



## Original research article

## Ellipticity pivot star method for autonomous star identification

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## ARTICLE INFO

## Article history:

Received 30 November 2016

Accepted 20 February 2017

## Keywords:

Star identification

Star catalog

Singular value decomposition

Star tracker

Attitude determination

## ABSTRACT

A new fast algorithm for autonomous star identification in the general lost-in-space case is developed based on optimized catalog. The proposed method takes advantage of singular value decomposition method and the accuracy angular separation information. The central idea is that a unique pattern is created for each of guide star so that the pattern recognition is simple and straightforward and the index entry reaches minimum. The unique pattern is comprised of ellipticity and angular separations between pivot star and adjacent stars. The method for selecting adjacent stars of pivot star and how to rank them is presented. Three series of simulations each of which included more than ten thousand star tracker orientations were performed by dividing the entire celestial sphere into small regions. The results support the validity of the proposed method that achieves higher identification rate with 99.99%.

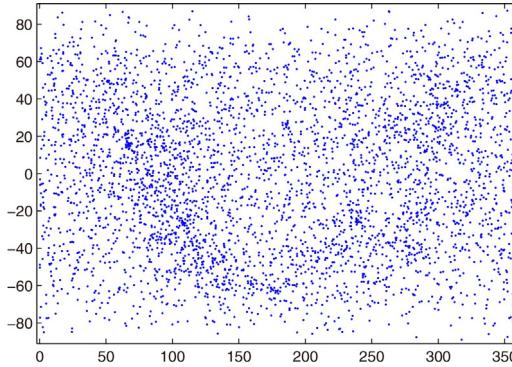
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## 1. Introduction

Star tracker is intentionally invented for attitude determination in both orbiting and interplanetary spacecraft by sensing constellation. It has been one of the most critical components for spacecraft's guidance, navigation and control. Conventionally, the process of attitude determination consists of these main steps: star centroiding, feature extraction, star identification and attitude determination. Star identification is the key point of the star tracker, and many individuals have developed algorithms for star identification [1]. Most of the algorithms identify each of the stars in the image, and then convert the locations of the identified stars to unit vectors for attitude estimation of satellites. The crux of star identification is to create features that independent on coordinate transformation. The first candidate of the invariant features selected by many researchers is angular distance, for instance, Gottlieb's polygon method [2], Liebe's method [3], Mortari's Pyramid [4], Kolomenkin's geometric voting algorithm [5], and some neural networks methods [6,7]. Pattern recognition was introduced by Padgett and Kreutz-Delgado [8], who developed the grid algorithm which using star pattern matching method by dividing the image into a grid and a matrix is formed with zeros and ones, depending on whether a star exists in each grid element. This method is different from the above methods because they use the star "pattern" not the traditional angular distance as the feature. But the grid method requires more observed stars in image, has to rotate star image and has larger catalog. The aforementioned methods have some common points: (1) large catalog; (2) the matching algorithm is complicated. In 2004, Juang et al. [9] presented a simple and inexpensive attitude initialization method for solving the lost-in-space problem by introducing

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**Fig. 1.** Guide star distribution.

singular values. The first version mission catalog was generated by choosing the 4 brighter stars at each bore sight direction. But pattern recognition may fail due to magnitude ranking error or lack of bore sight direction. Then in 2012, the second version mission catalog [10] was constructed by expanding the number of bore sight direction to avoid experiencing lack of bore sight direction. Yin et al. [11] also created a catalog based on cost function defined by  $M_i/D_i$ , where  $M_i$  is brightness and  $D_i$  is the distance between reference star and pivot star. But Yin also uses the magnitude information of stars since star magnitude information is relatively inaccurate. In this paper, a new catalog is generated based on ellipticity and angular separation that creating unique patterns for each of  $M$  guide stars which require less memory. Following this section a brief introduction of singular value decomposition (SVD) is presented. Selecting stars method based on angular separation and uniqueness of star pattern are investigated in Section 3. Later, simulation results are given and conclusions are provided.

## 2. Singular value decomposition method

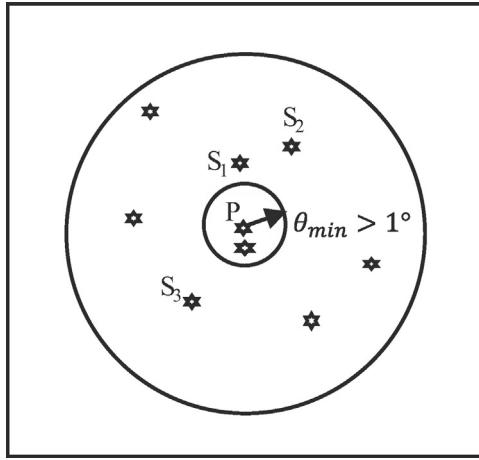
The singular value decomposition method is one of the most robust estimators in the attitude determination developed by Markley [12]. Then it was introduced by Juang et al. [9] for attitude initialization. The advantage of this method is that the pattern recognition and optimal attitude estimation can be performed simultaneously. The idea of this method lies that the singular values are invariant under coordinate transformation. The linear algebra tells us that if  $A \in C^{m \times n}$  has rank  $r$ , then it can be written in the form  $A = P\Lambda Q^T$ , where  $P \in C^{m \times m}$  and  $Q \in C^{n \times n}$  are orthogonal. The  $\Lambda$  is diagonal matrix with singular values  $\sigma_i > 0$  ( $i = 1, 2, \dots, r$ ). The columns of  $P$  and  $Q$  are eigenvectors of  $AA^T$  and  $A^TA$ , respectively. So if  $A = RB$ , where  $R$  is orthogonal matrix, then  $A$  and  $B$  have identical singular values and right singular vectors except for its signs. For more details see [9,10].

## 3. Pattern recognition and star catalog generation

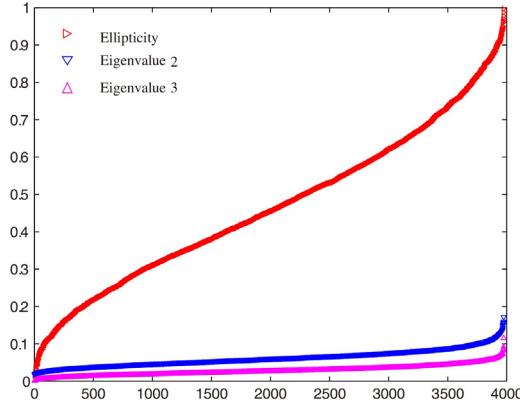
The pattern recognition process is simple and straightforward for original SVD method. Only singular values are compared with those in the catalog. The key point is for the method that how to select stars that must be robust to measurement noise guaranteeing the right rank of observed stars and cover the entire celestial sphere. Juang and Wang [10] choose four brighter stars within FOV to begin the identification. After performing the SVD of the selected for brighter stars, the second- and third-singular-value sensitivities are matched with those candidates in the mission catalog. However, the ranking errors of the four selected stars are fatal to attitude determination especially when the magnitudes of stars are close together. The star magnitude information is relatively inaccurate due to the noise or stray light. This is one of the reasons why the original SVD method experiences failures. It is wise to create star patterns using accurate information, for instance angular separation, other than magnitude information.

### 3.1. Unique pattern features created

The proposed mission catalog is created by taking advantage of SVD method and accurate angular separation for large FOV star tracker (e.g.,  $20^\circ \times 20^\circ$ ). First, select brighter stars than 6.0 Mv from SAO star database. Then, double stars, changing stars and too close stars less  $0.2^\circ$  are discarded. The remaining  $N=3974$  stars comprise the elementary guide star catalog. Fig. 1 presents the elementary guide star distribution. Each of  $N=3974$  stars is considered as pivot star. Angular separations between pivot star and adjacent stars are calculated. Then, the first 3 closer adjacent stars no less than the minimum angle  $\theta_{min} > 1^\circ$  are chosen as shown in Fig. 2. The accuracy angular separation information guarantees the correct rank of the pivot star and the adjacent stars. A matrix  $A$  is formed by the direction of pivot star and that of the first 3 closer adjacent stars:  $A = [\vec{v}_1 \quad \vec{v}_2 \quad \vec{v}_3 \quad \vec{v}_4]$ , where  $\vec{v}_i$  are direction unit vectors of pivot stars and adjacent stars, respectively. Perform SVD for matrix  $A$ , then 3 singular values are obtained. The largest singular values of all  $3 \times 4$  matrixes make no contribution to pattern recognition because they all are close to 2. The variation range of the second and third singular values are also small



**Fig. 2.** Illustration of selecting adjacent stars.



**Fig. 3.** Distribution of ellipticity and eigenvalues.

which make it difficult to select threshold and accomplish identification task (see Fig. 3). As it is explained by [9] that these three singular values are three semi-axis lengths of along their principle axes. Here an ellipticity is defined by the two latter singular values:

$$\epsilon = \frac{\sigma_3}{\sigma_2} \quad (1)$$

The distribution of ellipticity and eigenvalues are shown in Fig. 3. The ellipticity and angular separation are organized as pattern features (Table 1). When performing pattern recognition, the matching range is narrowed down by the ellipticity and these four features make up unique star patterns. The threshold of ellipticity and angular separation is 0.001 and 0.02°, respectively. The total storage requirement of mission catalog of the proposed method and the original SVD method is shown in Table 2. Table 2 shows that the storage requirement of proposed method is less than that of original method [9].

### 3.2. Pattern recognition

When observed stars are obtained, choose a pivot star which locates at the nearer center of image. This process is very simple because the direction of bore sight is always  $\bar{w}_0 = [0 \ 0 \ 1]^T$  and that of observed star is  $\bar{v}_i =$

**Table 1**  
Pattern features.

$\epsilon$	$\theta_{p1}$	$\theta_{p2}$	$\theta_{p3}$	Pivot star	Adjacent star 1	Adjacent star 2	Adjacent star 3
$\epsilon_1$	$\theta_{p11}$	$\theta_{p12}$	$\theta_{p13}$	$p_1$	$s_{11}$	$s_{12}$	$s_{13}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\epsilon_N$	$\theta_{pN1}$	$\theta_{pN2}$	$\theta_{pN3}$	$p_N$	$s_{N1}$	$s_{N2}$	$s_{N3}$

**Table 2**

Storage requirement of mission catalog.

	Original method	Proposed method
Guide star	$N \times 3$	$N \times 3$
Index matrix	$\sim 2 \times N \times 4$	$N$
Pattern features	$\sim 2 \times N \times 4$	$N \times 4$

**Table 3**

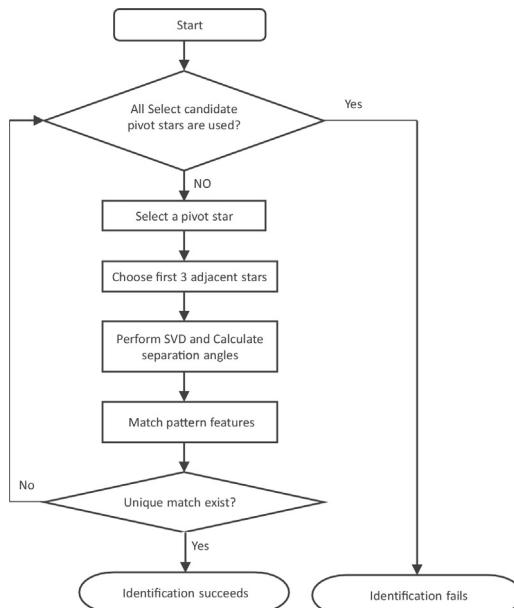
Simulation results.

$\Delta\text{RA}$	$\Delta\text{DE}$	No. of simulations	No. of fails	No. adjacent stars < 3	Success rate
1°	0.5°	128,877	0	14	100%
0.5°	0.1°	1,284,101	1	160	99.99999%
0.1°	0.05°	12,823,161	2	160	99.99999%

$[-x_i \ -y_i \ f]^T / \sqrt{x_i^2 + y_i^2 + f^2}$ , so the nearest star to the bore sight is the star that has the smallest value of  $\sqrt{x_i^2 + y_i^2 + f^2}$ . Then select the first 3 closer adjacent stars. Note that separation angles between the pivot stars and the bore sight are no larger than a critical value, for example 4°. A set of four pattern stars which consist of a pivot star and three adjacent stars are obtained. Perform the SVD of the four pattern stars and register the angles between the pivot stars and the three adjacent stars. The ellipticity and angular separations are matched with those candidates in the mission catalog. The four pattern features guarantee the unique set of four mission catalog stars. When the pattern recognition is accomplished, the attitude can be estimated by the left singular vectors of the set of four pattern stars and the unique set of four mission catalog stars. Fig. 4 shows the flow chart of pattern recognition. To make this algorithm more robust to fake stars, more than 3 adjacent stars can be chosen (e.g., 4–5 adjacent stars) since the matching process is very fast.

### 3.3. Simulation results

Three series of simulations have been performed to test the identification rate. The Monte Carlo simulations are not selected, instead the entire sphere is divided into grids by the right ascension and the declination. The intersections of axes of the right ascension and the declination are chosen as the bore-sights. The grids are so small that the FOV of star camera covers several grids and when the bore-sight moves to the next grid node it makes little difference to the FOV of star camera. The first series of simulations were performed 128,877 times with each region has an interval of 1° between two adjacent right ascensions and 0.5° between two adjacent declinations. The attitude estimation was performed via Euler angles. For simplicity, the declination range is –89° to 89°. The results show that pattern recognition is successfully performed for all the simulations. Among these 128,877 success pattern recognitions the number of the first selected pivot stars that have no

**Fig. 4.** Flow chart of pattern recognition.

more than 3 adjacent stars is 14. The identification rate is 100% for 128,877 bore sights. The two other series of simulations are listed in [Table 3](#). The results show that the identification rate of proposed method are better than the conventional SVD method [9] and Yin's method [11].

#### 4. Conclusion

A new algorithm is developed for star identification in general “lost space” case. This algorithm is based on a set of unique star pattern features which are created by ellipticity and the accuracy of star position information. It is independent of brightness and requires less storage and the matching process is simple and very fast. To validate the proposed method, three series of simulations were conducted and the results show that the identification rate of proposed method achieves higher identification rate with 99.999%.

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