Generation of Airy beams using a phase-only Fresnel holographic lens

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A method to generate Airy beam by combining the Fresnel holographic lens and the cubic phase of Airy beam is proposed. The detailed theoretical derivation to express the optical transform principle of the proposed method is presented. And excellent experimental results are demonstrated. It is shown that this approach works well and simplifies the experimental facility effectively, especially reducing the optical system length to half of that of the conventional method. In addition, the proposed method can realize the beam propagation trajectory control of Airy beam and generate Airy beam array.

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Airy beam, one of non-diffractive beams, presents diffraction-free, self-healing and self-bending features in propagation. Due to these, Airy beam is widely applied to vacuum electron[1], optical trapping[2], microparticle clearing[3], surface plasmon polaritons[4], and free space optical communication[5]. It is well known that the Airy beam theory was first described by Balazs and Berry as a solution of the Schrödinger equation within the context of optics in 1979[6]. And the first experimental demonstration of Airy beams was given by Siviloglou in 2007[7], as they found the angular Fourier spectrum of Airy beam is Gaussian and involves a cubic phase. This type of spectrum implicates that an Airy beam can be generated through a Fourier transformation of a broad Gaussian beam when a cubic phase is imposed.

The conventional generation of an Airy beam is based on the Fourier transform method as shown in Fig.1(a), which defines phase plane and image plane, respectively. The distance between them and the Fourier lens is equal to the focal length of Fourier lens (f). In experiments, the cubic phase is loaded on the liquid crystal on silicon (LCoS) device to encode the input Gaussian beam at the phase plane, and then through the optical Fourier transform, a desired Airy beam is obtained at the image plane.

However, the optical system of the Fourier transform method has a rather large length of $2f$, and usually f is 1 m in experiment. The quality of optical Fourier lens is strictly required, which means a large cost. Recently, a generation method by directly encoding the Fresnel diffraction lens together with the Fourier transform phase of Airy beam onto the LCoS is demonstrated as described in Fig.1(b). Such an approach has been experimentally illustrated in Refs.[8, 9]. This type of method works well and reduces the system length to f, removing an expensive Fourier lens. However, no theoretical analysis on the generation principle of the formed Airy beams has been presented still now.

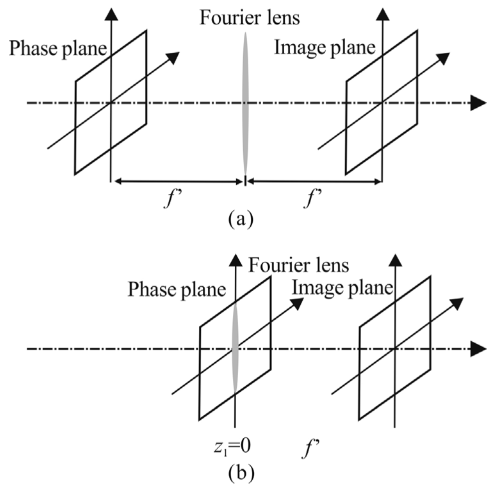

Fig.1 Schematic of optical systems: (a) The conventional Fourier transform method; (b) The Fresnel lens method

In this paper, the detailed theoretical derivation is given to express the principle of the Fresnel method. And the corresponding experimental demonstration based on this method only using a reflective LCoS device is presented. In addition, the further generation of Airy beam array is also presented. We perform our theoretical derivation from the finite power Airy beams expressed as

$$\varphi(s, \xi = 0) = Ai(s)\exp(as), \tag{1}$$

where φ stands for the electric field envelope of Airy beam, a

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is the decay factor which is a small positive parameter associated with the effective aperture of the system, ξ is a normalized propagation distance, s is the dimensionless transverse coordinate, and $Ai(\ )$ is the Airy function. The Fourier spectrum in k-space of Eq.(1) is given by

$$\Phi(k)\alpha\exp(ak^2)\exp(\frac{\mathrm{i}}{3}k^3). \quad (2)$$

From Eq.(2), we can see that the Fourier spectrum of Airy beam is Gaussian shape and consists of a cubic phase k^3. In a classical optical system as shown in Fig.1(a), the phase plane and the image are the corresponding Fourier transform planes. Thus, the Airy beam can be obtained in the image plane when the cubic phase is put in the phase plane.

As below, we derive the optical transform of a Fourier lens to show that when the phase plane overlaps with the Fourier lens, as shown in Fig.1(b), the image plane is also the Fourier transform plane. For general, as described in Fig.2, the phase plane P1 is assigned in front of the Fourier lens with a distance of z_1, the image plane is behind the Fourier lens with a distance of z_2, while P2 and P3 stand for the planes close to the surface of the Fourier lens.

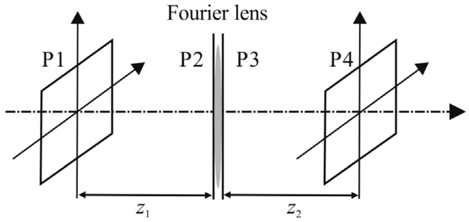


Fig.2 Schematic of the optical transform for a Fourier lens

The input beam in plane P1 is $U_1(x_1, y_1)$, and the propagation from P1 to P4 is divided into three processes. Firstly, the propagation from P1 to P2 can be described by the Fresnel diffraction theory as

$$U_2(x_2,y_2)=\int_{\infty}^{+\infty}\int U_1(x_1,y_1)\frac{1}{\mathrm{j}\lambda z}\exp(\mathrm{j}kz_1)\times$$
$$\exp\left\{\mathrm{j}\frac{\pi}{\lambda z_1}\left[(x_2-x_1)^2+(y_2-y_1)^2\right]\right\}\mathrm{d}x_1\mathrm{d}y_1. \quad (3)$$

Secondly, the propagation from P2 to P3 can be expressed as

$$U(x_3,y_3)=t(x_2,y_2)U(x_2,y_2)P(x_2,y_2)=$$
$$\exp\left[-\mathrm{j}\frac{k}{2f}(x_2^2+y_2^2)\right]U(x_2,y_2), \quad (4)$$

where $t(x,y)=\exp\left(\mathrm{j}kn\varDelta_0\right)\exp[-\mathrm{j}k(x^2+y^2)/2f]$ stands for the optical function of a Fourier lens with a focal length of f. $P(x_2, y_2)$ stands for the pupil function, where we take $P(x_2, y_2)=1$ for simplify. Finally, the propagation from P3 to P4 can be also described by the Fresnel diffraction theory as

$$U_4(x_4,y_4)=\int_{\infty}^{+\infty}\int U_3(x_3,y_3)\frac{1}{\mathrm{j}\lambda z_2}\exp(\mathrm{j}kz_2)\times$$
$$\exp\left\{\mathrm{j}\frac{\pi}{\lambda z_2}\left[(x_4-x_3)^2+(y_4-y_3)^2\right]\right\}\mathrm{d}x_3\mathrm{d}y_3. \quad (5)$$

Thus, the output beam can be described as

$$U_4(x_4,y_4)=-\frac{1}{\lambda^2 z_1 z_2}\exp\left[\mathrm{j}k(z_1+n\varDelta_0+z_2)\right]\times$$
$$\iint\iint_{-\infty}^{\infty}U_1(x_1,y_1)P(x_2,y_2)\times$$
$$\exp\left\{\mathrm{j}\frac{\pi}{\lambda z_1}\left[(x_2-x_1)^2+(y_2-y_1)^2\right]\right\}\times$$
$$\exp\left[-\mathrm{j}\frac{\pi}{\lambda f}(x_2^2+y_2^2)\right]\times$$
$$\exp\left\{\mathrm{j}\frac{\pi}{\lambda z_2}\left[(x_4-x_2)^2+(y_4-y_2)^2\right]\right\}\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}x_2\mathrm{d}y_2. \quad (6)$$

When the positions of the phase plane P1 and the Fourier lens overlap, namely z_1=0, the image plane P2 is at the focal plane of the Fourier lens, namely z_2=f. In this case, Eq.(6) can be further simplified to

$$U_4(x_4,y_4)=-\lambda f\exp[\mathrm{j}\frac{\pi}{\lambda f}(x_4^2+y_4^2)]\times$$
$$\mathcal{F}\left\{U_1(x_1,y_1)\right\}_{f_x=\frac{x}{\lambda f},f_y=\frac{y}{\lambda f}}. \quad (7)$$

Eq.(7) represents a Fourier transform function, whose phase is modulated by $\exp\left[-\mathrm{j}\pi(x_4^2+y_4^2)/\lambda f\right]$, where λf is a constant. For generation of Airy beam in image plane, we consider the amplitude term only as the phase term of Eq.(7) can be removed. Therefore, the image plane and the phase plane close to the Fourier lens in Fig.2 are in the relationship of Fourier transform, which is suitable for generation of Airy beam as interpreted above.

A Fresnel holographic lens uses the phase modulation method to realize the function of an optical lens[10-12]. The combination of the Fresnel phase function and the cubic phase of Airy beam can make the phase plane and the Fourier transform plane overlap. The Fresnel phase function is written as

$$\varphi=2\pi\times(x^2+y^2)\times z/\lambda f^2, \quad (8)$$

where f is the focal length, and z is the displacement of the image plane relative to the focus of the lens. Fig.3 illustrates the phase masks used to generate Airy beam, which are modulated into 2π, white stands for 255, while black stands for 0. The modulating depth of cubic phase is from -35π to 35π, and 1 920×1 080 pixels are sampled to support the LCoS device.

Fig.4 shows the schematic of experimental setup, and the operation of the optical system can be described as follows. Firstly, the He-Ne laser (wavelength is 632.8 nm) goes through a spatial filter system to expand the Gaussian beam waist to 50 mm. Then, the expanded beam passes through a polarizer P, which ensures the polarized direction of the incident beam parallel to the long axis of the liquid crystal molecules. Following that, the beam reaches the LCoS device[13,14], and the incident angle is about 5°. In phase-only modulation mode, the incident beam is modulated by the spatial light modulator (SLM), which is loaded with the phase mask in Fig.3. Finally, a 2D Airy beam can be captured by a CCD camera, which can axially scan along the propagation direction. The size of the captured image is 3 mm ×3 mm in this paper.

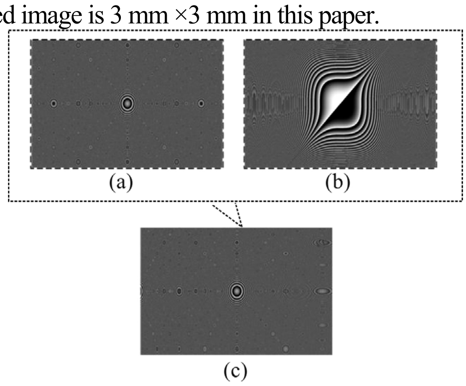
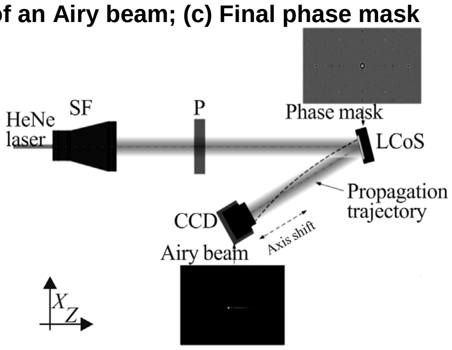


Fig.3 Illustration of phase masks to generate Airy beam: (a) Phase mask of a Fresnel lens; (b) Phase mask of an Airy beam; (c) Final phase mask



SF: spatial filter; P: polarizer; LCoS: reflective liquid crystal on silicon spatial light modulator with 1 920×1 080 pixels (The black dotted line denotes the propagation trajectory of the Airy beam.)

Fig.4 Schematic of the experimental setup to generate Airy beam

Fig.5 shows the captured intensity patterns of Airy beam in experiment. As expected, the Airy beam begins to be generated at 0.150 m utilizing the Fresnel holographic lens method. It is obvious to see that the beam remains almost diffraction-free during propagation and presents the quadratic accelerating. Fig.6 depicts the normalized intensity distributions of Airy beam at 0.150 m, 0.190 m and 0.213 m along longitudinal direction, respectively. It is noting that the profile of the Airy beam meets the Airy function basically, which directly proves the generation method we proposed is accurate. Fig.7 illustrates the transverse acceleration of the generated Airy beam. Clearly, the parabolic trajectory is the result of the "self-bending" and well meets the theoretical relation.

In most applications, we need to control propagation trajectory of Airy beam to achieve desired performance. Through adjusting the central position of the mask in Fig.3(c), the original position of the generated Airy beam can be controlled. In this optical control mechanism, no mechanical movement device is required. Fig.8 presents the experimental results of the control propagation trajectory of Airy beam, where (a1) and (b1) are the intensity patterns of a Fresnel lens, which we use as a reference. In this case, the moving directions of the original positions are the same with the mask, but the propagation trajectories are not changed as shown in Fig.7.

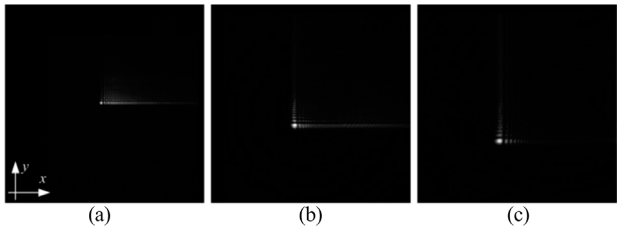


Fig.5 Experimental results of a single Airy beam: (a)—(c) Corresponding intensity patterns at 0.150 m, 0.190 m and 0.213 m behind the LCoS, respectively

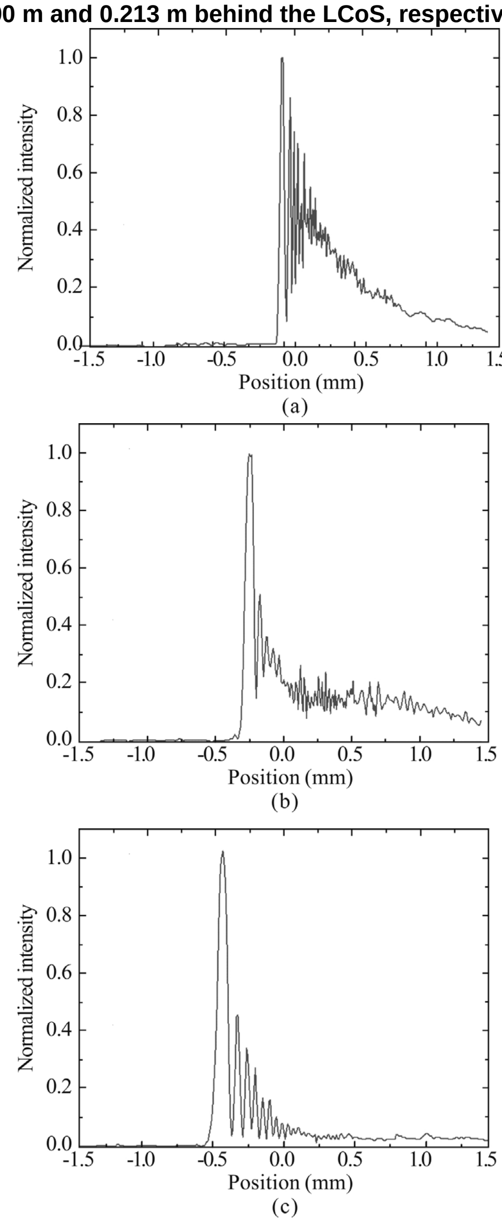


Fig.6 Intensity distributions of Airy beams at (a) 0.150 m, (b) 0.190 m and (c) 0.213 m along the longitudinal direction, respectively

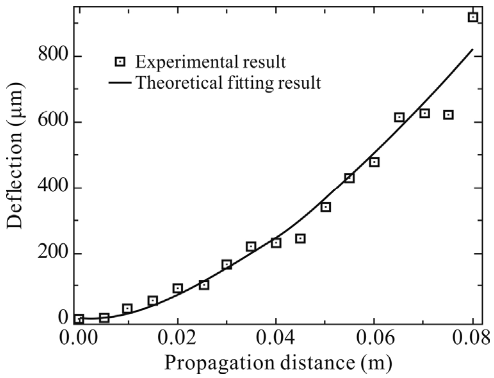


Fig.7 Transverse acceleration of the generated Airy beam

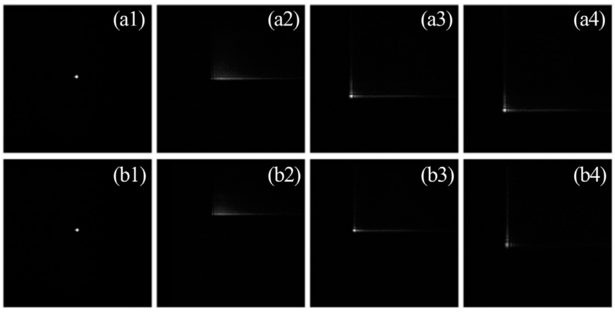


Fig.8 Experimental results of the control propagation trajectory of Airy beam: (a1) and (b1) The intensity patterns of a Fresnel lens with focal length of 150 mm; (a2)—(a4) The intensity patterns when removing the central mask position from origin to left by 1.44 mm; (b2)—(b4) The intensity patterns when removing the central mask position from origin to left by 1.44 mm then to upper by 1.44 mm

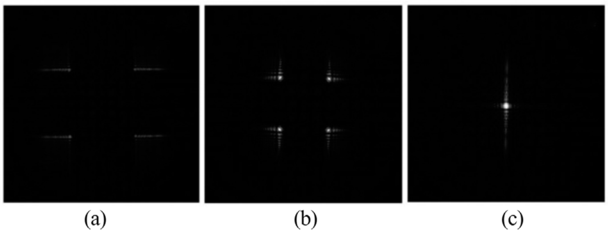


Fig.9 Experimental results of 4-Airy-beamlet array: (a)—(c) Corresponding intensity patterns at 0.150 m, 0.190 m and 0.213 m behind the LCoS, respectively

Airy beam array is suggested as one of the efficient solutions for scintillation reduction of beam propagation through atmospheric turbulence[15]. Fig.9 shows the experimental results of Airy beam array based on dividing the phase mask into four same small masks (960×540 pixels, generation method and calculation parameters are the same with those of Fig.3) and rotating them by 0°, 90°, 180° and 270°, respectively. It can be seen that with the propagation distance increases, the main lobe tends to accelerate towards the origin. And at the position of 0.213 m, the main lobes of four Airy beamlets overlap.

In conclusion, a method is proposed to experimentally generate Airy beam based on directly encoding the Fresnel diffraction lens together with the Fourier transform phase of Airy beam onto the LCoS. Corresponding theoretical derivation to express the optical transform principle of the proposed method is presented. The experimental demonstrations of a single Airy beam generation, propagation trajectory control of Airy beam and Airy beam array are given. Experimental results prove that this approach can work well. The generation method of Airy beam is useful to deeply study Airy beam in laboratory.

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