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# Correction Optimization of Lens Radial Distortion with Bending Measurement Function

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**Abstract:** In this paper, a distortion correction method with reduced complexity is proposed. With the single-parameter division model, the initial approximation of distortion parameters and the distortion center can be calibrated. Based on the distance from the image center to the fitting lines of the extracted curves, a bending measurement function with a weighted factor is proposed to optimize the initial value. Simulation and experiments verify the proposed method.
**Keywords:** arc fitting; camera calibration; distortion correction; division model; field angle

Lens radial distortion is a common problem in digital image analysis. Various methods have been proposed for this problem, which can be classified into three types: traditional camera calibration, auto-calibration and line-based calibration. The first type methods[1-3] are reliable and accurate, but they require multiple images and accurate coordinates of 3D-point correspondences, which is complex and time-consuming. The second type methods use multiple views taken by a moving or rotating camera, and they do not require a calibration pattern[4-6], but the process is tedious and not always possible. In contrast, line-based methods just use a single image that contains abundant lines in human-made environment. Ahmed and Farg[7], Devernay and Faugeras[8] used the principle that straight lines in the 3D world plane should be projected into straight lines in a 2D image plane under any perspective projection. Our method is similar to that of Strand and Hayman[9], and Wang *et al* [10], which estimates the distortion parameters based on the property that distorted straight lines can be circularly modeled by a single-parameter division model. Moreover, the radial distortion can be corrected with circle fitting method[11].

In this paper, the initial approximation of distortion coefficient and the center in the process of circle fitting are calibrated with the extracted curved lines. As pixels move continually from the image's center, the curve line's bending degree enlarges, which means that the distortion increases accordingly. According to the imaging characteristics of camera lens, a bending measurement function with weighted factors is proposed as the objective function.

## 1 Distortion model

In a real camera system, due to the non-negligible distortion of lens, the linear pinhole model is not valid, which makes the actual pixel point $P_d(x_d, y_d)$ deviate from the ideal pixel point $P_u(x_u, y_u)$. Geometry distortion can be divided into radial distortion and tangential distortion. Usually, radial distortion is much bigger than tangential distortion, thus the latter can be ignored[3,7,8]. This paper only considers radial distortion and adopts Fitzgibbon's division model[12], as shown by the following equations.

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$$x_u = \frac{x_d}{1+\lambda_1 r_d^2 + \lambda_2 r_d^4 + \cdots} \tag{1}$$

$$y_u = \frac{y_d}{1+\lambda_1 r_d^2 + \lambda_2 r_d^4 + \cdots} \tag{2}$$

For most lens distortions, it is sufficient to consider a single-parameter division model. More parameters lead to increasing calculation without improving the calculation accuracy. Actually, distortion center $P_0(x_0, y_0)$ is not the image center. Thus, we simplify Eq. (1) and Eq. (2) as

$$x_u = x_0 + \frac{x_d - x_0}{1+\lambda r_d^2} \tag{3}$$

$$y_u = y_0 + \frac{y_d - y_0}{1+\lambda r_d^2} \tag{4}$$

where $r_d^2 = (x_d - x_0)^2 + (y_d - y_0)^2$.

## 2 Methodology

### 2.1 Initial estimation

In an ideal projection model, the straight lines in the 3D world are projected into the straight lines on a 2D image plane. However, due to the introduction of radial distortion, the straight lines in the 3D world plane are instead projected into the circular arcs on the 2D image plane. Let the equation of straight lines of undistorted points be

$$L_u : ax_u + by_u + c = 0 \tag{5}$$

Assume that $a^2 + b^2 > 0$ and exclude the case of $a = b = 0$. By substituting Eq. (3) and Eq. (4) into Eq. (5), we obtain

$$x_d^2 + y_d^2 + (\frac{a}{c\lambda} - 2x_0)x_d + (\frac{b}{c\lambda} - 2y_0)y_d + (x_0^2 + y_0^2 - \frac{a}{c\lambda}x_0 - \frac{b}{c\lambda}y_0 + \frac{1}{\lambda}) = 0 \tag{6}$$

where,

$$A = \frac{a}{c\lambda} - 2x_0$$
$$B = \frac{b}{c\lambda} - 2y_0 \tag{7}$$
$$C = x_0^2 + y_0^2 - \frac{a}{c\lambda}x_0 - \frac{b}{c\lambda}y_0 + \frac{1}{\lambda}$$

By referring to Eq. (6), we have

$$x_d^2 + y_d^2 + Ax_d + By_d + C = 0 \tag{8}$$

Eq. (8) can be viewed as the general expression of circles. Thus, the curved line extracted from the image can be considered as a circle. Let $A$ be multiplied by $x_0$, $B$ multiplied by $y_0$, and $Ax_0$, $By_0$, and $C$ stacked together. Accordingly, we can express the distortion center as follows:

$$x_0^2 + y_0^2 + Ax_0 + By_0 + C - \frac{1}{\lambda} = 0 \tag{9}$$

First, we extract $n\,(n \geqslant 3)$ curved lines $L_i\,(i = 1,2,\cdots, n)$ from the distorted image, obtain the parameters $(A_i, B_i, C_i)$ by fitting $L_i$ into a circle, and calculate the center of the radial distortion $P(x_0, y_0)$ as follows:

$$\begin{cases}(A_1 - A_2)x_0 + (B_1 - B_2)y_0 + (C_1 - C_2) = 0\\(A_1 - A_3)x_0 + (B_1 - B_3)y_0 + (C_1 - C_3) = 0\\(A_2 - A_3)x_0 + (B_2 - B_3)y_0 + (C_2 - C_3) = 0\end{cases} \tag{10}$$

Next, we substitute the distortion center into Eq. (9) and calculate the distortion parameter according to Eq. (11),

$$\frac{1}{\lambda} = x_0^2 + y_0^2 + Ax_0 + By_0 + C \tag{11}$$

To fit the distorted straight lines into circles, we adopt the Levenberg-Marquardt method[13,14]. Generally, the circle's equation can be written as

$$(x - x_c)^2 + (y - y_c)^2 = R^2 \tag{12}$$

Assume that there are $N$ characteristic points $(x_i, y_i)$ on the extracted curved line, and the distance from each feature point to the fitting arc is $d_i$. The objective function is

$$f(R_i, x_{ci}, y_{ci}) = \sum_{i=1}^{N} d_i^2 = \sum_{i=1}^{N}(\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} - R)^2 \tag{13}$$

To find a local minimum, we use the iterative Levenberg-Marquardt nonlinear optimization algorithm, obtain $R_i$, $x_{ci}$, and $y_{ci}$, and then obtain the optimization circle parameters $(A, B, C)$.

### 2.2 Optimization algorithm

#### 2.2.1 Linear fitting

The algorithm described in Section 2.1 uses only part of the circular arc instead of the whole curved line from the entire image. Thus, the derived parameters have considerable estimation errors and they are sensitive to noises. To solve this problem, a specific function with a weighted factor is put forward in this paper.

First, we randomly select a long curved line, and then correct all $n$ distorted points $P_{di}(x_{di}, y_{di})$ to the undistorted points using the following equations:

$$x_{ui} = x_0 + \frac{x_{di} - x_0}{1+\lambda r_{di}^2} \tag{14}$$

$$y_{ui} = y_0 + \frac{y_{di} - y_0}{1+\lambda r_{di}^2} \tag{15}$$

Ideally, if correct distortion parameters are found, then all undistorted points $P_{ui}(x_{ui}, y_{ui})$ should lie on the same line. Therefore, a fitting straight line of undistorted points can be formulated as $L_i: a_i x + b_i y + c_i = 0$, where $a_i$, $b_i$ and $c_i$ can be calculated by linear regression.

Due to the inaccurate estimation of distortion parameters, there is some deviation between the undistorted points $P_{ui}(x_{ui}, y_{ui})$ and their associated fitting line $L_i: a_i x + b_i y + c_i = 0$. Thus, the deviation between point $P_{ui}(x_{ui}, y_{ui})$ and $L_i$ is

$$d_{i,j} = \left|a_i x_{ui,j} + b y_{ui,j} + c_i\right| \Big/ \sqrt{a_i^2 + b_i^2} \tag{16}$$

If there are $m$ feature points on the curved line, then the sum of the squared distances between the undistorted points and the fitting line can be defined as follows:

$$e_m = \sum_{j=1}^{m} d_{i,j}^2 \tag{17}$$

2.2.2 Optimal function

According to the rules of camera imaging characteristics, the radial distortion increases with the field of view. The lines close to the image center bend slightly, while those far away bend seriously. The algorithm proposed in this paper introduces the weighted factor $t_n$, which depends on the distance between $L_m$ and the image center. The weighted factor is defined as the length of vertical distance from the image center to the fitting line, as shown in Fig. 1.The curved lines close to the image center have small weighted factors, while those far away have large values. The normalized weighted factor equals 0 when $d_m$ is 0, and equals 1 when $d_m$ is the distance between the center and vertex of the image. The weighted factor is formulated as:

$$t_n = \frac{2d_m}{\sqrt{w^2 + h^2}} \tag{18}$$

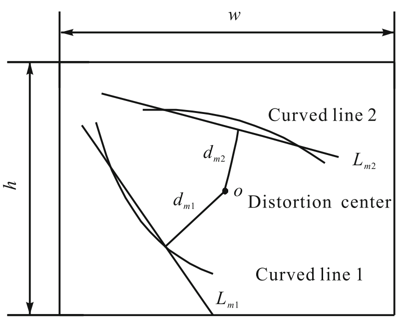



**Fig. 1 The distance between image center and fitting line**

where $w$ is the width of the image; $h$ is the height; and $d_m$ is the vertical distance from the image center to the fitting line $L_m$. For each curved line extracted from the image, we calculate the correlation fitting lines of each curved line. Using Eq. (16), we also calculate the squared distance between the undistorted points and the fitting line. To search for optimal parameters, different weighted factor values are applied. Then the bending measure function is constructed by $n$ curved lines extracted from the entire image as follows:

$$F(x_0, y_0, \lambda) = t_n e_n \tag{19}$$

Thus, the final cost function is defined as:

$$e_{\text{final}} = \min\{\sum_{i=1}^{n} F(x_0, y_0, \lambda)\} = \min\{\sum_{i=1}^{n}(t_1 e_1 + t_2 e_2 + t_3 e_3 + \cdots + t_n e_n)\} \tag{20}$$

This cost function is a nonlinear function of $x_0$, $y_0$ and $\lambda$, and we use the whole curved lines extracted from the image. To estimate $x_0$, $y_0$ and $\lambda$, this cost function is reduced with the Levenberg-Marguardt method[15], and the distortion parameters from Section 2.1 are applied as the initial values.

## 3 Experiment on synthetic images

### 3.1 Tests on synthetic images and analysis

3.1.1 Varying noise level

We conduct a series of quantitative evaluation on synthetic images, which can provide exact information about the distortion center, line positions and distortion parameter[10]. A sample of the synthetic images consists of five horizontal straight lines and five vertical straight lines. The original image size is 640 × 480. Using known distortion parameter $\lambda = -1.0 \times 10^{-6}$ and distortion center (320, 240), the straight line is distorted by using the single-parameter division model. In order to simulate the error in the feature extraction process, a zero-mean Gaussian noise with standard deviation $\sigma$ (varying from 0 to 3 pixels) is inserted at each image point. Twenty random experiments are conducted on different levels of $\sigma$ to eliminate the influence of randomness.

Since each pixel coordinate on the original image can be accurately calculated, we use root mean square error (RMSE) to measure the correction effect of the synthetic image.

$$D_{\text{RMSE}} = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left[(\hat{x}_{ij} - x_{ij})^2 + (\hat{y}_{ij} - y_{ij})^2\right]}{m \times n}} \tag{21}$$

The relationship between RMSE and various noise levels is shown in Fig. 2. For the method without optimization proposed by Wang *et al*[10], the initial value is sensitive to noise simply by using arc fitting. As the noise level increases, the performance of Wang's method[10] deteriorates and the RMSE increases robustly as well. The optimization method without weighting factor uses the entire curved lines in the image, and it greatly improves the stability and accuracy in both the low and high noise zones. However, the optimization algorithm with weighted factor proposed in this paper obtains even more accurate results. When the noise level is less than 2 pixels, the RMSE of the corresponding coordinates can be controlled within 0.4 pixels. Compared with the method without weighted factor, the accuracy increases by 0.2. These results demonstrate that the proposed algorithm is feasible and effective.

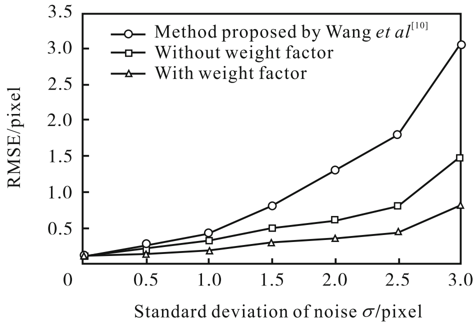



**Fig. 2 Results from the synthetic image experiments at various noise levels**

### 3.1.2 Varying $\lambda$

At a fixed noise level $\sigma$ of 1 pixel, we vary $\lambda$, and fix the distortion center at (320, 240). The distortion varies from extreme barrel to extreme pin-cushion. Twenty random experiments were performed on different levels of $\lambda$. The relationship between RMSE and various values of $\lambda$ obtained from the three algorithms is given in Fig. 3. It can be seen that the proposed method is more accurate in finding the distortion parameter at moderate $\lambda = -1.0\times10^{-6}$ or even larger. However, when the absolute value of the distortion parameter is small, the RMSE is very large, thus the algorithm will fail. The amount of distortion is so slight that it is greatly affected by the noises introduced by the process of extracting curve[10].

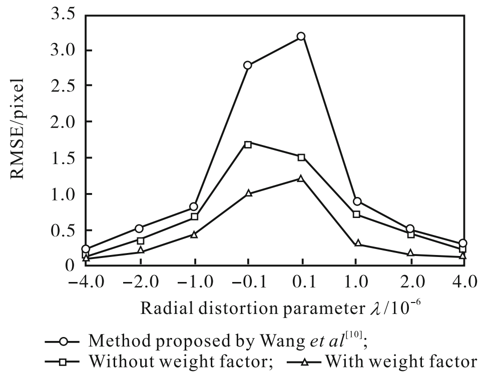
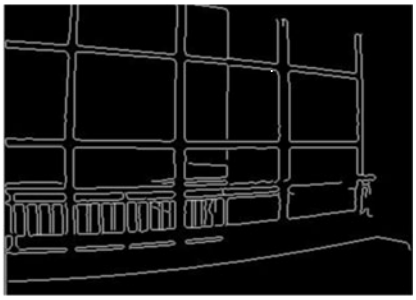



**Fig. 3 Results from the synthetic image experiments with various values of $\lambda$**

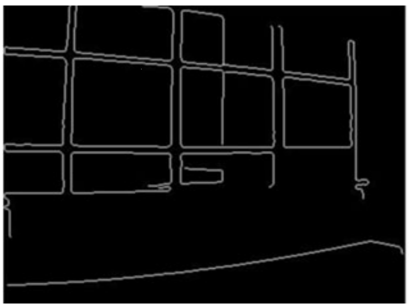
## 3.2 Experiment on real-image

To verify the stability and reliability of our proposed algorithm, experiments are conducted on images obtained using NIKON D7000. One group of images are used to show the step-by-step results, as shown in Fig. 4. Since the image distortion is often less than a pixel, an edge detection method is used with sub-pixel accuracy to extract the edge information[16, 17] (Fig. 4(b)). The edges

(a) Original image



(b) Detected edges



(c) Extracted contours

(d) Corrected image

**Fig. 4 Experimental result**

obtained by the algorithm can be broken, closed or short. Since the short edges contain less information and are sensitive to noise, the edges less than 50 pixels in length are removed. Then the long pixel subsequences that can be fit by circular arcs are also studied (Fig. 4(c)). Two groups of experimental calibration results are listed in Tab. 1. It is observed that the proposed algorithm can achieve a good corrective effect. Moreover, the other original images and corrected images can also be seen in Fig. 5, showing that the distortion can be effectively removed.

**Tab. 1 Experiment calibration data**

| Method | $x_0$/ pixel | $y_0$/pixel | $\lambda$ | Total number of arcs |
|---|---|---|---|---|
| Wang *et al*[10] | 1 023.196 4 | 766.892 1 | $-5.501\,8\times10^{-7}$ | 35 |
| This paper | 1 024.801 6 | 768.249 7 | $-5.976\,1\times10^{-7}$ | 35 |

(a) Distorted image

(b) Corrected image

**Fig. 5 Distorted images and corrected results**

## 4 Conclusions

In this paper, the initial values of the distortion parameters and distortion center are calibrated. In addition, a bending measurement function with a weighted factor is proposed to optimize the distortion parameter and distortion center. This method only requires straight-line segments on a single image, and avoids the complex process of calculating parameters and highly precise calibration pattern, making it straightforward to implement.

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