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A New Hybrid Reliability Index Definition and Its Application to the Structure Buckling Reliability Analysis of Supercavitating Projectiles

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Abstract: As structure buckling problems easily arise when supercavitating projectiles operate with high underwater velocity, it is necessary to perform structure buckling reliability analysis. Now it is widely known that probabilistic and non-probabilistic uncertain information exists in engineering analysis. Based on reliability comprehensive index of multi-ellipsoid convex set, probabilistic uncertain information is added and transferred into non-probabilistic interval variable. The hybrid reliability is calculated by a combined method of modified limit step length iteration algorithm (MLSLIA) and Monte-Carlo method. The results of engineering examples show that the convergence of MLSLIA is better than that of limit step length iteration algorithm (LSLIA). Structure buckling hybrid reliability increases with the increase of ratio of base diameter to cavitator diameter, and decreases with the increase of initial launch velocity. Also the changes of uncertain degree of projectile velocity and cavitator drag coefficient affect structure buckling hybrid reliability index obviously. Therefore, uncertain degree of projectile velocity and cavitator drag coefficient should be controlled in project for high structure buckling reliability.
Key words: supercavitating projectile, structure buckling, hybrid reliability, modified limit step length iteration algorithm (MLSLIA), Monte-Carlo method
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0 Introduction

There are different forms of uncertain information in engineering structure system analysis, such as random or uncertain-but-bounded information. When sample data of structure are sufficient, accurate probability distribution can be obtained and uncertain variables are described as random variable reasonably. Otherwise, there are large difference results of probabilistic reliability between few differences of probability distribution[1]. Because exact probability distribution cannot be obtained when sample data of structure are insufficient, probabilistic reliability analysis results are not satisfactory. Boundary of uncertain variable is easy determined, so it is more appropriate to use non-probabilistic variable to describe uncertain information at this time. However, there are widely probabilistic and non-probabilistic variables at the same time in engineering. Therefore, it is necessary to perform probabilistic and non-probabilistic hybrid reliability analysis of structure. Different definitions and solving methods of hybrid reliability index were proposed[2-4].

Structure buckling problems arise easily when supercavitating projectiles, whose slenderness ratio is high, undergo high longitudinal force caused by high velocity in the submarine. Gu et al.[5] investigated structure buckling probabilistic reliability of supercavitating projectile which was simplified as variable cross-section beam. Under different ratios of base diameter to cavitator diameter, An et al.[6] investigated the change tendency of buckling non-probabilistic failure degree along with the variety of speed. Zhou et al.[7-8] performed structure buckling probabilistic reliability analysis and buckling load non-probabilistic interval analysis of supercavitating projectiles.

In the early stage of product development, there are both of probabilistic and non-probabilistic variables. Because data samples of some variables are not enough,

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it is necessary to perform structural buckling probabilistic and non-probabilistic hybrid reliability analysis.

1 Probabilistic and Non-Probabilistic Hybrid Reliability

Structural reliability is defined as the capacity of structure which completes the required performance under specified conditions and time. Probabilistic measure of reliability is called probabilistic reliability P_{r} and is expressed as

$$P_{\mathrm{r}} = P\{M > 0\} \approx \Phi(\beta), \tag{1}$$

where M is safety margin function, β is probabilistic reliability index, and $\Phi(\cdot)$ is standard normal distribution function. When accurate probabilistic distribution of uncertain variables can be obtained, probabilistic reliability method can be used. Probabilistic reliability index β can be solved by advanced first order second moment (AFOSM), second order second moment (SOSM) and Monte-Carlo method[9].

When accurate probabilistic distribution of uncertain variables cannot be obtained but their boundaries are known, non-probabilistic reliability method can be used. A new non-probabilistic reliability comprehensive index and the corresponding solving algorithm are presented in Ref. [10]. The new multi-ellipsoid convex set non-probabilistic reliability comprehensive index κ is expressed as[10]

$$\kappa = \begin{cases} \eta, & \eta > 1 \\ R_{\mathrm{set}}, & 0 \leqslant \eta \leqslant 1 \end{cases}, \tag{2}$$

where R_{set} is non-probabilistic degree, and η is non-probabilistic reliability index. The non-probabilistic reliability index is expressed as

$$\begin{aligned} \eta = & \\ & \mathrm{sgn}(g'(0)) \min_{\Delta \boldsymbol{v}} \left[\max_{i=1,2,\cdots,k} \delta_i = \sqrt{(\Delta \boldsymbol{v}_i)^{\mathrm{T}} \Delta \boldsymbol{v}_i} \right], \quad (3) \\ & \text{s.t.} \quad g'(\Delta \boldsymbol{V}) = g(\boldsymbol{Y}) = 0, \end{aligned}$$

where $\mathrm{sgn}(\cdot)$ is sign function; $\Delta \boldsymbol{v}_i$ is the ith standard super-sphere space vector; $\Delta \boldsymbol{V}$ is standard super-sphere space vector; $\boldsymbol{Y}$ is multi-ellipsoid space vector; $g'(\Delta \boldsymbol{V})$ is limit state function in standard super-sphere space, $g'(0)$ is limit state function when $\Delta \boldsymbol{V} = \boldsymbol{0}$; $g(\boldsymbol{Y})$ is limit state function in super-ellipsoid space; δ_i is the ith equivalent interval variable.

When structure uncertain parameters are described by multi-ellipsoid convex set, non-probabilistic reliability index η is a min-max value problem. However, sample data of some variables are sufficient and others are lack in engineering. Therefore, it is more common that probabilistic and non-probabilistic variables exist at the same time in engineering. When there are both probabilistic variables and multi-ellipsoid convex set non-probabilistic variables, safety margin function is expressed as

$$\begin{aligned} M = & g(\boldsymbol{X}, \boldsymbol{Y}) = \\ & g(X_1, \cdots, X_i, \cdots, X_m, \boldsymbol{Y}_1, \cdots, \boldsymbol{Y}_j, \cdots, \boldsymbol{Y}_n), \end{aligned} \tag{4}$$

where $\boldsymbol{X}$ is probabilistic vector, X_i is the ith probabilistic variable, and $\boldsymbol{Y}_j$ is the jth super-ellipsoid space vector. Generally random variables can be truncated according to 3σ principle in engineering, namely random variable x_i can be transferred into interval variable y'_i as follows:

$$y'_i \in [\mu_i - 3\sigma_i, \mu_i + 3\sigma_i], \tag{5}$$

where μ_i is mean value of random variables, and σ_i is standard deviation of random variables. After random vector $\boldsymbol{X}$ is transferred into equivalent interval vector $\boldsymbol{Y}'$, safety margin function is transferred as

$$\begin{aligned} M = & g(\boldsymbol{Y}', \boldsymbol{Y}) = \\ & g(Y'_1, \cdots, Y'_i, \cdots, Y'_m, \boldsymbol{Y}_1, \cdots, \boldsymbol{Y}_j, \cdots, \boldsymbol{Y}_n). \end{aligned} \tag{6}$$

In summary, hybrid reliability of probabilistic and non-probabilistic, φ, can be expressed as

$$\varphi = \begin{cases} \eta'(\boldsymbol{X}, \boldsymbol{Y}), & \eta' > 1 \\ R_{\mathrm{set}}(\boldsymbol{X}, \boldsymbol{Y}), & 0 \leqslant \eta' \leqslant 1 \end{cases}. \tag{7}$$

And hybrid reliability index η' can be expressed as

$$\begin{aligned} \eta' = & \mathrm{sgn}(g(0)) \min_{(\Delta \boldsymbol{v}', \Delta \boldsymbol{v})} \max_{\substack{i=1,2,\cdots m \\ j=1,2,\cdots n}} (\delta_i, \delta_j), \quad (8) \\ \text{s.t. } & g(\Delta \boldsymbol{V}', \Delta \boldsymbol{V}) = g(\boldsymbol{Y}', \boldsymbol{Y}) = 0, \\ & \delta_i = \sqrt{(\Delta \boldsymbol{v}'_i)^{\mathrm{T}} \Delta \boldsymbol{v}'_i}, \\ & \delta_j = \sqrt{(\Delta \boldsymbol{v}_j)^{\mathrm{T}} \Delta \boldsymbol{v}_j}, \end{aligned}$$

where $\Delta \boldsymbol{V}'$ is equivalent standard super-sphere space vector which is transformed from random vector $\boldsymbol{X}$.

2 Solving Method of Hybrid Reliability Index

Before calculating hybrid reliability φ, hybrid reliability index η' should be obtained. And the min-max value problem of η' which is expressed in Eq. (8) can be converted to the shortest distance problem from origin to limit state surface in the standard super-sphere space, namely

$$\eta' = \mathrm{sgn}(g(0))\frac{1}{\sqrt{m+n}}\min_{(\Delta \boldsymbol{v}', \Delta \boldsymbol{v})}\sqrt{\sum_{i=1}^{m}\delta_i'^2+\sum_{j=1}^{n}\delta_j^2}, \quad (9)$$

$$\text{s.t. } \mathrm{G}(\Delta \boldsymbol{V}', \Delta \boldsymbol{V}) = \frac{1}{2}\Big\{g^2(\Delta \boldsymbol{V}', \Delta \boldsymbol{V})+ C\Big[\sum_{i=1}^{m-1}(\delta_i' - \delta_{i+1}')^2 + (\delta_1 - \delta_m')^2 + \sum_{j=1}^{n-1}(\delta_j - \delta_{j+1})^2 + (\delta_n - \delta_1')^2\Big]\Big\} = 0.$$

We can see that η' can be obtained by similar iteration method such as AFOSM which is used to solve probabilistic reliability index. There are only the differences of iteration space and constraint function of limit state equation. Considering iterative convergence problem, modified limit step length iteration algorithm (MLSLIA) is used to guarantee convergence.

The solving steps of hybrid reliability φ are as follows.

Step 1 Firstly normal random variable X_i is converted into interval variable Y_i' by Eq. (5), and then Y_i' is converted into standard super-sphere space $\Delta v_i'$:

$$\Delta v_i' = \frac{Y_i' - \mu_i}{3\sigma_i}. \quad (10)$$

Step 2 Multi-ellipsoid convex set vector $\boldsymbol{Y}'$ is converted into standard super-sphere space $\Delta \boldsymbol{V}$:

$$\Delta \boldsymbol{v}_j = \frac{1}{\alpha_j}\boldsymbol{\Lambda}_j^{\frac{1}{2}}\boldsymbol{P}_j(\boldsymbol{Y}_j - \boldsymbol{Y}_{j0}), \quad (11)$$

where α_j is a known positive real number that determines the size of the jth super-ellipsoid convex set, $\boldsymbol{\Lambda}_j$ is a diagonal matrix, and $\boldsymbol{P}_j$ is an orthogonal matrix, also $\boldsymbol{W}_j = \boldsymbol{P}_j^{\mathrm{T}}\boldsymbol{\Lambda}_j\boldsymbol{P}_j$.

Step 3 The shortest distance problem in the standard super-sphere space can be solved by MLSLIA and its iterative formulae are

$$\boldsymbol{\alpha}^{(k+1)} = \frac{\Delta \boldsymbol{V}^{(k)} - \lambda^{(k)}\nabla g_{\Delta \boldsymbol{V}}(\Delta \boldsymbol{V}^{(k)})}{\|\Delta \boldsymbol{V}^{(k)} - \lambda^{(k)}\nabla g_{\Delta \boldsymbol{V}}(\Delta \boldsymbol{V}^{(k)})\|}, \quad (12)$$

$$\eta'^{(k+1)} = -\frac{g_{\Delta \boldsymbol{V}}(\Delta \boldsymbol{V}^{(k)}) - (\nabla g_{\Delta \boldsymbol{V}}(\Delta \boldsymbol{V}^{(k)}))^{\mathrm{T}}\Delta \boldsymbol{V}^{(k)}}{(\boldsymbol{\alpha}_{\Delta \boldsymbol{V}}^{(k+1)})^{\mathrm{T}}\nabla g_{\Delta \boldsymbol{V}}(\Delta \boldsymbol{V}^{(k)})}, \quad (13)$$

$$\Delta \boldsymbol{V}^{(k+1)} = \boldsymbol{\alpha}_{\Delta \boldsymbol{V}}^{(k+1)}\eta'^{(k)}, \quad (14)$$

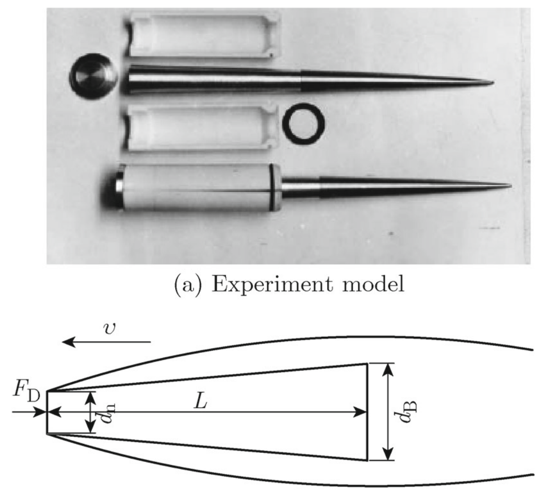
where the superscript k indicates the number of iteration steps. In the iteration process, the optimal step length $\lambda^{(k)}$ can be searched through the golden section method. Evaluation standard is determined by constructing a new evaluation function $m(\Delta \boldsymbol{V})$ which is based on the extreme value conditions of augmented Lagrange function[10].

Step 4 If $\eta' > 1$, then $\varphi = \eta'$. If $0 \leqslant \eta' \leqslant 1$, then $\varphi = R_{\mathrm{set}}$, and R_{set} can be calculated by Monte-Carlo method[10].

3 Structure Buckling Hybrid Reliability of Supercavitating Projectile

3.1 Safety Margin Equation and Partial Derivatives Formula

A sketch of supercavitating projectile is shown in Fig. 1, where d_{n} is cavitator diameter, d_{B} is base diameter, L is projectile length, v is projectile velocity, and F_{D} is cavitator drag.



(a) Experiment model

(b) Force and geometric dimensions sketch

Fig. 1 Supercavitating projectile

Set structural critical buckling load F_{Dcr} as strength, and cavitator drag F_{D} as stress. Safety margin equation of structure buckling of supercavitating projectile is

$$M = g(\boldsymbol{X}, \boldsymbol{Y}) = F_{\mathrm{Dcr}}(E, d_{\mathrm{n}}, L) - F_{\mathrm{D}}(C_{\mathrm{x}}, v, d_{\mathrm{n}}), \quad (15)$$

where E is elasticity modulus, and C_{x} is cavitator drag coefficient. Reference [11] provides solving method of F_{Dcr} and F_{D} in details.

First order partial derivatives of structure buckling performance function to uncertain variables will be used by MLSLIA, namely

$$\frac{\partial g(\Delta \boldsymbol{V}', \Delta \boldsymbol{V})}{\partial \Delta v_i'} = \frac{\partial g(\boldsymbol{X}, \boldsymbol{Y})}{\partial \Delta v_i'} = \Big(\frac{\partial F_{\mathrm{Dcr}}}{\partial Y_i'} - \frac{\partial F_{\mathrm{D}}}{\partial Y_i'}\Big)\frac{\partial Y_i'}{\partial \Delta v_i'}, \quad (16)$$

$$\frac{\partial g(\Delta \boldsymbol{V}', \Delta \boldsymbol{V})}{\partial \Delta v_j} = \frac{\partial g(\boldsymbol{X}, \boldsymbol{Y})}{\partial \Delta v_j} = \Big(\frac{\partial F_{\mathrm{Dcr}}}{\partial \boldsymbol{Y}_j} - \frac{\partial F_{\mathrm{D}}}{\partial \boldsymbol{Y}_j}\Big)\frac{\partial \boldsymbol{Y}_j}{\partial \Delta v_j}. \quad (17)$$

The partial derivatives of F_D to Y'_i and $\boldsymbol{Y}_j$ are simple. But F_{Dcr} is a complex implicit function, and the partial derivatives of F_{Dcr} to Y'_i and $\boldsymbol{Y}_j$ can be expressed respectively as

$$\frac{\partial F_{Dcr}}{\partial Y'_i}=\frac{\boldsymbol{C}^{T}\Big(\dfrac{\partial \boldsymbol{Q}}{\partial Y'_i}F_{Dcr}-\dfrac{\partial \boldsymbol{P}}{\partial Y'_i}F_{Dcr}-\dfrac{\partial \boldsymbol{A}}{\partial Y'_i}\Big)\boldsymbol{C}}{\boldsymbol{C}^{T}(\boldsymbol{P}-\boldsymbol{Q})\boldsymbol{C}}, \quad (18)$$

$$\frac{\partial F_{Dcr}}{\partial \boldsymbol{Y}_j}=\frac{\boldsymbol{C}^{T}\Big(\dfrac{\partial \boldsymbol{Q}}{\partial \boldsymbol{Y}_j}F_{Dcr}-\dfrac{\partial \boldsymbol{P}}{\partial \boldsymbol{Y}_j}F_{Dcr}-\dfrac{\partial \boldsymbol{A}}{\partial \boldsymbol{Y}_j}\Big)\boldsymbol{C}}{\boldsymbol{C}^{T}(\boldsymbol{P}-\boldsymbol{Q})\boldsymbol{C}}, \quad (19)$$

where vector $\boldsymbol{C}$ and the partial matrix of $\boldsymbol{A}$, $\boldsymbol{P}$, $\boldsymbol{Q}$ to E, d_n, L, C_x, υ will be used and its expression can be obtained in Ref. [11].

3.2 Numerical Example

For uncertain variables of supercavitating projectile, there are sufficient sample data of material elasticity modulus; then E can be treated as a normal distribution random variable with mean value $\mu_E = 210\,\text{GPa}$ and standard deviation $\sigma_E = 3.3\,\text{GPa}$. There are insufficient sample data of geometric dimension L and d_n, but the tolerance of them can be provided by designer. Therefore, L and d_n can be treated as non-probabilistic variables, and their expressions are

$$\Big(\frac{L-0.160}{0.005\gamma}\Big)^2+\Big(\frac{d_n-0.002\,5}{0.000\,1\gamma}\Big)^2\leqslant 1, \quad (20)$$

where γ is degree of uncertainty.

Uncertain degree of cavitator drag coefficient and projectile velocity should be determined by experiment data. Due to the strong correlation and the limit of test data, C_x and υ are described by super-ellipsoid convex set:

$$(\boldsymbol{Y}-\boldsymbol{Y}_0)^{T}\boldsymbol{W}(\boldsymbol{Y}-\boldsymbol{Y}_0)\leqslant 0.2\gamma, \quad (21)$$

where

$$\boldsymbol{Y}=\begin{bmatrix}C_x\\ \upsilon\end{bmatrix},\quad \boldsymbol{Y}_0=\begin{bmatrix}1.0\\ 1\,200\end{bmatrix},\quad \boldsymbol{W}=\begin{bmatrix}1 & 0.2\\ 0.2 & 1\end{bmatrix}.$$

Random variable E and multi-ellipsoid convex set variables L, d_n, C_x and υ are converted into standard super-sphere space $\Delta\boldsymbol{V}$:

$$E = 3\Delta v'_1\sigma\gamma + \mu_E, \quad (22)$$
$$L = 0.160 + 0.005\Delta v_1\gamma, \quad (23)$$
$$d_n = 0.002\,5 + 0.000\,1\Delta v_2\gamma. \quad (24)$$

Diagonal matrix $\boldsymbol{\Lambda}$ and orthogonal matrix $\boldsymbol{P}$ can be obtained through eigenvalue decomposition:

$$\boldsymbol{\Lambda}=\begin{bmatrix}0.8 & 0\\ 0 & 1.2\end{bmatrix},\quad \boldsymbol{P}=\begin{bmatrix}-0.707\,1 & 0.707\,1\\ 0.707\,1 & 0.707\,1\end{bmatrix}. \quad (25)$$

And $\boldsymbol{Y}$ can be expressed by $\Delta\boldsymbol{V} = [\Delta v_3 \;\; \Delta v_4]$:

$$\boldsymbol{Y} = 0.2\gamma\boldsymbol{P}^{-1}\boldsymbol{\Lambda}^{-\frac{1}{2}}\Big(\Delta\boldsymbol{V}+\frac{1}{0.2\gamma}\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{P}\boldsymbol{Y}_0\Big). \quad (26)$$

Limit state equation in standard super-sphere space can be obtained by substituting Eqs. (22)—(24) and Eq. (26) into Eq. (15). And then hybrid reliability index η' can be solved by MLSLIA.

The iterative solving process of η' is compared by MLSLIA and limit step length iteration algorithm (LSLIA)[12] in Fig. 2 (the ratio of base diameter to cavitator diameter $\alpha = 2.5$). The iteration results of two methods are the same and the hybrid reliability indexes η' are both equal to 1.888. There is oscillation phenomenon in the LSLIA iterative process and convergence point arises after 25 iteration steps. Whereas convergence point arises only after 10 iteration steps and there is no oscillation phenomenon in the MLSLIA iteration process. It shows that MLSLIA has better and fast iterative convergence.

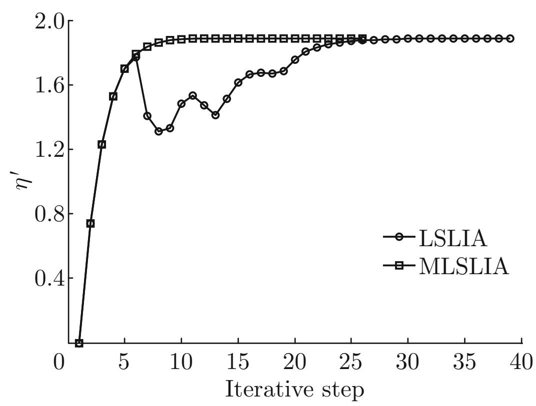


Fig. 2 Iterative solving process

Table 1 shows the change of structure critical buckling load F_{Dcr}, hybrid reliability φ, and hybrid reliability index η' with the change of α ($\upsilon = 1.2\,\text{km/s}$). Table 2 shows the change of cavitator drag F_D, hybrid reliability φ and hybrid reliability index η' with the change of projectile velocity υ ($\alpha = 2.5$). When υ is fixed, nominal value of structure critical buckling load F_{Dcr}

Table 1 The change of F_{Dcr}, φ and η' with α

α	Nominal value of F_{Dcr}/kN	φ	η'
2.0	3.842	0.752 2	0.238 8
2.5	6.534	1.888 4	1.888 4
3.0	10.63 1	3.551 3	3.551 3
3.5	16.571	5.142 2	5.142 2
4.0	24.859	6.623 1	6.623 1
4.5	36.066	7.978 5	7.978 5
5.0	50.831	9.200 5	9.200 5

Table 2 The change of F_D, φ and η' with υ

$\upsilon/(\mathrm{km\cdot s^{-1}})$	Nominal value of F_D/kN	φ	η'
1.0	2.454	3.126 7	3.126 7
1.1	2.970	2.469 8	2.469 8
1.2	3.534	1.888 4	1.888 4
1.3	4.148	1.369 9	1.369 9
1.4	4.811	0.999 9	0.905 4
1.5	5.522	0.935 9	0.490 0

increases with the increase of α, and hybrid reliability φ and hybrid reliability index η' also increase. When α is fixed, cavitator drag F_D increases with the increase of υ, but hybrid reliability φ and hybrid reliability index η' decrease.

Figure 3 shows that hybrid reliability index η' changes with the change of uncertain degree factor γ of each variable. When uncertain degree factor γ of each variable increases, hybrid reliability indexes η' are all declining. The change of uncertain degree factor γ of C_x and υ affects the change of η' greatly, and then uncertain degree factor γ of d_n, E, L in turn.

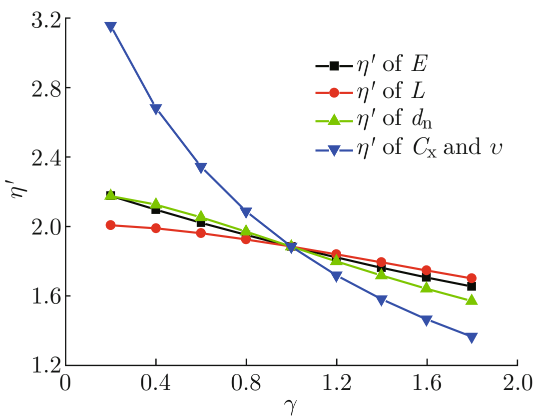


Fig. 3 The change of η' with γ

4 Conclusion

Hybrid reliability definition and solving method are presented in this paper when both probabilistic and non-probabilistic variables are present.

Iteration step number of MLSLIA is less than that of LSLIA, and convergence of MLSLIA is better.

Structure critical buckling load F_{Dcr}, hybrid reliability φ and hybrid reliability index η' increase with the increase of α. Cavitator drag F_D increases with the increase of υ, but hybrid reliability φ and hybrid reliability index η' decrease.

The change of uncertain degree factor γ of C_x and υ affects the change of η' greatly. Therefore, uncertain degree of C_x and υ should be controlled in project for high structure buckling reliability.

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