# Improved simulation method for urban free-space optical links based on the finite Markov state model

**GUO Shao-ai** (郭少艾)[1] **and LI Ya-tian** (李亚添)[2,3]*

*1.Chang Guang Satellite Technology Co., Ltd., Changchun 130001, China*

*2. Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun 130033, China*

*3.University of Chinese Academy of Sciences, Beijing 100049, China*

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An improved method is proposed to simulate the scintillation introduced by the turbulence, based on the finite Markov state model. As a contrast to the literatures, uniformly distributed variables take place during a certain state, which contributes to equivalent simulation of the intensity fluctuations with fewer states than the traditional Markov model. It's also discovered that the proposed Markov model with 20 states provides a satisfactory approximation to the experimental results in the auto-covariance analysis. Moreover, the outage probability and mean fading time are more accurate than those of the traditional Markov model with equivalent states.

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Recently, the free space optics (FSO) has aroused wide attention, which may be an efficient alternative to optical fiber communications. Due to the obvious advantages of enormous bandwidth, license-free operation and high security, it has various applications compared with radio frequency (RF) communications[1-3]. However, the FSO systems are seriously degraded by the environmental conditions, such as absorption, diffraction, scattering, etc., especially the atmospheric turbulence. For the sake of revealing the turbulence features, kinds of mathematical models were proposed, such as lognormal, Gamma-Gamma, and exponential turbulence[4-6]. Moreover, some turbulence models have been discussed latterly, containing but not limited to double-generalized Gamma model, model, and double-Weibull model[7-9].

The finite-state Markov chains have been considered as an effective tool to model communication channels with correlated fading for a long time[10]. A two-state continuous-time Markov model was employed to experimental power measurement obtained from a 250 m FSO link. It displayed a satisfactory fitting to the observed behavior[11]. The finite Markov model was evaluated in Ref.[12], where auto-variance results are accordant between the Markov model and experimental results. According to Ref.[13], a novel Markov model was established by the consideration of information-theory. However, in order to achieve a precise result, the number of states has to be large enough by both the regimes in Ref.[12] and Ref.[13].

Motivated by these, an improved Markov model is proposed to obtain an equivalent simulation of intensity scintillations with smaller state amount. Both the intensity and the auto-covariance are analyzed, compared with those of the schemes from Refs.[12,13]. Besides, the outage performance and the mean fading time are also evaluated by the improved Markov model. In particular, the simulation results provide a good accordance with the experimental results.

In the experiment, profuse channel samples were collected over the period from 15th July to 25th July in 2016. The system structure is shown in Fig.1, and the transmitted power intensity could be manipulated by the computer connected to the laser amplifier. The beam radius at transmitter $W_0$ is equal to 10 mm, while the receiving aperture diameter $D_G$ is 150 mm. In order to measure the laser's intensity with high accuracy, a 3 dB fiber splitter ensures to separate the received intensity into two equal parts. One is connected to the Agilent power meter with the multimode fiber, while the other is linked to the avalanche photodiode device (APD), which is followed by a transimpedance amplifier (TIA). The voltage out of the TIA is sampled by an oscilloscope with the rate of 100 kHz. The results in the power meter are stored in the computer through an Ethernet cable within the range of 1 kHz.

Assuming the turbulence to be a stationary random process[12], a finite Markov model is utilized to simulate the irradiance. Let $\boldsymbol{\xi}=\{\xi_1,\xi_2,...,\xi_M\}$ be the finite set of $M$ states with the same interval $\Delta_\xi$, which is defined as $\Delta_\xi=(h_{\max}-h_{\min})/M$. For any integer $k$ ($1\le k\le m$), $\xi_k$ is

* E-mail: gsha06@163.com

equal to $(k-0.5)\Delta_{\xi}$. And $S_n$ is defined as the state in the $n$-th time interval.

For brevity, the flow chart of the improved simulation method is shown in Tab.1. Assume that $\boldsymbol{h}_{sa}$ denotes the samples of channel gain from the experiment. And $\boldsymbol{h}_L=[h_1,h_2,...h_L]$ is the channel gain simulated by the proposed Markov model with the length of $L$. As shown in Tab.1, the main difference between this paper and traditional Markov model is that the variables produced in each state obey the uniform distribution. It helps to enhance the accuracy with the same state number of $M$.

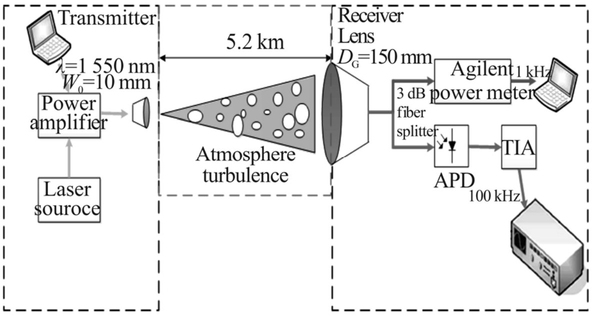


**Fig.1 Structure of the experimental system**

For an $M$-state Markov process, the element $t_{i,j}$ in the state-transition matrix $\boldsymbol{T}_{M\times M}$ is derived from the collected samples, which is

$$t_{i,j}=\Pr\left(S_{n+1}=\xi_j \left| S_n=\xi_i\right.\right)\frac{N_{i,j}}{\sum_{l=1}^{M}N_{l,j}}, \tag{1}$$

where $N_{i,j}$ represents the number of samples from state $S_i$ to state $S_j$. It needs attention that the samples have to be as many as possible, in order to ensure $t_{i,j}$ in Eq.(1) with high accuracy.

**Tab.1 Flow chart of the proposed Markov model**

| Step | Operation |
|---|---|
| 1: | Input: $M$, $L$, $\boldsymbol{h}_{sa}$ |
| 2: | Output: $\boldsymbol{h}_L$ |
| 3: | Initialization: |
| 4: | Set the counter $l$=0 |
| 5: | Compute the states $\boldsymbol{\xi}=\{\xi_1,\xi_2,...,\xi_M\}$ |
| 6: | Derive the $M$-order state-transition matrix $\boldsymbol{T}_{M\times M}$ |
| 7: | Generating procedure: |
| 8: | While $l<L$ do |
| 9: | Derive $h_l$ from the state $S_l$ |
| 10: | Compute next state $S_{l+1}$ from $S_l$ and $\boldsymbol{T}_{M\times M}$ |
| 11: | Update the iterative variable $l=l+1$ |

The matrices $\boldsymbol{T}_{5\times5}$ and $\boldsymbol{T}_{20\times20}$ below are derived from the experiment. Noticing the giant matrix $\boldsymbol{T}_{20\times20}$ in the limited space, it's assumed that $\boldsymbol{T}_m^{\rho}$ denotes the non-zero values in the $m$-th column with the position $\rho$. For example, $\boldsymbol{T}_2^{[3,4]}=\left[0.1,\ 0.2\right]$ denotes $t_{3,2}$=0.1, $t_{4,2}$=0.2. All the other elements in the 2nd column are zero except the 3rd and 4th ones.

$$\boldsymbol{T}_{5\times5}=\begin{bmatrix}0.935\,2 & 0.426\,5 & 0 & 0 & 0\\ 0.064\,8 & 0.560\,9 & 0.317\,2 & 0 & 0\\ 0 & 0.012\,5 & 0.649\,2 & 0.051\,3 & 0\\ 0 & 0 & 0.032\,6 & 0.820\,5 & 0.391\,2\\ 0 & 0 & 0.001\,1 & 0.128\,2 & 0.607\,8\end{bmatrix}. \tag{2}$$

$$\begin{array}{l}\boldsymbol{T}_1^{[1,2,3]}=\left[0.773,\ 0.182\ ,0.045\right],\quad \boldsymbol{T}_2^{[2,3,4,5]}=[0.865,0.130,0.004,0.002],\quad \boldsymbol{T}_3^{[2,3,4,5]}=[0.018,0.846,0.135,0.001]\\ \boldsymbol{T}_4^{[3,4,5]}=[0.047,0.844,0.109],\quad \boldsymbol{T}_5^{[4,5,6]}=[0.073,0.813,0.114],\quad \boldsymbol{T}_6^{[5,6,7]}=[0.106,0.772,0.122],\quad \boldsymbol{T}_7^{[6,7,8]}=[0.122,0.751,0.127]\\ \boldsymbol{T}_8^{[7,8,9]}=[0.151,0.723,0.126],\quad \boldsymbol{T}_9^{[7,8,9,10,11]}=[0.001,0.173,0.694,0.132,0.001],\quad \boldsymbol{T}_{10}^{[8,9,10,11,12]}=[0.001,0.203,0.655,0.139,0.001]\\ \boldsymbol{T}_{11}^{[9,10,11,12]}=[0.002,0.203,0.622,0.153],\quad \boldsymbol{T}_{12}^{[11,12,13,14]}=[0.27,0.598,0.13,0.002],\boldsymbol{T}_{13}^{[11,12,13,14,15]}=[0.008,0.294,0.527,0.169,0.002]\\ \boldsymbol{T}_{14}^{[12,13,14,15]}=[0.009,0.336,0.523,0.132],\boldsymbol{T}_{15}^{[13,14,15,16]}=[0.011,0.28,0.489,0.22],\boldsymbol{T}_{16}^{[14,15,16,17,18]}=[0.014,0.414,0.479,0.079,0.014]\\ \boldsymbol{T}_{17}^{[15,16,17,18]}=[0.048,0.286,0.452,0.214],\quad \boldsymbol{T}_{18}^{[17,18,19]}=[0.381,0.561,0.048],\quad \boldsymbol{T}_{19}^{[18,19]}=[0.667,0.333],\quad \boldsymbol{T}_{20}^{[19,20]}=[0.75,0.25]\end{array}. \tag{3}$$

It's observed that the elements in the leading diagonal are larger than the ones anywhere else. That is to say, it's proper to characterize the turbulence as a quasi-static model.

Fig.2 illustrates the histograms of the samples simulated by the Markov process compared with the ones in the experiment. It could be obtained that the Markov model provides a perfect approximation in both $M$=5 and $M$=20. However, they differ from each other in the auto-covariance function, which could be obtained in Fig.3. The coherent time $\tau_c$ of $M$=5 is 3.3 ms. However, it's 5.2 ms when $M$=20, which is similar to the experimental results of $\tau_c$ = 5.2 ms.

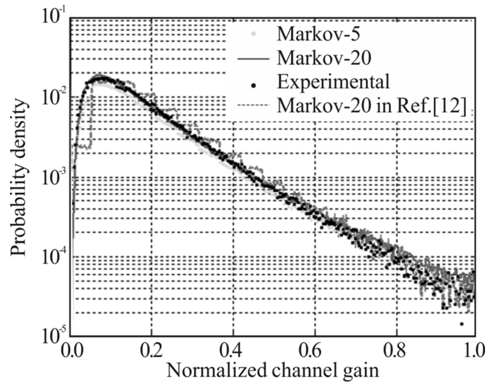


**Fig.2 The histograms of the normalized channel gains obtained from the measurement and the realization generated by the Markov model**

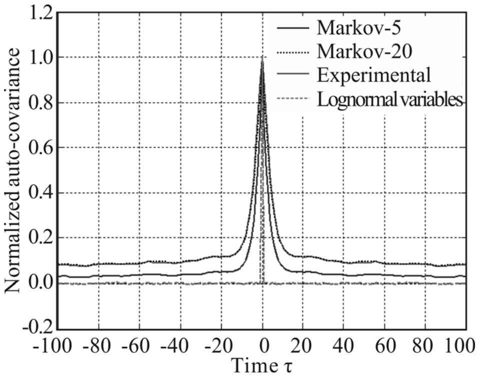


**Fig.3 Normalized auto-covariance analysis**

On the basis of the quasi-static model of the turbulence, the coherent time is assumed to measure the time duration between the contiguous samples by amplitude correlation, which is always considered as 0.1—10 ms. And it could be calculated from the auto-covariance function $R_{hh}$,

$$R_{hh}\left(m\right)=\sum_{i=1}^{N-|m|}h_i h_{i+|m|}, \tag{4}$$

where $N$ is the total samples' number. And the normalized auto-covariance function $\hat{R}_{hh}$ is

$$\hat{R}_{hh}\left(m\right)=\frac{R_{hh}\left(m\right)}{R_{hh}\left(0\right)}. \tag{5}$$

Any two arbitrary samples are considered as not coherent when $\hat{R}_{hh}$ is smaller than $e^{-1}$. Thus, the coherent time $\tau_c$ could be defined in Eq.(5), which is

$$\hat{R}_{hh}\left(\tau_c \cdot R_s\right)=e^{-1}, \tag{6}$$

where $R_s$ is the sampling rate.

As described above, the characteristics of the turbulence channel are analyzed, as well as the noise. Thus, the fading features are discussed, which are outage probability and the mean fading time, respectively.

After the maximum likelihood (ML) estimation of the samples of channel gain, the channel gain follows the distribution with the probability density function (PDF) $f_h(h)$ of turbulence

$$f_h\left(h\right)=\frac{1}{\sqrt{2\pi}\sigma_{\ln I}h}\exp\left[-\frac{\left(\ln h-u_{\ln I}\right)^2}{2\sigma_{\ln I}^2}\right], \tag{7}$$

where $u_{\ln I}$ and $\sigma_{\ln I}$ represent the mean value and the standard deviation, respectively.

Considering the PDF model in Eq.(7), the probability of fade describes the percentage of time when the irradiance at the receiver is below the specified threshold value $I_T$. According to Ref.[3], the outrage probability $P_{I_T}^{out}$ for an arbitrary positive $I$ is revealed as

$$P_{I_T}^{out}=\Pr\left(I<I_T\right)=\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{0.5\sigma_I^2-0.23F_T}{\sqrt{2}\sigma_I}\right)\right], \tag{8}$$

where $F_T$ is the fading threshold, denoting the irradiance level below the threshold $I_T$. Note that $\sigma_I^2$ represents the normalized variance of the optical power at the photo-detector rather than the one at the receiver lens. Fig.4 shows how the outage probability $P_{I_T}^{out}$ changes with the threshold $I_T$ by the Markov model, compared with the measured results and the theoretical ones by Eq.(8).

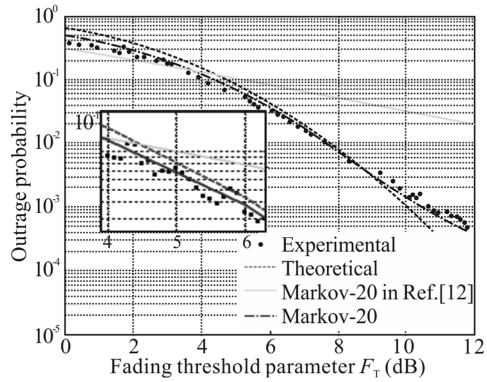


**Fig.4 Outage probability versus $F_T$**

Fig.4 provides that the Markov model with 20 states coincides with the experimental outage performance. It's also obtained that the traditional Markov model with 20 states differs far from the experimental results upon the outage performance. The tiny distinction between the theoretical and the experimental performance results from the fact that noise is not considered in the theoretical analysis.

After discussing the statistical performance of the fading, the temporal characteristics of the fading are analyzed. Suppose that $\left\langle t\left(I_T\right)\right\rangle$ denotes the mean fading time when the irradiance stays below the threshold $I_T$. $\left\langle t\left(I_T\right)\right\rangle$ could be calculated by the quotient of the outage probability $P_{I_T}^{out}$ and the expected number of fading $\left\langle n\left(I_T\right)\right\rangle$, which is shown in Eq.(9)[3].

$$\left\langle t\left(I_T\right)\right\rangle=\frac{P_{I_T}^{out}}{\left\langle n\left(I_T\right)\right\rangle}=\frac{\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{0.5\sigma_I^2-0.23F_T}{\sqrt{2}\sigma_I}\right)\right]}{\upsilon_0\exp\left[-\frac{\left(0.5\sigma_I^2-0.23F_T\right)^2}{2\sigma_I^2}\right]}, \tag{9}$$

where $\upsilon_0$ stands for the quasi-frequency, representing the standard deviation of the normalized irradiance temporal spectrum. The experimental results and simulation performance are given in Fig.5. It could be derived that the mean fading time's magnitude stays at $10^{-3}$ with lower $F_T$, which validates the coherent time $\tau_c$ above. What's more, it's found that the proposed Markov model approaches the experimental results more accurately than the traditional one. The reasons are illustrated as follows. The turbulence-introduced channel could be regarded as the specific distribution of the receiving power. In order to simulate the distribution, the traditional Markov method is required to have enough number of states. It somehow acts like the conjoint short segments to describe a curve. In this paper, uniformly distributed variables in a fixed state would surely supply more situations than a certain value. That's why this paper gives a better simulation than the traditional ones.

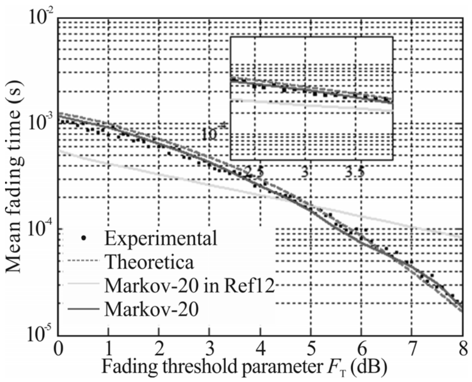


**Fig.5 Mean fading time analysis**

As a whole, the conclusion could be drawn that an improved method has been proposed in order to simulate the intensity scintillation caused by the turbulence. By comparing with current literatures, it requires fewer states but provides equivalent simulated results of both the channel gain's intensity and auto-covariance analysis. That is to say, with identical number of states, the proposed method contributes more accuracy to the simulation. Besides, in the outage performance, the outage probability and the mean fading time has been studied. It's also discovered that the simulation by the proposed Markov model with 20 states coincides with the experimental results, while the traditional model with 20 states differs from the experiments vastly.

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