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Entanglement Dynamics of Electrons and Photons

Xiang-Yao Wu[1] · Xiao-Jing Liu[1,2] · Jing-Bin Lu[2] · Tian-Shun Li[1] · Si-Qi Zhang[2] · Yu Liang[1] · Ji Ma[1] · Hong Li[3]

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Abstract Entanglement is a fundamental feature of quantum theory as well as a key resource for quantum computing and quantum communication, but the entanglement mechanism has not been found at present. We think when the two subsystems exist interaction directly or indirectly, they can be in entanglement state. such as, in the Jaynes-Cummings model, the entanglement between the atom and the light field comes from their interaction. In this paper, we have studied the entanglement mechanism of electron-electron and photon-photon, which are from the spin-spin interaction. We found their total entanglement states are relevant both space state and spin state. When two electrons or two photons are far away, their entanglement states should be disappeared even if their spin state is entangled.

Keywords Entanglement states · Entanglement mechanism · Spin-spin interaction

1 Introduction

One of the most peculiar properties of quantum mechanics is entanglement, that is the possibility to construct quantum states of several subsystems that cannot be factorized into a product of individual states of each subsystem. Such entangled states are the most common in quantum mechanics, and they display correlations which cannot be seen in a classical world.

Quantum entanglement has been extensively studied in the past few years, both due to its fundamental significance in quantum theory [1, 2], and it is the basic concept of the

✉ Xiang-Yao Wu
wuxy2066@163.com

1 Institute of Physics, Jilin Normal University, Siping 136000 China

2 Institute of Physics, Jilin University, Changchun 130012 China

3 Institute of Physics, Northeast Normal University, Changchun 130012 China

quantum information processes, such as quantum computing [3], quantum teleportation [4], quantum cryptography [5] and quantum communication [6]. The atom-photon entanglement has been studied in atomic cascade systems [7, 8] as well as in trapped ions [9, 10]. The observation of the quantum entanglement between a single trapped $87Rb$ atom and a single photon at a wavelength suitable for low-loss communication has been reported [11]. Theoretical description of entanglement evolution between atom and quantized field in the Jaynes-Cummings model has been proposed [12–14]. However, it was shown that the induced entanglement between two interacting two-level quantum systems can be controlled by the relative phase of applied fields [15]. In another study it was shown that the atom-photon entanglement near a 3D anisotropic photonic band edge depends on the relative phase of applied fields [16].

Entangled states of photons are the basic resource in the successful implementation of quantum information processing applications. The standard method for generating entangled photon states nowadays is spontaneous parametric down conversion (SPDC), which is achieved by pumping one or more nonlinear crystals with a laser source. could have important benefits to applications in optical quantum information. Photonic quantum gates require pure states, which can be created by heralded sources producing pairs of spectrally decorrelated photons [17, 18]. On the other hand, long distance fiber based quantum communication and quantum metrology suffers from chromatic dispersion, which could potentially be improved with positive spectral correlations [19, 20].

In this paper, we have studied the entanglement mechanism of electron-electron and photon-photon. With the spin-spin interaction, we have given the spin entanglement states of two electrons, three electrons, two photons, and found their total entanglement states are relevant both space state and spin state. When two electrons or two photons are far away, their entanglement states should be disappeared even if their spin state is entangled.

2 The Entanglement Between the Atom and the Light Field

Entanglement can not be produced in isolation, when the two subsystems exist interaction directly or indirectly, they can be in entanglement state. In Jaynes-Cummings model, the entanglement between the atom and the light field is from their interaction. So what interaction lead to the entanglement of electron-electron and photon-photon? we think their spin-spin interaction form their entanglement. In the following, we should prove that the electron-electron and photon-photon spin entanglement states come from their spin-spin interaction.

For the two-photon Jaynes-Cummings model, the Hamiltonian is [21, 22]

$$H = \omega a^{+}a + \frac{1}{2}\omega_0\sigma_z + g(a^{+2}\sigma_- + a^2\sigma_+), \qquad (\hbar = 1) \tag{1}$$

describing the interaction of the light field with the two-level atom, where ω, ω_0 are the frequencies of light field and atom, $a(a^{+})$ denotes the annihilation (creation) operator of light field, $\sigma_z =| a >< a | - | b >< b |$, $\sigma_+ =| a >< b |$, $\sigma_- =| b >< a |$ are the atomic operators, $| b >$ ($| a >$) is the ground (excited) state of the atom, and g is the coupling strength of atom and light field, which reflects the interaction intensity of atom and light field.

At the initial time, the atom is at excited state and the quantized field photon number is N, the initial state is

$$| \psi(0) >=| a, n >, \tag{2}$$

with Schrodinger equation, we can obtain the state at any time t, it is

$$| \psi(t) >= c_1(t) \mid a, n > + c_2(t) \mid b, n+2 >, \tag{3}$$

where

$$c_1(t) = e^{-i\omega(n+1)t}[\cos(\frac{\sqrt{\delta^2+\omega_1^2}}{2}t) + i\delta(\delta^2+\omega_1^2)^{-1/2}\sin(\frac{\sqrt{\delta^2+\omega_1^2}}{2}t)], \tag{4}$$

$$c_2(t) = -ie^{i\omega(n+1)t}\omega_1(\delta^2+\omega_1^2)^{-1/2}\sin(\frac{\sqrt{\delta^2+\omega_1^2}}{2}t), \tag{5}$$

where $\delta = \omega_0 - 2\omega$ is the mismatching quantity and $\omega_1 = 2g\sqrt{(n+1)(n+2)}$ is the Rabi frequency.

From (5), we can find when the interaction intensity of atom and light field $g = 0$, the coefficient $c_2(t) = 0$, the entanglement of atom and light field should be disappeared. So, the entanglement of atom and light field is from the the interaction of atom and light field, i.e., the interaction between particles generate the entanglement between the particles. Similarly, the entanglement between the electrons as well as the photons come from the interaction between the electrons and the photons. In the following, we shall study the entanglement dynamics between the electrons and the photons.

2.1 The Entanglement Between the Electrons

The two electrons Hamiltonian operator is

$$\begin{aligned}\hat{H} &= -\frac{-\hbar^2}{2m}\nabla_1^2 - \frac{-\hbar^2}{2m}\nabla_1^2 + V(r_1, r_2) + g_e\vec{s_1}\cdot\vec{s_2} \\ &= \hat{H}_1 + \hat{H}_2 + \hat{H}_I(r_1, r_2) + \hat{H}_I(s_1, s_2),\end{aligned} \tag{6}$$

where $\hat{H}_1 = -\frac{-\hbar^2}{2m}\nabla_1^2$, $\hat{H}_2 = -\frac{-\hbar^2}{2m}\nabla_2^2$, $\hat{H}_I(r_1, r_2) = V(r_1, r_2)$, $\hat{H}_I(s_1, s_2) = g_e\vec{s_1}\cdot\vec{s_2}$, $V(r_1, r_2)$ is the potential energy $g_e\vec{s_1}\cdot\vec{s_2}$ is the spin-spin interaction energy of two electrons, g_e is electrons spin coupling strength, $\vec{s_1}$ and $\vec{s_2}$ are the spins of electron 1 and 2.

The eigen equation of (6) is

$$(\hat{H}_1 + \hat{H}_2 + \hat{H}_I(r_1, r_2) + \hat{H}_I(s_1, s_2))\psi(r_1, r_2, s_{1z}, s_{2z}) = E\psi(r_1, r_2, s_{1z}, s_{2z}), \tag{7}$$

where E is the total energy of two electrons, the (7) can be solved by separation variable method, it is

$$\psi(r_1, r_2, s_{1z}, s_{2z}) = \psi(r_1, r_2)\cdot\chi_{SM_s}(s_{1z}, s_{2z}), \tag{8}$$

substituting (8) into (7), we can obtain

$$(\hat{H}_1 + \hat{H}_2 + \hat{H}_I(r_1, r_2))\psi(r_1, r_2) = E_r\psi(r_1, r_2), \tag{9}$$

and

$$\hat{H}_I(s_1, s_2)\chi_{SM_s}(s_{1z}, s_{2z}) = g_e\vec{s_1}\cdot\vec{s_2}\chi_{SM_s}(s_{1z}, s_{2z}) = E_s\chi_{SM_s}(s_{1z}, s_{2z}), \tag{10}$$

where $E_s = E - E_r$, the space and spin wave functions of two electrons can be solved by (9) and (10), respectively. So, the spin entanglement states of two electrons can be obtained by (10).

Since $\vec{s_1}\cdot\vec{s_2} = \frac{1}{2}(\vec{S}^2 - \frac{3\hbar^2}{2})$, the (10) can be written as

$$\frac{1}{2}g_e(\vec{S}^2 - \frac{3\hbar^2}{2})\chi_{SM_s}(s_{1z}, s_{2z}) = E_s\chi_{SM_s}(s_{1z}, s_{2z}), \tag{11}$$

where $\vec{S}=\vec{s_1}+\vec{s_2}$ is the total spin of the two electrons. In quantum mechanics, the (11) has four spin eigenfunctions, they are

(1) $S=1$ two electrons spin symmetrical states

$$\chi^{(1)}_{SM_s}=\chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z}) \qquad S=1, \quad M_s=1, \tag{12}$$

$$\chi^{(2)}_{SM_s}=\chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z}) \qquad S=1, \quad M_s=-1, \tag{13}$$

$$\chi^{(3)}_{SM_s}=\frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})+\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{1z})] \qquad S=1, \quad M_s=0, \tag{14}$$

(2) $S=0$ two electrons spin antisymmetrical state

$$\chi^{0}_{SM_s}=\frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})-\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{1z})] \qquad S=0, \quad M_s=0. \tag{15}$$

where $M_s=s_{1z}+s_{2z}$. The states $\chi^{(3)}_{SM_s}$ and $\chi^{(0)}_{SM_s}$ are the spin entanglement states of two electrons.

For the three electrons, their Hamiltonian operator of electrons spin interaction is

$$\hat{H}_I(s_1,s_2,s_3)=g_e(\vec{s_1}\cdot\vec{s_2}+\vec{s_2}\cdot\vec{s_3}+\vec{s_3}\cdot\vec{s_1}). \tag{16}$$

In three electrons system, the spin of two-electron and three-electron $\vec{S}_{12}$ and $\vec{S}$ are

$$\vec{S}_{12}=\vec{s}_1+\vec{s}_2, \tag{17}$$

and

$$\vec{S}=\vec{s}_1+\vec{s}_2+\vec{s}_3=\vec{S}_{12}+\vec{s}_3, \tag{18}$$

their square are

$$\vec{S}^2_{12}=\frac{3}{2}\hbar^2+2\vec{s}_1\cdot\vec{s}_2, \tag{19}$$

and

$$\vec{S}^2=\frac{9}{4}\hbar^2+2(\vec{s}_1\cdot\vec{s}_2+\vec{s}_2\cdot\vec{s}_3+\vec{s}_3\cdot\vec{s}_1), \tag{20}$$

substituting (20) into (16), we have

$$\hat{H}_I(s_1,s_2,s_3)=\frac{g_e}{2}(\vec{S}^2_{12}-\frac{9}{4}\hbar^2), \tag{21}$$

the operators $\{\vec{S}^2,\vec{S'}^2,s_z\}$ common eigenfunctions $\chi_{S'SM_s}$ are the eigenfunctions of (21), and the corresponding quantum numbers S' and S are taken as

$$S=\frac{3}{2}, \qquad S'=1, \tag{22}$$

$$S=\frac{1}{2}, \qquad S'=1,0. \tag{23}$$

In quantum mechanics, the (21) has eight spin eigenfunctions, they are

(1) $S'=1$, $S=\frac{3}{2}$ spin wave functions $\chi_{S'SM_s}$

$$\chi_{1\frac{3}{2}\frac{3}{2}}=\chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}), \tag{24}$$

$$\begin{aligned}\chi_{1\frac{3}{2}\frac{1}{2}}=&\frac{1}{\sqrt{3}}[\chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z})+\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z})\\&+\chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z})],\end{aligned} \tag{25}$$

$$\chi_{1\frac{3}{2}-\frac{1}{2}} = \frac{1}{\sqrt{3}}[\chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) + \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) + \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z})], \quad (26)$$

$$\chi_{1\frac{3}{2}-\frac{3}{2}} = \chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}). \quad (27)$$

(2) $S' = 1, S = \frac{1}{2}$ spin wave functions $\chi_{S'SM_s}$

$$\chi_{1\frac{1}{2}\frac{1}{2}} = \frac{1}{\sqrt{6}}[2\chi_{\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) - \chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) - \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z})], \quad (28)$$

$$\chi_{1\frac{1}{2}-\frac{1}{2}} = \frac{1}{\sqrt{6}}[\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) + \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) - 2\chi_{-\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z})]. \quad (29)$$

(3) $S' = 0, S = \frac{1}{2}$ spin wave functions $\chi_{S'SM_s}$

$$\chi_{0\frac{1}{2}\frac{1}{2}} = \frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z}) - \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{\frac{1}{2}}(s_{3z})], \quad (30)$$

$$\chi_{0\frac{1}{2}-\frac{1}{2}} = \frac{1}{\sqrt{2}}[\chi_{\frac{1}{2}}(s_{1z})\chi_{-\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z}) - \chi_{-\frac{1}{2}}(s_{1z})\chi_{\frac{1}{2}}(s_{2z})\chi_{-\frac{1}{2}}(s_{3z})]. \quad (31)$$

Obviously, the three electrons spin states $\chi_{1\frac{3}{2}\frac{1}{2}}$, $\chi_{1\frac{3}{2}-\frac{1}{2}}$, $\chi_{1\frac{1}{2}\frac{1}{2}}$ and $\chi_{1\frac{1}{2}-\frac{1}{2}}$ are their spin entanglement states.

3 The Entanglement Between the Photons

In Section 2, the entanglement between the electrons come from their Hamiltonian operator of electrons spin interaction. For the two photons spin entanglement, we can assume they are from the Hamiltonian operator of photons spin interaction, they are

$$\hat{H}_I(s_1, s_2) = g_\gamma \vec{s_1} \cdot \vec{s_2}, \quad (32)$$

where g_γ is photons spin coupling strength, $\vec{s_1}$ and $\vec{s_2}$ are the spins of photon 1 and 2. The single photon spin quantum number $s = 1$, and the total spin square of two-photon is

$$\vec{S}^{\,2} = (\vec{s}_1 + \vec{s}_2)^2 = \vec{s}_1^{\,2} + \vec{s}_2^{\,2} + 2\vec{s}_1 \cdot \vec{s}_2 = 4 + 2\vec{s}_1 \cdot \vec{s}_2, \quad (33)$$

and the total spin quantum numbers S are

$$S = 0, 1, 2. \quad (34)$$

the (32) can be written as

$$\hat{H}_I(s_1, s_2) = \frac{1}{2} g_\gamma (\vec{S}^{\,2} - 4), \quad (35)$$

the spin eigenequation of two photons is

$$\frac{1}{2} g_\gamma (\vec{S}^{\,2} - 4)\chi_{SM_s} = E_s \chi_{SM_s}. \quad (36)$$

In Ref. [21], we have given the spin eigenequation of two photons, they are

(1) $S = 2$ two-photon spin symmetrical states

$$\chi_{22} = \chi_1(s_{1z})\chi_1(s_{2z}), \tag{37}$$

$$\chi_{21} = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_1(s_{2z}) + \chi_0(s_{2z})\chi_1(s_{1z})], \tag{38}$$

$$\chi_{20} = \frac{1}{\sqrt{6}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) + 2\chi_0(s_{1z})\chi_0(s_{2z}) + \chi_{-1}(s_{1z})\chi_1(s_{2z})], \tag{39}$$

$$\chi_{2-1} = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_{-1}(s_{2z}) + \chi_{-1}(s_{1z})\chi_0(s_{2z})], \tag{40}$$

$$\chi_{2-2} = \chi_{-1}(s_{1z})\chi_{-1}(s_{2z}). \tag{41}$$

(2) $S = 1$ two-photon spin antisymmetrical states

$$\chi_{11} = \frac{1}{\sqrt{2}}[\chi_1(s_{1z})\chi_0(s_{2z}) - \chi_0(s_{1z})\chi_1(s_{2z})], \tag{42}$$

$$\chi_{10} = \frac{1}{\sqrt{2}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) - \chi_{-1}(s_{1z})\chi_1(s_{2z})], \tag{43}$$

$$\chi_{1-1} = \frac{1}{\sqrt{2}}[\chi_0(s_{1z})\chi_{-1}(s_{2z}) - \chi_{-1}(s_{1z})\chi_0(s_{2z})]. \tag{44}$$

(3) $S = 0$ two-photon symmetrical spin states

$$\chi_{00} = \frac{1}{\sqrt{3}}[\chi_1(s_{1z})\chi_{-1}(s_{2z}) - \chi_0(s_{1z})\chi_0(s_{2z}) + \chi_{-1}(s_{1z})\chi_1(s_{2z})], \tag{45}$$

the single photon spin states χ_0, χ_1 and χ_{-1} are [23]

$$\chi_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \chi_1 = -\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \quad \chi_{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}. \tag{46}$$

Obviously, the three photons spin states χ_{21}, χ_{20}, χ_{2-1}, χ_{11}, χ_{10}, χ_{1-1} and χ_{00} are their spin entanglement states.

For the three photons, the Hamiltonian operator of their spin interaction are

$$\hat{H}_I(s_1, s_2) = g_\gamma(\vec{s_1} \cdot \vec{s_2} + \vec{s_1} \cdot \vec{s_3} + \vec{s_2} \cdot \vec{s_3}). \tag{47}$$

Similarly, we can obtain the three photons spin entanglement states.

4 The Total Entanglement States of two Electrons and two Photons

For the Bose (Fermi) systems, The total states should be symmetrical (antisymmetrical). The total entanglement states between two electrons can be written as

$$\psi^A(\vec{r}_1, \vec{r}_2, s_{1z}, s_{2z}, t) = \psi^S(\vec{r}_1, \vec{r}_2, t) \otimes \chi_{00}^A(s_{1z}, s_{2z}), \tag{48}$$

and

$$\psi^A(\vec{r}_1, \vec{r}_2, s_{1z}, s_{2z}, t) = \psi^A(\vec{r}_1, \vec{r}_2, t) \otimes \chi_{11}^S(s_{1z}, s_{2z}), \tag{49}$$

where $\psi^S(\vec{r}_1, \vec{r}_2, t)$ and $\psi^A(\vec{r}_1, \vec{r}_2, t)$ are space symmetrical and antisymmetrical states, $\chi_{11}^S(s_{1z}, s_{2z})$ and $\chi_{00}^A(s_{1z}, s_{2z})$ are spin symmetrical and antisymmetrical entanglement states of two electrons, i.e., the full entanglement states of two electrons are the direct product of space states and spin entanglement states. When the space state $\psi^S(\vec{r}_1, \vec{r}_2, t) \to 0$ or

$\psi^A(\vec{r}_1, \vec{r}_2, t) \to 0$, the entanglement of two electrons should be disappeared even if they are in the spin entanglement state $\chi^A_{00}(s_{1z}, s_{2z})$ or $\chi^S_{11}(s_{1z}, s_{2z})$. So, the entanglement state of two electrons only exist in the limited spatial scope. When they are far away, the space state $\psi^S(\vec{r}_1, \vec{r}_2, t) \to 0$ or $\psi^A(\vec{r}_1, \vec{r}_2, t) \to 0$, the total entanglement state $\psi^A(\vec{r}_1, \vec{r}_2, s_{1z}, s_{2z}, t)$ should approach to zero, the two electrons will not be in entanglement state even if their spin state is entangled. In experiments [24, 25], the authors have found the two electrons entanglement exit within a limited space range, rather than an arbitrary distance, which is agreement with the theory result.

The total entanglement states between two photons can be written as

$$\psi^S(\vec{r}_1, \vec{r}_2, s_{1z}, s_{2z}, t) = \psi^S(\vec{r}_1, \vec{r}_2, t) \otimes \chi^S_{2M_s}(s_{1z}, s_{2z}), \qquad (M_s = 1, 0. -1) \tag{50}$$

$$\psi^S(\vec{r}_1, \vec{r}_2, s_{1z}, s_{2z}, t) = \psi^A(\vec{r}_1, \vec{r}_2, t) \otimes \chi^A_{1M_s}(s_{1z}, s_{2z}), \qquad (M_s = 1, 0. -1) \tag{51}$$

and

$$\psi^S(\vec{r}_1, \vec{r}_2, s_{1z}, s_{2z}, t) = \psi^S(\vec{r}_1, \vec{r}_2, t) \otimes \chi^S_{00}(s_{1z}, s_{2z}), \tag{52}$$

where $\psi^S(\vec{r}_1, \vec{r}_2, t)$ and $\psi^A(\vec{r}_1, \vec{r}_2, t)$ are space symmetrical and antisymmetrical states, $\chi^S_{2M_s}(s_{1z}, s_{2z})$, $\chi^S_{00}(s_{1z}, s_{2z})$ and $\chi^A_{1M_s}(s_{1z}, s_{2z})$ are spin symmetrical and antisymmetrical entanglement states of two photons. Similarly, When two photons are far away, they will not be in entanglement state even if their spin state is entangled.

5 Conclusion

In this paper, we have studied the entanglement mechanism of electron-electron and photon-photon. With the spin-spin interaction, we have given the spin entanglement states of two electrons, three electrons, two photons, and found their total entanglement states are relevant both space state and spin state. When two electrons or two photons are far away, their entanglement states should be disappeared even if their spin state is entangled.

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