviations at plane RP$_2$ show that the peak intensity axis is transferred from the z' axis to the z axis, and that the intensity center moves to center of the exit plane of the actual element. We can define this part of the deviations as the axis deviation $\boldsymbol{\Delta z}$. The x' or y' direction can be represented by the subscript r, and the component of $\boldsymbol{\Delta z}$ in either direction can be given by

$$\Delta_{zr} = \begin{pmatrix} \varepsilon_r + l\varepsilon_r' \\ n_2\varepsilon_r' \end{pmatrix} \tag{45}$$

The second terms of the deviations at plane RP$_2$ show that the virtual element compensates for the peak intensity axis. Treating $(-\varepsilon_r, -n_1\varepsilon_r')^{\mathrm{T}}$ as the misalignment of the z' axis relative to the z axis, the compensations and the misalignments satisfy the matrix $\underrightarrow{\mathbf{M}}$. We can define this part of the deviations as the element compensation $\boldsymbol{\Delta e}$. Similarly, the component of $\boldsymbol{\Delta e}$ in either direction can be given by

$$\Delta_{er} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -\varepsilon_r \\ -n_1\varepsilon'_r \end{pmatrix} \tag{46}$$

To obtain the optical field distribution in the actual system, plane RP$_3$ can be set to be floating along z' axis, as well as parameter v in the virtual system. Thus, a point(x, y, z) in the coordinate system $o_2 \times_2 y_2 z$ behind the exit plane RP$_{2m}$ of the actual element can be set in plane RP$_3$. This point can then correspond to point (x_2, y_2, 0) in the coordinate system $o_3'x_3'y_3'z'$. The coordinates in the two coordinate systems meet the coordinate transformation relation R(ζ, θ_z, α_y):

$$\begin{pmatrix} x' \\ y' \\ v + u + l \end{pmatrix} = R(\zeta, \theta_z, \alpha_y)\begin{pmatrix} x - a_0 \\ y - b_0 \\ z + u_0 + l \end{pmatrix} = Z'(\zeta)Y'_s(\theta_z)Z(\alpha_y)\begin{pmatrix} x - a_0 \\ y - b_0 \\ z + u_0 + l \end{pmatrix} \tag{47}$$

where α_y,θ_z, ζ, as mentioned before, are the precession, nutation, and spin angles of the tilted beam coordinate system $o'x'y'z'$ relative to the coordinate system $oxyz$. Z(α_y), Ys'(θ_z), and Z'(ζ) are the rotation operators for the rotations around z, y_s', and z' axes by α_y, θ_z, and ζ, respectively. R(ζ, θ_z, α_y) is as follows:

$$R(\zeta, \theta_z, \alpha_y) = Z'(\zeta)Y_s'(\theta_z)Z(\alpha_y) = \begin{pmatrix} \cos\zeta\cos\theta_z\cos\alpha_y - \sin\zeta\sin\alpha_y & \cos\zeta\cos\theta_z\sin\alpha_y + \sin\zeta\cos\alpha_y & -\cos\zeta\sin\theta_z \\ -\sin\zeta\cos\theta_z\cos\alpha_y - \cos\zeta\sin\alpha_y & -\sin\zeta\cos\theta_z\sin\alpha_y + \cos\zeta\cos\alpha_y & \sin\zeta\sin\theta_z \\ \sin\theta_z\cos\alpha_y & \sin\theta_z\sin\alpha_y & \cos\theta_z \end{pmatrix} \tag{48}$$

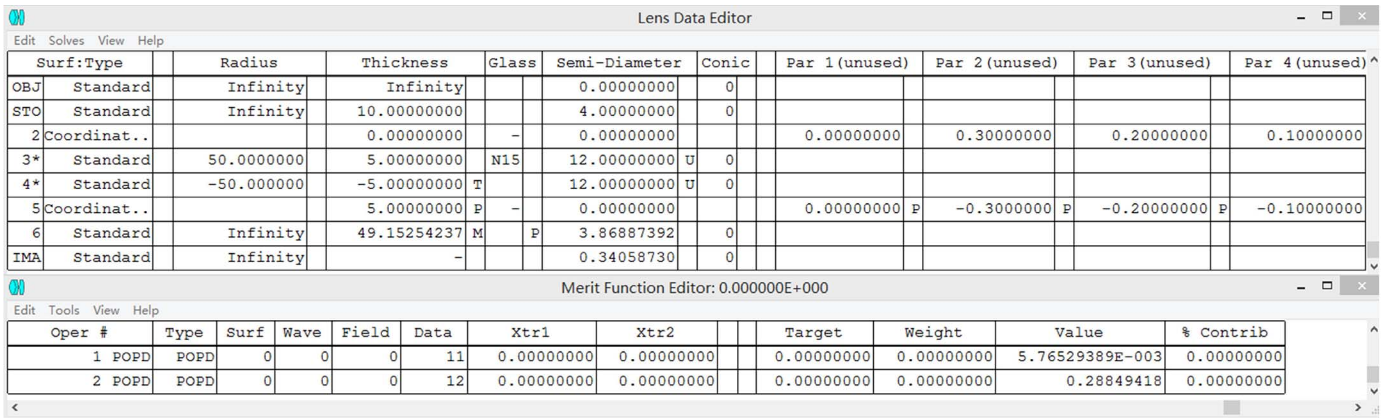
Substituting Formulas (47) and (48) into Formula (40), we can obtain the optical field distribution E_2 (x, y, z) for any point (x, y, z) in the coordinate system $o_2 \times_2 y_2 z$ behind the actual element.

5. Simulation analysis

By using ZEMAX, we can easily create a virtual optical system along the beam propagation direction to simulate a tilted beam passing through a thick lens system exposed to air. The tilted beam, after passing through the thick lens, is transformed into a decentered Gaussian beam according to the conclusion in Section 4. We can obtain the simulation results of the position deviations for the decentered Gaussian beam at the receiver plane of the virtual system. The calculations of Formula (44) are validated through these results, and the relative errors between them are presented.

In ZEMAX, the default system axis is aligned with the default beam propagation direction. By using the native function “Tilt/Decenter Element” of ZEMAX, the thick lens becomes tilted relative to the default system axis. Thus, we obtain two systems. One is the system along the symmetry axis of the tilted element, which is the actual system. The other is the system along the beam propagation direction, which is the required virtual system. In this system, the values of all the parameters, including the misalignments, can be directly set in the Lens Date Editor. By using Dates 11 and 12 in operand POPD in the Merit Function Editor, we can generate the coordinates of the spot center in the local coordinate system for the receiver plane of the virtual system. According to Formulas (40) and (43), the coordinates of the spot center are the position deviations (Fig. 6). Data of the thick lens and Merit Function in ZEMAX simulation are shown in Fig. 6.

ε_x,ε_y range in [−0.5, 0.5]mm with step size 0.1 mm, and ε_x',ε_y'



Lens Data Editor

Edit Solves View Help

Surf:Type	Radius	Thickness	Glass	Semi-Diameter	Conic	Par 1(unused)	Par 2(unused)	Par 3(unused)	Par 4(unused)
OBJ Standard	Infinity	Infinity		0.00000000	0				
STO Standard	Infinity	10.00000000		4.00000000	0				
2 Coordinat..		0.00000000	-	0.00000000		0.00000000	0.30000000	0.20000000	0.10000000
3* Standard	50.0000000	5.00000000	N15	12.00000000 U	0				
4* Standard	-50.000000	-5.00000000 T		12.00000000 U	0				
5 Coordinat..		5.00000000 P	-	0.00000000		0.00000000 P	-0.3000000 P	-0.20000000 P	-0.10000000
6 Standard	Infinity	49.15254237 M	P	3.86887392	0				
IMA Standard	Infinity	-		0.34058730	0				

Merit Function Editor: 0.000000E+000

Edit Tools View Help

Oper #	Type	Surf	Wave	Field	Data	Xtr1	Xtr2	Target	Weight	Value	% Contrib
1 POPD	POPD	0	0	0	11	0.00000000	0.00000000	0.00000000	0.00000000	5.76529389E-003	0.00000000
2 POPD	POPD	0	0	0	12	0.00000000	0.00000000	0.00000000	0.00000000	0.28849418	0.00000000

Fig. 6. Lens and Merit Function Data. In the Lens Data Editor, Thicknesses in Surf 1, 3 and 6 are *u, l and v* in the virtual system, respectively; Radii in Surf 3 and 4 represent the radii of curvature R_1 and R_2 of the thick lens, respectively; Par1, Par4, Par2, Par3 in Surf 2 correspond to the misalignments ε_x, ε_x', ε_y, -ε_y', respectively. In the Merit Function Editor, Values in the first and second rows are the simulation results $xd_s(z)$, $yd_s(z)$ for position deviations at the meridional and sagittal planes, respectively.

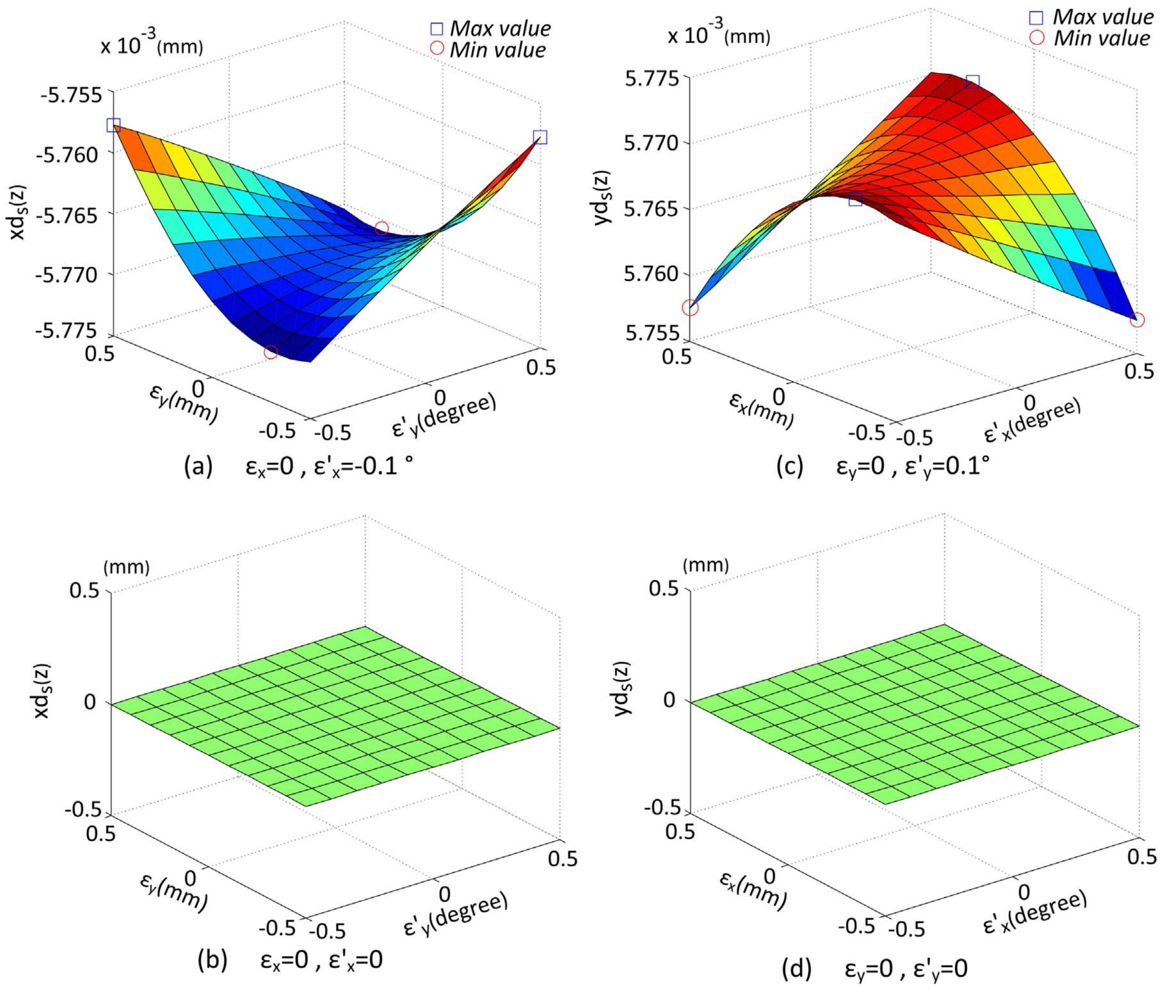


Fig. 7. Simulation results of position deviations. (a) and (b) show $xd_s(z)$ with the changes in ε_y, ε_y' when ε_x=0, ε_x'=−0.1° and ε_x=0, ε_x'=0. (c) and (d) show $yd_s(z)$ with the changes in ε_x, ε_x' when ε_y=0, ε_y'=0.1°and ε_y=0, ε_y'=0.

range in [−0.5, 0.5]° with step size 0.1°. When the misalignments ε_x, ε_x' at the meridional plane are fixed, the changes in misalignments ε_y, ε_y' at the sagittal plane can cause a small fluctuation in the simulation results $xd_s(z)$. Therefore, we can acquire 121 arrays of different ε_x, ε_x', with each group having 121 simulation results $xd_s(z)$ for the position deviations at the meridional plane. Similarly, we can generate 121 groups, with each group having 121 simulation results $yd_s(z)$ for the position deviations at the sagittal plane. As shown in Fig. 7, xd_s(z) and yd_s(z) are similar in distribution and fluctuation.

The thick lens is exposed to the air. Thus ZEMAX provides us with an accurate refractive index of air n_{air}=1.000269, as well as other required parameters. Based on the data of ZEMAX, the refractive indices on the two sides of the thick lens are n_1=n_2=1.000269; the refractive index of the thick lens is n =1.5; the radii of curvature for the front and back surf of the thick lens are R_1=0.05 m and R_2=−0.05 m, respectively; the thicknesses of v and l are v=0.04915254 m,

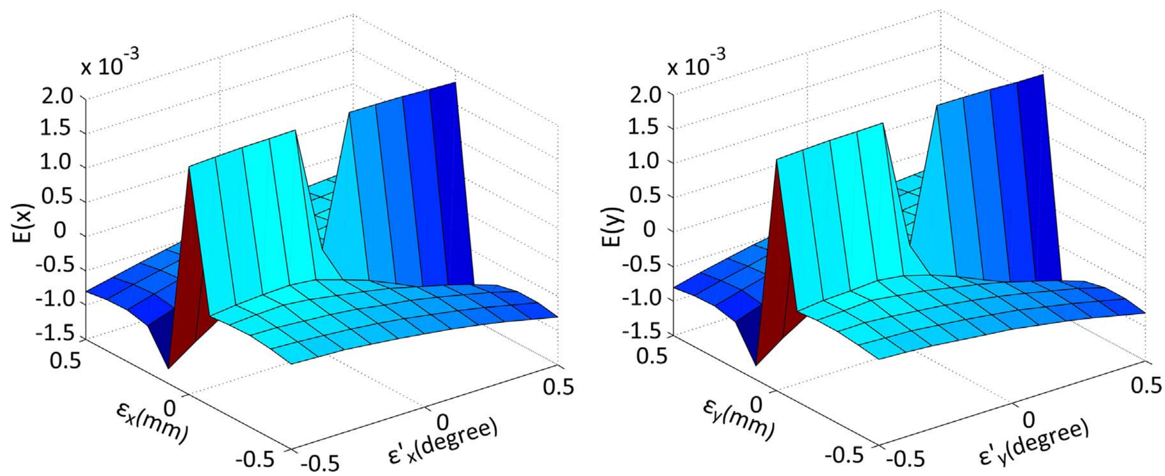


Fig. 8. Errors between the simulation results and the theoretical calculations of the position deviations at the meridional and sagittal planes.

l=0.005 m. The ray transfer matrix for the thick lens [19] is thus

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a_1 & b_1/n_1 \\ n_2 c_1 & n_2 d_1/n_1 \end{pmatrix} = \begin{pmatrix} a_1 & b_1/n_1 \\ n_2 c_1 & d_1 \end{pmatrix}, \tag{49}$$

where

$$a_1 = 1 - \frac{(n-1)l}{nR_1},\ \ b_2 = \frac{l}{n},\ \ c_1 = (n-1)\left(\frac{1}{R_2} - \frac{1}{R_1} - \frac{(n-1)l}{nR_2R_1}\right),$$
$$d_1 = 1 + \frac{(n-1)l}{nR_2}. \tag{50}$$

ε_x, ε_x', ε_y, ε_y' in the theoretical calculation are the same with those in the simulation. Then all the required parameters in the virtual optical system of the thick lens are acquired. Substituting these parameters into the Formula (44), we obtain 121 theoretical calculations $xd_t(z)$ of the position deviations at the meridional plane and 121 theoretical calculations $yd_t(z)$ for position deviations at the sagittal plane.

We identify the maximum and minimum values in each group of $xd_s(z)$, and analyze the errors between the two selected simulation results and the theoretical calculation with the same ε_x, ε_x'. In each group of ε_x, ε_x', the error whose absolute value is larger is taken as a member of $E(x)$. $E(x)$ denotes the errors between the simulation results and the theoretical calculations of the position deviations at the meridiona plane.

$$E(x) = \begin{cases} \dfrac{\max(xd_s(z)) - xd_t(z)}{\max(xd_s(z))} \\ \dfrac{\min(xd_s(z)) - xd_t(z)}{\min(xd_s(z))} \end{cases} \tag{51}$$

Similarly, we generate $E(y)$ for the errors between the simulation results and the theoretical calculations of the position deviations at the sagittal plane.

$$E(y) = \begin{cases} \dfrac{\max(yd_s(z)) - yd_t(z)}{\max(yd_s(z))} \\ \dfrac{\min(yd_s(z)) - yd_t(z)}{\min(yd_s(z))} \end{cases} \tag{52}$$

Substituting the values in $xd_s(z)$, $xd_t(z)$, $yd_s(z)$, $yd_t(z)$ into Formulas (51) and (52), we can obtain the range of $E(x)$ and $E(y)$:

$$-1.286‰ \le E(x) \le 1.867‰, \quad -1.301‰ \le E(y) \le 1.905‰$$

As shown in Fig. 8 the errors between the simulation results and the theoretical calculations of the position deviations are less than 2‰ when the misalignments range between [−0.5, 0.5] mm and [−0.5,0.5]° at the meridional and sagittal planes.

The errors are caused by the aberration, and can be further reduced by reducing the misalignments. Thus we can obtain favorable results for a tilted beam with the tilt and off-axis of the first order approximation by using the theory of virtual optical system.

6. Conclusion

In this study, we establish a virtual optical system that is aligned with the propagation direction of the tilted beam under the premise of maintaining the transformation properties of the beam. Thus, we can easily study the propagation of a tilted beam in the virtual system instead of the actual system. For beams within the first order approximation of the tilt and off-axis, the theory can be generalized to a complicated optical system that composed by two or more elements, and that may be aligned or not. And the theory can also be used to acquire a more reliable precision analysis of incoherent beam combination, such as wavelength multiplexing.

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