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A new quantum approach of one-dimensional photonic crystals

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ABSTRACT

In this paper, we have presented a quantum theory to study one-dimensional photonic crystals, and give the quantum transform matrix and quantum transmissivity. We calculate the quantum transmissivity with defect layer, which include absorbing medium and active medium, and obtain some valuable results. The quantum approach can be used to study two-dimensional and three-dimensional photonic crystals.

1. Introduction

In 1987, E. Yablonovitch and S. John had pointed out that the behavior of photons can be changed when propagating in the material with periodical dielectric constant, and termed such material Photonic Crystal [1,2]. Photonic crystal important characteristics are: Photon Band Gap, defect states, Light Localization and so on. These characteristics make it able to control photons, so it may be used to manufacture some high performance devices which have completely new principles or can not be manufactured before, such as high-efficiency semiconductor lasers, right emitting diodes, wave guides, optical filters, high-Q resonators, antennas, frequency-selective surface, optical wave guides and sharp bends [3,4], WDM-devices [5,6], splitters and combiners [7,8]. optical limiters and amplifiers [9,10]. The research on photonic crystals will promote its application and development on integrated photoelectron devices and optical communication. To investigate the structure and characteristics of band gap, there are many methods to analyze Photonic crystals including the plane-wave expansion method [11], Greens function method, finite-difference time-domain method [12–14] and transfer matrix method [15–17]. All of methods are come from classical Maxwell equations. In this paper, We should study the 1D Photonic crystals by the quantum wave equations of photon [18,19], and give quantum transmissivity. In numerical calculation, we calculate the quantum transmissivity with and without defect layer, which include absorbing medium and active medium, and obtain some valuable results. The quantum approach can be used to study two-dimensional and three-dimensional photonic crystals.

2. The quantum wave equation and probability current density of photon

The quantum wave equations of free and non-free photon have been obtained in Refs. [18,19], they are

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t) = c\hbar\nabla\times\vec{\psi}(\vec{r},t), \tag{1}$$

and

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t) = c\hbar\nabla\times\vec{\psi}(\vec{r},t) + V\vec{\psi}(\vec{r},t), \tag{2}$$

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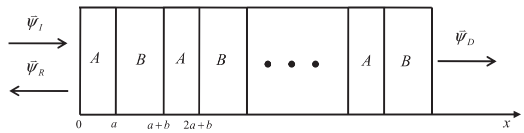

Fig. 1. The structure of one-dimensional photonic crystals.

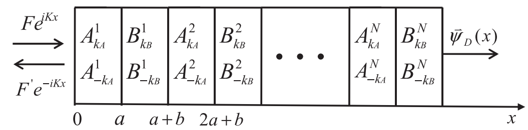

Fig. 2. The quantum structure of one-dimensional photonic crystals.

where $\vec{\psi}(\vec{r},t)$ is the vector wave function of photon, and V is the potential energy of photon in medium. In the medium of refractive index n, the photon's potential energy V is [18,19]

$$V=\hbar\omega(1-n). \tag{3}$$

By Eq. (2), we obtain

$$\frac{\partial\rho}{\partial t}+\nabla\cdot J=0, \tag{4}$$

where

$$\rho=\vec{\psi}^{*}\cdot\vec{\psi}, \tag{5}$$

and

$$J=ic\vec{\psi}\times\vec{\psi}^{*}, \tag{6}$$

are the probability density and probability current density, respectively. By the method of separation variable

$$\vec{\psi}(\vec{r},t)=\vec{\psi}(\vec{r})f(t), \tag{7}$$

the time-dependent Eq. (2) becomes the time-independent equation

$$c\hbar\nabla\times\vec{\psi}(\vec{r})+V\vec{\psi}(\vec{r})=E\vec{\psi}(\vec{r}), \tag{8}$$

where E and V are the energy and potential energy of photon in medium, respectively.

3. The quantum transmissivity

We consider the photon travels along with the x axis in one-dimensional Photonic crystals, which is shown in Fig. 1. The wave vector $k_y=k_z=0$ and $k_x\neq 0$. Since the photon wave is transverse wave, we have

$$\begin{cases}\psi_x=0\\ \psi_y=\psi_{0y}e^{i(kx-\omega t)}\\ \psi_z=\psi_{0z}e^{i(kx-\omega t)}\end{cases}, \tag{9}$$

where $k=\frac{\omega}{c}n$ is the wave vector of photon in medium. Substituting Eq. (9) into Eq. (8), we get

$$\begin{cases}-ik\psi_z=\dfrac{\omega}{c}n\psi_y=k\psi_y\\ ik\psi_y=\dfrac{\omega}{c}n\psi_z=k\psi_z\end{cases}, \tag{10}$$

their solution are

$$\psi_y=\psi_{0y}e^{i\frac{\omega}{c}nx}=\psi_{0y}e^{ikx} \tag{11}$$

and

$$\psi_z=\psi_{0z}e^{ikx}=i\psi_y=i\psi_{0y}e^{ikx} \tag{12}$$

the total wave function of photon in medium is

$$\vec{\psi}=\psi_y\vec{j}+\psi_z\vec{k}=\psi_{0y}e^{ikx}\vec{j}+\psi_{0z}e^{ikx}\vec{k}. \tag{13}$$

the total wave function of photon in vacuum is

$$\vec{\psi}=\psi_y\vec{j}+\psi_z\vec{k}=\psi_{0y}e^{iKx}\vec{j}+\psi_{0z}e^{iKx}\vec{k}. \tag{14}$$

where $K=\frac{\omega}{c}$ is the wave vector of photon in vacuum.

For one-dimensional Photonic crystals, we should define and calculate its quantum transmissivity and quantum reflectivity. The one-dimensional photonic crystals structure is shown in Fig. 1.

In Fig. 1, $\vec{\psi}_I$, $\vec{\psi}_R$, $\vec{\psi}_D$ are the wave functions of incident, reflection and transmission photon, respectively, they can be written as

$$\vec{\psi}_I=F_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j}+F_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{15}$$

$$\vec{\psi}_R=F'_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j}+F'_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{16}$$

$$\vec{\psi}_D=D_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j}+D_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{17}$$

where F_y, F_z, F'_y, F'_z, D_y, and D_z are incident, reflection and transmission amplitudes of y and z components.

By Eq. (13), the probability current density can be written as

$$J=ic\vec{\psi}\times\vec{\psi}^{*}=2c|\psi_z|^2\vec{i}=2c|\psi_{0z}|^2\vec{i}, \tag{18}$$

where ψ_{0z} is the amplitude of $\psi_z=\psi_{0z}e^{i(\vec{k}\cdot\vec{r}-\omega t)}$.

For the incident, reflection and transmission photon, their probability current density J_I, J_R, J_T are

$$J_I=2c|F_z|^2,\qquad J_R=2c|F'_z|^2,\qquad J_T=2c|D_z|^2, \tag{19}$$

We can define quantum transmissivity T is

$$T=\frac{J_T}{J_I}=\left|\frac{D_z}{F_z}\right|^2, \tag{20}$$

By the amplitudes of z component F_z and D_z, we can calculate the quantum transmissivity.

4. The photon wave function and quantum transform matrix in one-dimensional photonic crystals

In Fig. 2, we give the simplification form of wave function in every medium, such as symbols $A^1_{k_A}$ and $A^1_{-k_A}$ express simplifying wave function of medium A in the first period, it express wave function

$$\psi^1_A(x)=A^1_{k_A}e^{ik_Ax}+A^1_{-k_A}e^{ik_Aa+ik_A(a-x)}, \tag{21}$$

in medium B of first period, the symbols $B^1_{k_B}$ and $B^1_{-k_B}$ express wave function

$$\psi^1_B(x)=B^1_{k_B}e^{ik_Aa+ik_B(x-a)}+B^1_{-k_B}e^{ik_Aa+ik_Bb+ik_B(a+b-x)}, \tag{22}$$

similarly, in medium A of Nth period, the symbols $A^N_{k_A}$ and $A^N_{-k_A}$ express wave function

$$\psi^N_A(x)=A^N_{k_A}e^{ik_A(N-1)a+ik_B(N-1)b+ik_A(x-(N-1)a-(N-1)b)}$$
$$+A^N_{-k_A}e^{ik_ANa+ik_B(N-1)b+ik_A(Na+(N-1)b-x)}, \tag{23}$$

in medium B of Nth period, the symbols $B^N_{k_B}$ and $B^N_{-k_B}$ express wave function

$$\psi^N_B(x)=B^N_{k_B}e^{ik_ANa+ik_B(N-1)b+ik_B(x-Na-(N-1)b)}$$
$$+B^N_{-k_B}e^{ik_ANa+ik_BNb+ik_B(Na+Nb-x)}. \tag{24}$$

In the incident area, the total wave function $\psi_{tot}(x)$ is the superposition of incident and reflection wave function, it is

$$\psi_{tot}(x)=\psi_I(x)+\psi_R(x)=Fe^{iKx}+F'e^{-iKx}, \tag{25}$$

where K is the wave vector of incident, reflection, and transmission photon.

In the following, we should use the condition of wave function and its derivative continuation at interface of two mediums.

(1) At $x = 0$, by the continuation of $\psi_{tot}(x)$, $\psi_A^1(x)$ and its derivative, we have

$$F + F' = A_{k_A}^1 + A_{-k_A}^1 e^{ik_A 2a}, \tag{26}$$

$$iKF - iKF' = ik_A A_{k_A}^1 - ik_A A_{-k_A}^1 e^{ik_A 2a}, \tag{27}$$

the Eqs. (26) and (27) can be written as matrix form

$$\begin{pmatrix} A_{k_A}^1 \\ A_{-k_A}^1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 + K/k_A & 1 - K/k_A \\ (1 - K/k_A)e^{-ik_A 2a} & (1 + K/k_A)e^{-ik_A 2a} \end{pmatrix}\begin{pmatrix} F \\ F' \end{pmatrix} = M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix}, \tag{28}$$

where M_A^1 is the quantum transform matrix of the first period medium A, it is

$$M_A^1 = \frac{1}{2}\begin{pmatrix} 1 + K/k_A & 1 - K/k_A \\ (1 - K/k_A)e^{-ik_A 2a} & (1 + K/k_A)e^{-ik_A 2a} \end{pmatrix}, \tag{29}$$

(2) At $x = a$, by the continuation of $\psi_A^1(x)$, $\psi_B^1(x)$ and its derivative, we have

$$A_{k_A}^1 e^{ik_A a} + A_{-k_A}^1 e^{ik_A a} = B_{k_B}^1 e^{ik_A a} + B_{-k_B}^1 e^{ik_A a} e^{ik_B 2b}, \tag{30}$$

$$k_A A_{k_A}^1 e^{ik_A a} - k_A A_{-k_A}^1 e^{ik_A a} = k_B B_{k_B}^1 e^{ik_A a} - k_B B_{-k_B}^1 e^{ik_A a} e^{ik_B 2b}, \tag{31}$$

the Eqs. (30) and (31) can be written as matrix form

$$\begin{pmatrix} B_{k_B}^1 \\ B_{-k_B}^1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 + k_A/k_B & 1 - k_A/k_B \\ (1 - k_A/k_B)e^{-ik_B 2b} & (1 + k_A/k_B)e^{-ik_B 2b} \end{pmatrix}\begin{pmatrix} A_{kA}^1 \\ A_{-kA}^1 \end{pmatrix} = M_B^1 \begin{pmatrix} A_{kA}^1 \\ A_{-kA}^1 \end{pmatrix}, \tag{32}$$

where M_B^1 is the quantum transform matrix of the first period medium B, it is

$$M_B^1 = \frac{1}{2}\begin{pmatrix} 1 + k_A/k_B & 1 - k_A/k_B \\ (1 - k_A/k_B)e^{-ik_B 2b} & (1 + k_A/k_B)e^{-ik_B 2b} \end{pmatrix}, \tag{33}$$

By the above calculation, we can obtain the results of transform matrixes:

(1) For the transform matrix M_A^1 of the first period medium A is independent form.
(2) For the transform matrixes M_A^N of the Nth period ($N \geq 2$), they can be written as

$$M_A^N = M_A = \frac{1}{2}\begin{pmatrix} 1 + k_B/k_A & 1 - k_B/k_A \\ (1 - k_B/k_A)e^{-ik_A 2a} & (1 + k_B/k_A)e^{-ik_A 2a} \end{pmatrix}, \tag{34}$$

(3) For the transform matrixes M_B^N of the Nth period ($N \geq 1$), they can be written as

$$M_B^N = M_B = \frac{1}{2}\begin{pmatrix} 1 + k_A/k_B & 1 - k_A/k_B \\ (1 - k_A/k_B)e^{-ik_B 2b} & (1 + k_A/k_B)e^{-ik_B 2b} \end{pmatrix}. \tag{35}$$

We can obtain the total quantum transform matrixes, it is:

$$\begin{pmatrix} B_{k_B}^N \\ B_{-k_B}^N \end{pmatrix} = (M_B M_A)^{N-1} M_B M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix} = M \begin{pmatrix} F \\ F' \end{pmatrix}, \tag{36}$$

where $M = (M_B M_A)^{N-1} M_B M_A^1 = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}$ is the total quantum transform matrix of N period.

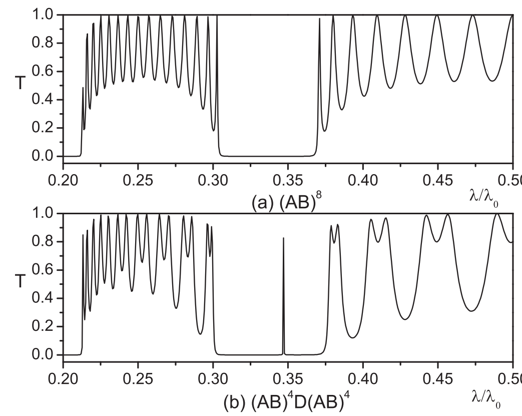

Fig. 3. Quantum transmissivity without and with defect layer.

By Eq. (36), we can give the wave function of Nth period in medium B, it is

$$\begin{aligned} \psi_B^N(x) &= B_{k_B}^N e^{ik_A Na + ik_B(N-1)b + ik_B(x - Na - (N-1)b)} \\ &\quad + B_{-k_B}^N e^{ik_A Na + ik_B Nb + ik_B(Na + Nb - x)} \\ &= (m_1 F + m_2 F')e^{ik_A Na + ik_B(N-1)b + ik_B(x - Na - (N-1)b)} \\ &\quad + (m_3 F + m_4 F')e^{ik_A Na + ik_B Nb + ik_B(Na + Nb - x)}. \end{aligned} \tag{37}$$

In Fig. 2, the transmission wave function is

$$\psi_D(x) = D \cdot e^{ik_A Na + ik_B Nb + K(x - Na - Nb)}. \tag{38}$$

At $x = N(a + b)$, by the continuation of wave function and its derivative, we have

$$m_1 F + m_2 F' + m_3 F + m_4 F' = D, \tag{39}$$

and

$$k_B(m_1 F + m_2 F') - k_B(m_3 F + m_4 F') = KD, \tag{40}$$

we can obtain

$$r = \frac{F'}{F} = \frac{m_1(k_B - K) - m_3(k_B + K)}{m_2(K - k_B) + m_4(K + k_B)}, \tag{41}$$

By Eqs. (39)–(41), we have

$$t = \frac{D}{F} = (m_1 + m_2 r) + (m_3 + m_4 r), \tag{42}$$

and the quantum transmissivity T is

$$T = |t|^2. \tag{43}$$

5. Numerical result

In this section, we report our numerical results of quantum transmissivity. The main parameters are: The medium A is SiO_2, its refractive indexes is $n_a = 1.45$, and its thickness is $a = 267$ nm, the medium B is *GaAs*, its refractive indexes is $n_b = 3.59$, and its thickness is $b = 108$ nm, the central wavelength is $\lambda_0 = 1.55$ μm, and the period number $N = 8$. With Eq. (43), we can calculate the quantum transmissivity. In Fig. 3(a) the photonic crystals structure is $(AB)^8$, and in Fig. 3(b) the structure of photonic crystals is $(AB)^4D(AB)^4$, i.e., we consider the effect of defect layer on quantum transmissivity, the defect layer D parameters are: the thickness

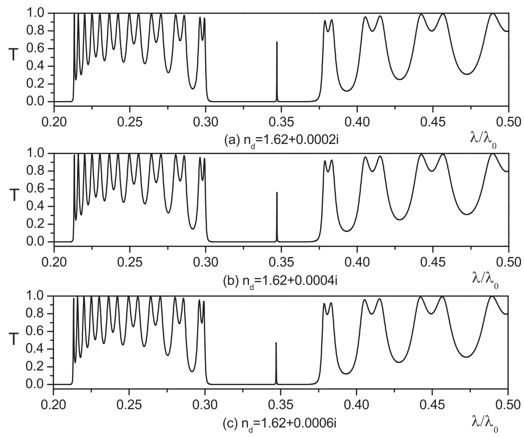

Fig. 4. Quantum transmissivity with defect layer ($(AB)^4D(AB)^4$).

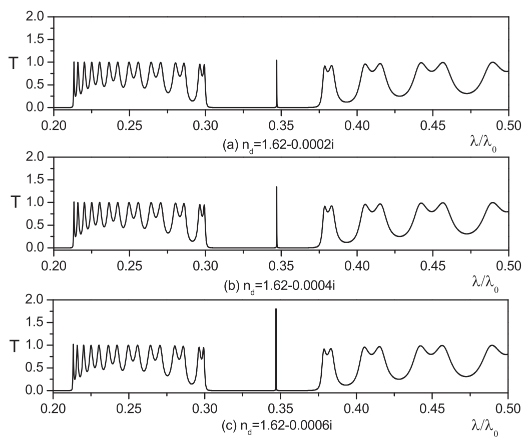

Fig. 5. Quantum transmissivity with defect layer ($(AB)^4D(AB)^4$).

$d = 106$ nm, the refractive indexes $n_d = 1.62$. Comparing Fig. 3(a) with (b), we find when the photonic crystals with defect layer, the width of band gap increase, and there is a defect mode in the band gap. In Figs. 4 and 5, we study the effect of absorbing medium and active medium defect layer on the quantum transmissivity. In Fig. 4(a), (b) and (c), the structure is $(AB)^4D(AB)^4$, and the refractive indexes of defect layer are $n_d = 1.62 + 0.002i$, $n_d = 1.62 + 0.004i$, $n_d = 1.62 + 0.006i$, respectively, i.e., the defect layer D is absorbing medium. We give the relation of quantum transmissivity and λ/λ_0. It can be found that when the defect layer D is absorbing medium, the defect model was decreased, with the imaginary part increase, the defect model decrease. In Fig. 5(a), (b) and (c), the structure is $(AB)^4D(AB)^4$, and the refractive indexes of defect layer are $n_d = 1.62 - 0.002i$, $n_d = 1.62 - 0.004i$, $n_d = 1.62 - 0.006i$, respectively, i.e., the defect layer D is active medium. We find when the defect layer D is active medium, the defect model was increased, with the absolute value of imaginary part increase, the defect model increase, and its transmissivity can be larger than 1.

6. Conclusion

In summary, we have presented a quantum theory to study one-dimensional photonic crystals, and give the quantum transform matrix and quantum transmissivity. We calculate the quantum transmissivity with and without defect layer. We can obtain the some results as follows: (1) When the photonic crystals with defect layer, the band gap width increase, and there is a defect mode in the band gap. (2) When the defect layer is absorbing medium the defect model was decreased. With the imaginary part increase, the defect model decrease. (3) When the defect layer is active medium the defect model was increased. With the absolute value of imaginary part increase, the defect model increase, and its quantum transmissivity can be larger than 1. The quantum approach can be used to study two-dimensional and three-dimensional photonic crystals further.

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