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# Note: Design and capability verification of fillet triangle flexible support

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By increasing the section thickness of a triangular flexible hinge, this study focuses on optimal selection of parameters of fillet triangle flexible hinges and flexible support. Based on Castigliano's second theorem, the flexibility expression of the fillet triangle flexible hinge was derived. Then, the case design is performed, and the comparison of three types of flexible hinges with this type of flexible hinge was carried out. The finite element models of fillet triangle flexible hinges and flexible support were built, and then the simulation results of performance parameters were calculated. Finally, the experiment platform was established to validate analysis results. The maximum error is less than 8%, which verifies the accuracy of the simulation process and equations derived; also the fundamental frequency fits the requirements of the system. The fillet triangle flexible hinge is proved to have the advantages of high precision and low flexibility. *Published by AIP Publishing.* https://doi.org/10.1063/1.5010229

With the development of space optical communication, the precision requirement for the fast steering mirror (FSM) system and flexible hinge becomes higher. As a kind of rotary kinematic pair, the flexible hinge rotates by taking advantages of elastic deformation and self-recovery characteristics of the material.[1] The flexible hinge has the advantages of no mechanical friction, less clearance, compact structure, high sensitivity, and so on.[2] So it has been widely used in high precision systems such as the FSM system.[3]

For classical hinges, such as straight circular and triangle flexible hinges, there is much research. Lin *et al.*[4] did the finite element analysis of the straight circular flexible hinge, elliptic flexible hinge, and the fillet flexible hinge and inferred that the triangle flexible hinge is less sensitive to the deformation of the non-working direction. Smith *et al.*[5] acquired the stiffness expression of the circular hinge and used the finite element method to verify the theoretical derivation.

In many cases, such as fine tracking, the precision of the system is very high, and a flexible hinge with higher motion accuracy and lower flexibility is more suitable. By increasing the section thickness of the triangle flexible hinge, this study focuses on optimal selection of shape parameters of the fillet triangle flexible hinge and flexible support. It is a new kind of flexile hinge proposed in 2010[6] and is expected to have lower flexibility and higher accuracy than these flexible hinges widely used before under the same parameters.

As shown in Fig. 1, a schematic diagram of force and structure of three types of hinges widely used and fillet triangle flexible hinges is listed. Type A is a corner-filleted rectangular flexible hinge, type B is a elliptical flexible hinge, type C is a triangular flexible hinge, and type D is a fillet triangle flexible hinge.

The main parameters for these flexible hinges are as follows: width $b$, minimum thickness $t$, cutting depth $a$, length $L$, and radii $r$ and $R$. The main properties of flexible hinges include rotation flexibility, rotation accuracy, and maximum stress. Among them, rotation flexibility and rotation accuracy are considered more often.

Based on the working condition of the flexible hinge, it can be assumed that the deformation is very small and there is no interference between the deformation of different directions; the hinge can be regarded as a cantilever beam; the angle deformation of the hinge is produced by the bending moment and force; only the influence of an axial load is considered, and the shear and torsional effects are ignored.[7]

Small angle displacement $\theta_{1z}$ of the flexible hinge is engendered under the action of the bending moment $M_{1Z}$ and force $F_{1y}$. $u_{1x}$ and $u_{1y}$ are small linear displacement along the X axis and Y axis under the action of the bending moment $M_{1Z}$ and force $F_{1y}$. Based on Castigliano's second theorem, the relationship between the deformation of the hinge and the load is

$$\begin{Bmatrix} u_{1x} \\ u_{1y} \\ \theta_{1z} \end{Bmatrix} = \begin{bmatrix} C_{x-F_x} & 0 & 0 \\ 0 & C_{y-F_y} & C_{y-M_z} \\ 0 & C_{\theta-F_y} & C_{\theta-M_z} \end{bmatrix} \begin{Bmatrix} F_{1x} \\ F_{1y} \\ M_{1z} \end{Bmatrix}. \quad (1)$$

The stiffness matrix is a symmetric matrix, so $C_{\theta-F_y} = C_{y-M_z}$. Apply Castigliano's second theorem to displacement vector, $u_{1x} = \partial U/\partial F_{1x}$, $u_{1y} = \partial U/\partial F_{1y}$, and $\theta_{1z} = \partial U/\partial M_{1z}$. $U$ is the deformation energy of flexible hinge, and its expression is

$$U = \frac{1}{2}\left(\int \frac{F_x^2}{EA(x)} d_x + \int \frac{M_z^2}{EI_z(x)} d_x\right), \quad (2)$$

where E is the elastic modulus of the hinge; $F_x$ is the force along the X direction; $M_z$ is the torque around the Z axis; $I_z(x)$ is the moment of inertia around the Z axis: $I_z(x) = bh^3(x)/12$; $A(x)$ is the cross-sectional area of the hinge, $A(x) = bh(x)$; and $h(x)$ is the section thickness of the hinge.

$F_x$ and $M_z$ can be calculated by

$$F_x = F_{1x}, M_z = M_{z1} + F_{y1}(L - x). \quad (3)$$

Substituting Eqs. (2) and (3) into Eq. (1) and solving it, the solution of Eq. (1) is

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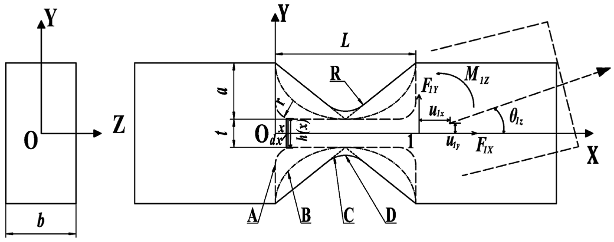


FIG. 1. Different types of flexure hinges under same parameters.

$$\begin{cases} C_{x-F_x} = \dfrac{1}{Eb}\displaystyle\int_0^L \dfrac{dx}{h(x)}, \\ C_{y-F_y}(\text{point 1}) = \dfrac{12}{Eb}\displaystyle\int_0^L \dfrac{x^2dx}{h(x)^3}, \\ C_{y-M_z} = C_{\theta-F_y} = -\dfrac{12}{Eb}\displaystyle\int_0^L \dfrac{xdx}{h(x)^3}, \\ C_{\theta-M_z} = \dfrac{12}{Eb}\displaystyle\int_0^L \dfrac{dx}{h(x)^3}(\text{rotational flexibility}). \end{cases} \quad (4)$$

Substitute thickness expression of fillet triangle flexible hinge into Eq. (4); however, the flexibility function is non-integrable, so only numerical results can be obtained when parameters are determined.

Under the action of the bending moment and tension force, the center of rotation will be shifted. The Y-axis flexibility of center point can represent its rotation accuracy, and it can be derived from Eq. (4),

$$C_j = C_{y-F_y}(\text{center point}) = \frac{12}{Eb}\int_0^{L/2} \frac{x^2dx}{h(x)^3}. \quad (5)$$

The maximum stress of the flexible hinge is located at the position which has the minimum cutting thickness. Taking the influence of stress concentration into account, assuming the stress concentration factor is k, according to the stress expression of the beam under bending load, the maximum stress can be obtained as follows:

$$\sigma_{y\max} = k\cdot\frac{M}{W_Z} = \frac{kM}{t^2b/6} = \frac{6kM}{t^2b}, \quad (6)$$

where $M$ is the bending load and $W_z$ is the flexural modulus of the rectangular section. Suppose that the maximum deflection angle is $\beta$, the bending load can be calculated by $M_z = K_{\theta_z,M_z}\cdot\beta = \beta/C_{\theta-M_z}$. k can be obtained from Ref. 8 $(\zeta+0.253)/(\zeta+0.097)$, where $\zeta = \rho/t$, $\rho$ is the curvature radius of the section curve which has the maximum stress.

The expression of the maximum stress can be obtained by substituting $k$ into Eq. (6),

$$\sigma_{y\max} = \frac{6\beta(\zeta+0.253)}{t^2bC_{\theta-M_z}(\zeta+0.097)}. \quad (7)$$

In a real case design, the flexible support rotating range should be large than ±3 mrad in the X-axis and the Y-axis, and the first order resonant frequency should be larger than 100 Hz. The material of the hinge is titanium alloy Tc10, its allowable stress is 160 MPa, and its elastic modulus is 106 GPa. According to the actual using background and processing capability, the range of parameters are determined: $L \geq 10$, $b \leq 8$ mm, $a \leq 5$ mm, and $t \leq 5$ mm.

Since the influence of parameter $b$ is the same to all kinds of hinges, the integral factor $12/Eb$ is outside the integral symbol, and when $b$ reaches its maximum value, the flexibility is the lowest, so $b$ should be 8 mm. The flexibility and rotational accuracy of different types of hinges under same $L$ and $a$ when $t$ = 3 mm are compared in Fig. 2 (though have not been mentioned, when $t$ changes, the relative magnitude of flexibilities of these hinge are similar).

The fillet triangle flexible hinge (type D) has the lowest flexibility and highest rotational accuracy. So it is more suitable for the FSM system of fine tracking. When $L$ = 10 mm and $a$ = 5 mm, the flexibility reaches its lowest value. So the parameters $b$, $L$, and $a$ can be determined.

According to the formulation derived, the influence of $R$ and $t$ to rotational flexibility rotary accuracy and maximum stress are shown in Fig. 3.

It can be concluded that the influence of $R$ and $t$ to rotational flexibility and rotary accuracy is similar, but to maximum stress it is opposite.

Considering the limit for the maximum stress, there exist numerous solutions. To make the maximum stress as large as possible, choose $R = 0.6$, $t = 0.6$, and at this time, the maximum stress was 137.28 MPa, a little less than allowable stress. Under these parameters the rotational flexibility is 0.052 rad $N^{-1}$ $m^{-1}$ and the rotary accuracy is 0.0064 mm $N^{-1}$ $m^{-1}$.

In order to verify the accuracy of the numerical solution, the finite element analysis of the fillet triangle flexible hinge and flexible support was performed. In the model, the torque was loaded at the free end and the angle deflection was obtained. The comparison of simulation results with numerical results of the flexible hinge is shown in Table I, and simulation

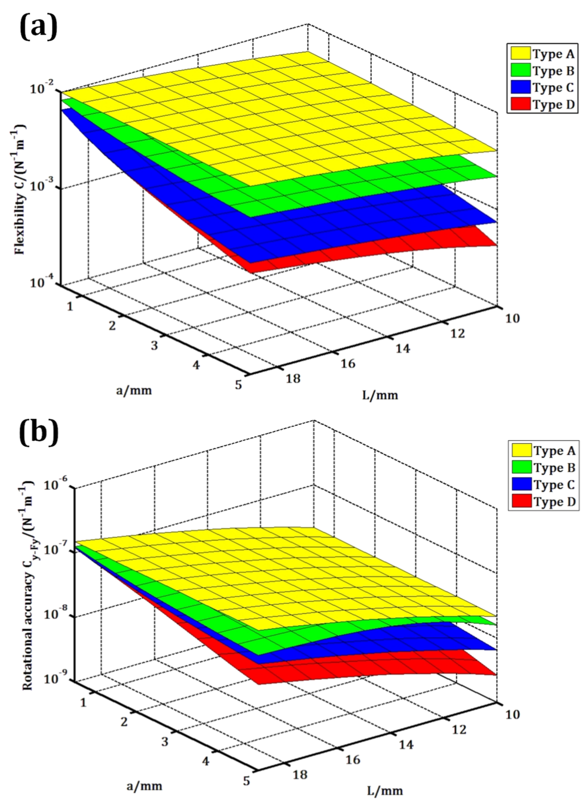


FIG. 2. The comparison of (a) flexibility, (b) rotational accuracy (when t = 3 mm).

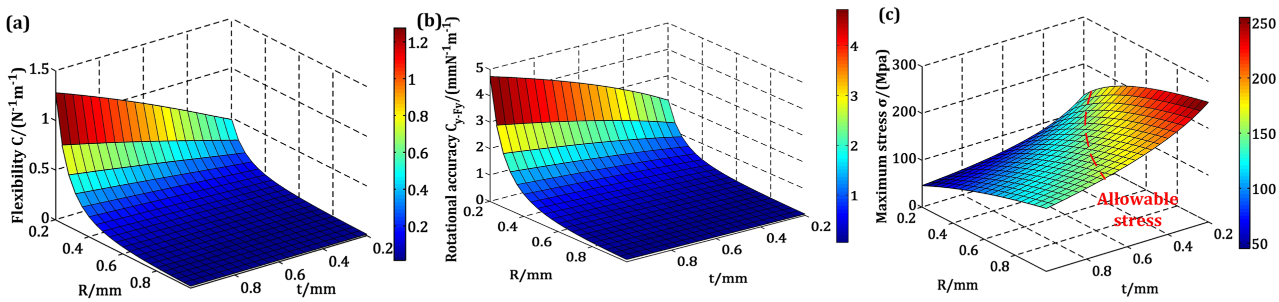


FIG. 3. The influence of R and t to (a) flexibility, (b) accuracy, (c) maximum stress.

TABLE I. Comparison of property parameters' simulation result with theoretical result.

| Property parameter | Numerical result of hinge | Simulation result of hinge | Error (%) | Simulation result of support |
|---|---|---|---|---|
| Rotational flexibility/rad $N^{-1}$ $m^{-1}$ | 0.052 | 0.0558 | 6.8 | 0.0305 |
| Rotary accuracy/mm $N^{-1}$ $m^{-1}$ | 0.0064 | 0.0068 | 5.9 | 0.0067 |
| Maximum stress/MPa | 137.28 | 141.99 | 3.4 | 141.68 |

results for flexible support are also listed. It can be seen from the table that the errors of simulation results of the flexible hinge with the theoretical results are within 8%, and the simulation results can confirm the numerical results.

In order to verify the accuracy of the simulation results, the flexibility of flexible support was verified experimentally. The main experimental equipments are the fillet triangle flexible support, base support, beam with a v-shaped groove, string, weight, and inductance micrometer. The experimental platform is shown in Fig. 4.

The principle of the experiment is to apply a bending moment with an amount of $mg\,L$ at the upper end of the flexible support. Under the action of the bending moment, the flexible support will have angle deformation. Because the deformation $\theta_x$ is very small, the rotational flexibility is $\Delta y/mgL^2$.

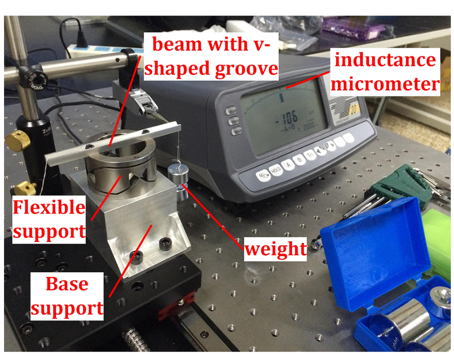


FIG. 4. Platform for flexible support flexibility test.

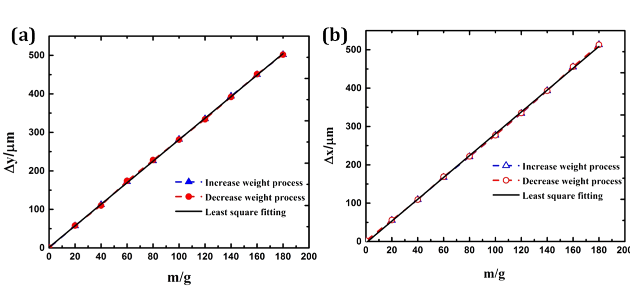


FIG. 5. Flexible support deformation around (a) X-axis and (b) Y-axis.

The X-axis deformation curve of the flexible support is fitted by the least squares method as shown in Fig. 5(a). The flexibility is $C_{\theta-Mz}$ = 0.0286 rad $N^{-1}$ $m^{-1}$. Also the Y-axis deformation curve is shown in Fig. 5(b). The flexibility is $C_{\theta-Mz}$ = 0.0290 rad $N^{-1}$ $m^{-1}$. Comparing the simulational results with the experimental results, the errors between the experimental results and the simulational results are 6.6% and 5.2%, also less than 8% and the errors are relatively small. This proves that the simulation results have high accuracy.

In order to verify the response frequency of the FSM system, the FSM structures were assembled and connected to the controller. The frequency scanning experiment was carried out. Load the sine sweep signal to the voice coil motor to drive the mirror in two working directions. The fast Fourier transform of the response signal can be obtained by the experiment. The first order resonant frequencies on the two working directions are 122 Hz and 160 Hz, respectively, and both meet the requirement of the FSM system.

In conclusion, the fillet triangle flexible hinge is proved to have a high degree of rotational accuracy and low flexibility and can better meet the engineering needs, which provides the reference for the design of the high precision flexible hinge.


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