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# High precision relative attitude measurement for inner payload of new photoelectric platform 

Y. Y. Wang, ${ }^{1,2}$ L. Wang, ${ }^{1,2}$ X. G. Bai, ${ }^{1}$ T. W. Ma, ${ }^{1}$ and M. Dai ${ }^{1,2}$<br>${ }^{1}$ Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Science, Changchun 130033, People's Republic of China<br>${ }^{2}$ University of Chinese Academy of Science, Beijing 100039, People's Republic of China

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#### Abstract

A new method for measuring the relative attitude of a load on an airborne photoelectric platform supported by three-axis universal joint structure is proposed. The mathematical model for angle measurement is established, based on which the linear relation between the angle and distance is derived by using the eccentric method. Furthermore, the relative attitude measurement method for a load rotating along the yaw and pitch directions is also proposed based on a unique eccentric structure. Finally, to validate the proposed new method, a comprehensive experiment for measurement angle from $-5^{\circ}$ to $+5^{\circ}$ in increments of $0.5^{\circ}$ was performed by using the angular capacitive sensor. The experimental results show that the precision of the angular measurement is better than $15^{\prime \prime}$ with maximum 1.24 kHz bandwidth, which meets the relative attitude measurement requirements of the internal load on the airborne photoelectric platform. © 2016 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4944476]


## I. INTRODUCTION

An airborne photoelectric platform is a real-time imaging system capable of carrying various measuring equipment. To perform properly during flight, the relative orientation of the optical axis of the platform needs to be controlled.

A conventional photoelectric platform utilizes the structure of two axes and four frames. ${ }^{1,2}$ Since the shaft and frames take up a lot of space, the development and applications of the photoelectric platform are limited. The external frame of the photoelectric platform is used for rough measurement, and the internal frame system is used for realizing the control precision of the platform. For the conventional structure, the accuracy of the internal frame system is influenced by the mechanical connection between the internal and external frames. Therefore, it is very necessary to propose a new structure for the photoelectric platform to achieve higher accuracy and better stability.

Instead of using the conventional structure, Lewis ${ }^{3}$ proposed a new structure with three-axis universal joint, which substantially increased the load capacity of the photoelectric platform. However, the photoelectric encoder, which is sensitive to the relative attitude, could not be installed in the universal joint structure. In order to overcome this shortcoming, more accurate methods for measuring the relative attitude between the internal load and the external frame are investigated. Experimentally, current spatial relative attitude measurement methods mainly focus on optical measurement and imaging measurement. Practically, due to the limited internal space of the photoelectric platform, ${ }^{4-7}$ the optical or imaging measurement methods could not be utilized.

In this paper, a new accurate method for measuring the relative attitude between the internal load and the external frame is proposed. The application of this new method will bring enormous economic benefit.

## II. ECCENTRIC MEASUREMENT METHOD

As shown in Figure 1, the center of the sphere is $O$, the radius of the sphere is $R$, the rotation center is $A$, and the distance between $O$ and $A$ is eccentricity $a$. The sphere is rotating around the center $O$, which also is moving along the red circle $\mathrm{OO}_{1} \mathrm{O}_{2}$. Build a coordinate system $\mathrm{O}_{\mathrm{xyz}}$ shown in Figure 1 and formulate the sphere $O$ as follows:

$$
\begin{equation*}
x^{2}+y^{2}+z^{2}=R^{2} . \tag{1}
\end{equation*}
$$

Assume that the sphere center $O$ rotates around the rotation center $A$ by an angle of $\theta$, sometimes known as "rotation angle," and reaches $O_{1}$ and $O_{2}$, respectively. Then the sphere $O$ can be expressed as follows:

$$
\begin{equation*}
x^{2}+(y-a \cos \theta)^{2}+(z-a \sin \theta)^{2}=R^{2} \tag{2}
\end{equation*}
$$

As shown in Figure 1, the coordinate of the measurement point $P$ is $P\left(x_{0}, y_{0}, z_{0}\right)$. Assume that $P$ is located within plane $y O z$, thus $P$ 's coordinate is $P\left(0, y_{0}, z_{0}\right)$. Spatially, straight line $P O$ is governed by the following equations:

$$
\begin{gather*}
x=0 \\
y=\frac{y_{0}}{z_{0}} z \tag{3}
\end{gather*}
$$

The coordinates of the intersection of the straight line $P O$ and the spherical surface, $T\left(x_{T}, y_{T}, z_{T}\right)$, can be obtained by substituting Equation (3) into Equation (2),

$$
\begin{gather*}
x_{T}=0  \tag{4}\\
y_{T}=\frac{y_{0}(A \pm \sqrt{K})}{z_{0} B},  \tag{5}\\
z_{T}=\frac{A \pm \sqrt{K}}{B} \tag{6}
\end{gather*}
$$



FIG. 1. Eccentric method for angle measurement.
where

$$
\begin{gather*}
K=\left[(1-\cos \theta) \frac{y_{o}}{z_{O}}+\sin \theta\right]^{2} \\
-4\left(1+\frac{y_{o}^{2}}{z_{o}^{2}}\right)\left[a^{2}\left(\sin ^{2} \theta-\cos ^{2} \theta-1+2 \cos \theta\right)-r^{2}\right]  \tag{7}\\
A=2 a\left[(1-\cos \theta) \frac{y_{O}}{z_{O}}+\sin \theta\right]  \tag{8}\\
B=2\left[1+\frac{y_{O}^{2}}{z_{O}^{2}}\right] \tag{9}
\end{gather*}
$$

It should be noticed that there are two intersections between line $P O$ and sphere $O$, thus both positive and negative signs should be applied in Equation (7).

The distance of $P T$ can be expressed as

$$
\begin{equation*}
d=\sqrt{\left(x_{T}-x_{O}\right)^{2}+\left(y_{T}-y_{O}\right)^{2}+\left(z_{T}-z_{O}\right)^{2}} \tag{10}
\end{equation*}
$$

Obviously, the maximum distance between the sphere $O_{1}$ and sphere $O_{2}$ is $2 a$, and the maximum value is $O A$. In order to accurately measure the angle variation, the distance variation should be maximized. Therefore, the sensor should be placed on the outer circumference of $O A$. Corresponding equations for spheres $O_{1}$ and $O_{2}$ are

$$
\left\{\begin{array}{l}
x^{2}+(y-a \cos \theta)^{2}+(z-a \sin \theta)^{2}=R^{2}  \tag{11}\\
x^{2}+(y-a \cos \theta)^{2}+(z+a \sin \theta)^{2}=R^{2}
\end{array}\right.
$$

and the reduced form is

$$
\left\{\begin{array}{l}
z=\delta+a \sin \theta  \tag{12}\\
z=\delta-a \sin \theta
\end{array},\right.
$$

which means that the maximum distance between spheres $O_{1}$ and $O_{2}$ is $2 a \sin \theta$. In addition, during the process of measurement, since the rotation angle could be positive or negative, the measurement value should be uniformly distributed, which


FIG. 2. Variation of the sphere displacement $d$ with the rotation angle $\pm \theta$.
means that the positive and negative direction movements of the sphere $O$ should be as uniformed as possible. According to the geometrical analysis, the maximum displacement of the sphere $O$ occurred at $\theta / 2$, and the maximum displacement is $2 a \sin (\theta / 2)$. In fact, the rotation angle should be denoted by $\pm \theta$ due to the symmetric property. As a matter of fact, the corresponding angle of $-\theta$ should be $360^{\circ}-\theta$. Shown in Figure 2 is the variation of the sphere displacement $d$ with the rotation angle $\pm \theta$, which is symmetric about $90^{\circ}$.

## III. ERROR ANALYSIS OF ECCENTRIC METHOD AND FEASIBILITY ANALYSIS OF ANGLE MEASUREMENT FOR DUAL-AXIS ROTATION

During the measurement process using the eccentric method, ${ }^{8,9}$ due to the eccentricity, the distance between the measuring surface and sensor is variable. As a result, the capacitance of the sensor ${ }^{10-12}$ is variable. Furthermore, uncertain factors, such as the manufacture deviation of the eccentric shaft, the machining accuracy of the sphere surface, the roughness of the sphere surface, and the precision of the sensor, will affect the eccentricity measurement results. Shown in Figure 3 is the relationship between the measurement distance and the rotation angle with different eccentric distances. The measurement range of the capacitance sensor (bounded by the red box in Figure 3) is $500 \mu \mathrm{~m}$. Thus, the maximum eccentricity is 2.5 mm .

In fact, the variation of the arc vertical distance is only related to the rotation angle and eccentricity, but it is unrelated to the sphere radius. The linear relation of the measurement results and the rotation angle is shown in Figure 4. Therefore, high precision capacitive sensors can be employed to measure the rotation angle.

For the photoelectric platform, there are two relative motions between the internal and external frames: yaw motion and pitch motion. ${ }^{13-16}$ The instrument mentioned above can only measure the angular displacement in single direction, but the yaw or pitch angle cannot be obtained simultaneously. A
reasonable design shown in Figure 5 can measure the yaw and pitch motions of the internal frame at the same time.

In the coordinate system shown in Figure 5, the rotation along the Z -axis is defined as yaw motion, and the rotation along the X -axis is defined as pitch motion. The center of the coordinates, $O$, also is the intersection of the two rotation axes. The center of the yaw rotation sphere ( $O_{\text {yaw }}$ ) is not in line with the center of the pitch rotation sphere $\left(O_{\text {pitch }}\right)$. Plus, the eccentricities of these two centers are equivalent $\left(O O_{\text {yaw }}=O O_{\text {yaw }}=a\right)$. It is obvious that the yaw rotation sphere $O_{\text {yaw }}$ is located on the yaw rotation axis $O Z$.

Assume that $O_{\text {yaw }}$ and $O_{\text {pitch }}$ are rotating along axis $O Z$, which means that the internal frame is producing yaw motion; the measurement value of the yaw sensor ( $S_{\text {yaw }}$ ) is constant; and due to the existence of the eccentricity $(a)$, the measurement value of the pitch sensor ( $S_{\text {pitch }}$ ) is variable. Similarly, if $O_{y a w}$ and $O_{\text {pitch }}$ are rotating along axis $O X$, which means that the internal frame is producing pitch motion; the measurement value of the pitch sensor ( $S_{\text {pitch }}$ ) is constant; and due to the existence of the eccentricity $(a)$, the measurement value of the yaw sensor ( $S_{\text {yaw }}$ ) is variable, which means that the yaw rotation angle is measurable,

$$
\left\{\begin{array}{l}
x^{2}+\left(y-a_{1} \sin \theta\right)^{2}+\left(z-a_{1} \cos \theta\right)^{2}=R_{1}^{2}  \tag{13}\\
\left(x-a_{2} \cos \theta \sin \varphi\right)^{2}+\left(y-a_{2} \sin \theta \cos \varphi\right)^{2}+\left(z-a_{2} \cos \theta\right)^{2}=R_{2}^{2}
\end{array}\right.
$$

The distance from $\mathrm{S}_{\text {yaw }}$ and $\mathrm{S}_{\text {pitch }}$ to the spherical surface is

$$
\left\{\begin{array}{l}
d_{\text {yaw }}=y_{0}-\sqrt{R_{1}^{2}-\left(x_{0}-a_{1} \cos \varphi\right)^{2}}-a \sin \varphi  \tag{14}\\
d_{\text {pitch }}=y_{0}-\sqrt{R_{2}^{2}-\left(z_{0}-a_{2} \cos \theta\right)^{2}}-a \sin \theta
\end{array}\right.
$$

Similarly, if $x_{0} \approx a_{1}$ and $z_{0} \approx a_{2}$, upper bounds can be simplified to

$$
\left\{\begin{array}{l}
d_{\text {yaw }}=y_{0}-R_{1}-a \sin \varphi  \tag{15}\\
d_{\text {pitch }}=y_{0}-R_{2}-a \sin \theta
\end{array} .\right.
$$

Since the yaw and pitch axes are in the orthogonal coordinate system, the rotation of each other will not affect the movement of other axes.

## IV. EXPERIMENTAL VALIDATION

## A. Experiment system

In order to measure the attitude angle of the internal frame using the eccentric method, the experimental equipment


FIG. 3. Relationship between the measurement distance and the rotation angle with different eccentric distances.
shown in Figure 6 is designed. Within this equipment, the photoelectric encoder is used to detect the measurement precision of the eccentric method, the distance between the shaft and the center of the sphere is the eccentricity, the high precision capacitive sensor is fixed with the base, and the center of the sensor is in line with the geometrical center of the sphere.

In this experiment, high precision capacitive encoder DS90 is used as the attitude angle detector. The relative angular rotation between the internal frame and the external frame is in the range of $-5^{\circ} \sim+5^{\circ}$, and the dominant components are low frequency components. The single pole capacitive sensor with high precision manufactured by Physik Instrument (PI) is made of stainless steel. For this capacitive sensor, the measurement distance is in the range of $20 \sim 100 \mu \mathrm{~m}$, when measured static measuring resolution less than $0.001 \%$ of measured values and dynamic measurement resolution not less than $0.002 \%$ of measured values, so the diameter of the sensor is $8 \sim 20 \mathrm{~mm}$, and other main parameters are shown in Table I.

Theoretically, the measurement accuracy of the photoelectric encoder should be one order of magnitude higher than that of the capacitive sensor designed in this paper. According


FIG. 4. Linear relation of the measurement results and the rotation angle.


FIG. 5. Double capacitive sensors for angle measurement of dual-axis rotation.
to the structure design requirements of the photoelectric platform, the minimum measurement precision of the rotation angle should be $20^{\prime \prime}$. In this experiment, photoelectric encoder with a precision better than $2^{\prime \prime}$ is utilized. Meanwhile, since the measurement precision is closely related to the manufacture deviation of the sphere and the eccentricity, it should be improved during the manufacture process. In addition, during the installation process of the capacitive sensor, the roughness of the sphere surface and the installation direction should be taken into consideration as the factors which will affect the measurement results.

The precision of the 20 bit photoelectric encoder is $1.2^{\prime \prime}$ (better than $2^{\prime \prime}$ ), and thus it can be utilized to detect the measurement precision of this equipment, in which the eccentricity


FIG. 6. Measuring equipment used to measure the attitude angle.

TABLE I. Main parameters of the single pole capacitive sensor.

| Parameters | Value |
| :--- | :---: |
| Channel | 1 |
| Sensor bandwidth $(\mathrm{Hz})$ | $1.24 \times 10^{3} / 112.4 / 10.0$ |
| Measurement range $(\mu \mathrm{m})$ | 500 |
| Static resolution $(\%)$ | $<0.002(\mathrm{RMS})$ |
| Dynamic resolution $(\%)$ | $<0.003(\mathrm{RMS})$ |
| Linearity at nominal range $(\%)$ | $< \pm 0.25$ |

$a=2 \mathrm{~mm}$. As observed from Figure 3, the measurement distance should be in the range of $\pm 0.1743115 \mathrm{~mm}$. As stated in Section III, angle measurement using the eccentric method is a linear measurement process. Therefore, in the range of $\pm 0.1743115 \mathrm{~mm}$, to ensure the precision of angle measurement better than $20^{\prime \prime}$, the distance measurement precision of the capacitive sensor should be better than 0.0049803 mm . In fact, the measurement precision of the selected capacitive sensor is $0.002 \%$ higher than that of the measurement value, which can meet above requirements.

## B. Measurement result analysis

Utilizing the measuring equipment shown in Figure 6 and the step size of $0.5^{\circ}$, the measurement accuracy of this system has been validated continuously. The experimental results are shown in Table II. In this experiment, the radius of the sensor $R$ is 240 mm , eccentricity $a=1 \mathrm{~mm}$, and the rotation angle is in the range of $-5^{\circ} \sim+5^{\circ}$. The true value is obtained by angle encoder, and the vertical distance is calculated by arc formula.

For the angle measurement using the method proposed in this paper, the precision is better than $15^{\prime \prime}$ and the average

TABLE II. Experimental results.

| Rotation angle <br> $(\mathrm{deg})$ | The vertical distance <br> from the surface (mm) | Linearity factor |
| :--- | :---: | :---: |
| 5 | 0.087156 | 0.017431 |
| 4.5 | 0.078459 | 0.017435 |
| 4 | 0.069756 | 0.017439 |
| 3.5 | 0.061049 | 0.017433 |
| 3 | 0.052336 | 0.017435 |
| 2.5 | 0.043619 | 0.017438 |
| 2 | 0.034899 | 0.017430 |
| 1.5 | 0.026177 | 0.017431 |
| 1 | 0.017452 | 0.017432 |
| 0.5 | 0.008727 | 0.017434 |
| 0 | 0 | 0 |
| -0.5 | -0.008727 | 0.017434 |
| -1 | -0.017452 | 0.017432 |
| -1.5 | -0.026177 | 0.017431 |
| -2 | -0.034899 | 0.017430 |
| -2.5 | -0.043619 | 0.017438 |
| -3 | -0.052336 | 0.017435 |
| -3.5 | -0.061049 | 0.017433 |
| -4 | -0.069756 | 0.017439 |
| -4.5 | -0.078459 | 0.017435 |
| -5 | -0.087156 | 0.017431 |

error is $10^{\prime \prime}$. Also, the bandwidth of the single polar sensor is 1.24 kHz , which means that the measurement bandwidth of the equipment fully meets the design requirements.

## V. CONCLUSIONS

In this paper, a new method to measure the relative attitude angle between the internal and external frames of the platform is proposed. This method is based on the basic principle of the eccentric method and the new three-axis universal joint structure. Experimental results show that using the proposed measurement method, the average error of the measurement is lower than $10^{\prime \prime}$, the measurement accuracy is better than $15^{\prime \prime}$, and the measurement bandwidth is 1.24 kHz . For the internal load of the airborne photoelectric platform, all of these criteria fully meet the relative attitude measurement requirements.
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