Ultrasonics 71 (2016) 172–176

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Band gap in tubular pillar phononic crystal plate

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ARTICLE INFO

Article history:
Received 19 December 2015
Received in revised form 23 May 2016
Accepted 22 June 2016
Available online 22 June 2016

Keywords:
Phononic crystal plate
Tubular pillar
Band gap opening
Enlargement

ABSTRACT

In this paper, a phononic crystal (PC) plate with tubular pillars is presented and investigated. The band structures and mode displacement profiles are calculated by using finite element method. The result shows that a complete band gap opens when the ratio of the pillar height to the plate thickness is about 1.6. However, for classic cylinder pillar structures, a band gap opens when the ratio is equal or greater than 3. A tubular pillar design with a void room in it enhances acoustic multiple scattering and gives rise to the opening of the band gap. In order to verify it, a PC structure with double tubular pillars different in size (one within the other) is introduced and a more than 2 times band gap enlargement is observed. Furthermore, the coupling between the resonant mode and the plate mode around the band gap is characterized, as well as the effect of the geometrical parameters on the band gap. The behavior of such structure could be utilized to design a pillar PC with stronger structural stability and to enlarge band gaps.

1. Introduction

Phononic crystals (PCs) [1,2] have drawn great attention during past two decades. The most notable property of the PCs is the existence of phononic band gaps (PBGs), making them excellent choices for applications such as noise control [3,4], acoustic filtering and confinement [5–8]. Because of the simple structure and being easy to fabricate, two kinds of the PC plates, i.e. the holed PC plate [5,9] and the pillared PC plate [10–13], have recently received increasing attention. However, the pillared PC plates possess more advantages. A high filling factor is usually required to obtain a wide band gap in holed PC plate, which results in fabrication challenges [14]. In contrast, pillar-based structures provide PBGs without stringent etching requirements [10]. Since they possess another degree of freedom in designing the PC, i.e. the height of the pillars. Furthermore, such kind of structure provides one side of perfect surface, a desirable platform for dealing with liquids or functioning process for immunological detection, showing great potential in bio-sensing applications [9].

The pillar PC plate is proposed and investigated independently by Wu et al. [10,13] and Pennec et al. [11,12] After then, much effort has devoted to it. The researchers reported that the PC plates with soft pillars [4,11,15,16] or composite pillars [17–19] support ultra-low frequency band gaps, while plates with double-side pillar [17], "necked" pillar [20,21] and hole-pillar structures [22] contribute to band gap enlargement. These works focused on gaps induced by locally resonance indicate great potential in sound insulation or acoustic control. Although pillar PC plates support Bragg band gaps as well [15,16], relative little attention has been paid to them [23]. Bragg gaps are of relative high frequency range and more suitable for sensing applications. Generally, hard pillars are required to obtain such gaps and tall pillars contribute to wide band gaps, indicating well acoustic energy confinement and high quality factors. As previous study reported, a band gap opens when the pillar height is triple the plate thickness and reaches maximal width when that ratio is ten [10,13]. However, tall pillars simultaneously increase the fabrication process cost and decrease the structural stability.

In this paper, we present an original structure composed of tubular pillars on a plate, by which we partially overcome the limitation for PBG generation condition, known as the ratio of pillar height to plate thickness. For classic cylindrical pillared PC, the ratio is about three. However, for tubular pillar PC, a complete PBG is observed with the ratio about 1.6. A tubular pillar design with a void room in it enhances acoustic multiple scattering and gives rise to the opening of the band gap. Based on such mechanism, a PC structure with double tubular pillars different in size (one within the other) is introduced and a more than 2 times band gap enlargement is observed. The coupling between the resonant mode and the plate modes on the PBG is characterized, as well as the effect of the geometrical parameters on the band gap. Such PC structures show great interest in complement of PC plate with stronger structural stability. Besides, they provide a new method of band gap enlargement.

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http://dx.doi.org/10.1016/j.ultras.2016.06.011

2. Model and simulation

The structure of the planer tubular pillared PCs realized in square lattices and irreducible Brillouin zones are shown in Fig. 1(a) and 1(b), respectively. The lattice constant is denoted as a and the plate thickness is e. The height, outer radius and inner radius of the tubular pillar are h, R and r, respectively. Both the substrate and the pillars are chose to be isotropic silicon (Si). The material properties are as follows, the mass density is 2329 kg/m^3, Young's modulus is 1.7×10^{11} Pa, and the Poisson's ratio is 0.28.

The numerical calculation of the PC is based on an efficient finite element method (FEM) [15] implemented using the COMSOL Multiphysics software. The band structures are calculated by applying the Floquet periodic boundary conditions to the interfaces between the nearest unit cells, and then solving eigenvalues with discrete wave vectors along the edge of the first irreducible Brillouin zone based on the physical model described as the following equation,

$$\nabla \cdot C : \left[\nabla^S u + \frac{i}{2}\left(\mathrm{k}^{\mathrm{T}} \otimes u + \mathrm{k} \otimes u^{\mathrm{T}}\right)\right] = -\rho\omega^2 u, \quad (1)$$

where C is the elasticity tensor, k is the wavevector, ρ is mass density, ω is the frequency, u is the Bloch displacement vector and the symbol $\otimes$ denotes the outer product.

3. Results and analysis

3.1. Mechanism of PBG creation

We calculated the band structure of the model shown in Fig. 1(a) with geometrical parameters as the following: $e = 0.1\ a$, $h = 0.2\ a$, and $R = 0.45\ a$. The filling factor β is chosen to be fixed to 0.385, the same value as that in studies reported that a PBG in cylinder pillared PC is formed when the pillar height is triple of the plate thickness [10,13]. The inner radius of tubular pillar $r = 0.283\ a$, which is determined by the relationship $\pi\,(R^2 - r^2)/a^2 = \beta$. Because of the periodicity of the structure, only one unit cell is considered and the structure is assumed to be infinite and periodic in both x and y directions. Fig. 2 represents the band structure calculated by FEM. A complete PBG with normalized frequency extending from 4612 to 4786 m/s is observed.

The band gap in Fig. 2(a) shows some characteristics, indicating it is a Bragg band gap. First, the normalized center frequency of the PBG is about 4700 m/s, nearing the velocity of transverse waves in the Si membrane (about 5000 m/s). That is, at the frequency of the band gap, the wavelength is comparable to the period of the PC. Second, dispersion branches at the edges of the band gap show

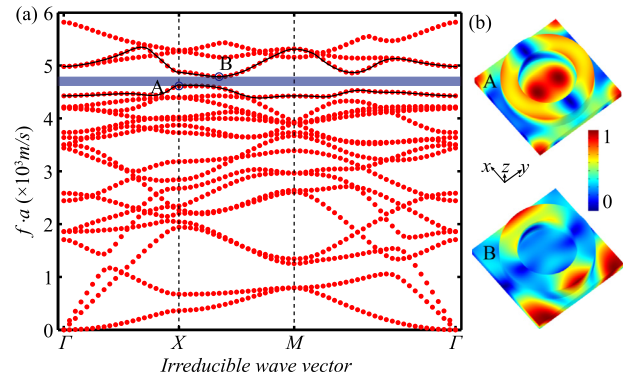

Fig. 2. (a) Band structure of the model in Fig. 1(a) with $e = 0.1\ a$, $h = 0.2\ a$, $R = 0.45\ a$, and $\beta = 0.385$. The shaded rectangular region represents the complete band gap. A and B denote the modes on the lower and upper edge of the PBG. (b) The displacement profiles of modes A and B. The blue (red) color in the color legend corresponds to the lower (higher) value of the displacement field.

sharp curve shape. The displacement profiles of modes A and B, located respectively on the lower and upper edges of the band gap, are calculated and shown in Fig. 2(b). Both of them have displacement not only in the pillar and but also in the membrane, indicating the group velocities are not zero.

The mechanism of the PBG opening in our model is similar to that in previous model, proposed by Wu et al. as mentioned in the introduction [13]. Oudich et al. have verified that the band gaps opening in their models are due to Bragg scattering. For structures constructed with a hard pillar on a plate, the coupling between the Lamb modes and the pillar modes is strong, a Bragg scattering procedure, which means that a periodic structure has to be used to obtain a PBG [16].

The key difference is that, compared to their model, a tubular pillar is used instead of the cylinder pillar. We argue that the designed structure consisting of a void room in pillar enhances the acoustic multiple scattering, and therefore, contributes to the band gap opening in structures with low height of pillars. To confirm that statement, a PC structure with double tubular pillars different in size stacked together (one within the other) is proposed. It is expected that a wider band gap will be observed since that the stacked tubular pillars provide even stronger multiple scattering of acoustic waves than a single tubular pillar.

The band structure of a double-pillar PC plate is calculated, with the geometrical parameters $R_1 = 0.4\ a$, $r_1 = 0.29\ a$, $R_2 = 0.24\ a$ and $r_2 = 0.1\ a$, where R_1 and r_1 are the outer and inner radius of the tubular pillar of large size, respectively. R_2 and r_2 correspond these of the smaller one. The filling factor and the height of the pillars keep remain. The result is shown in Fig. 3(a), there exists an

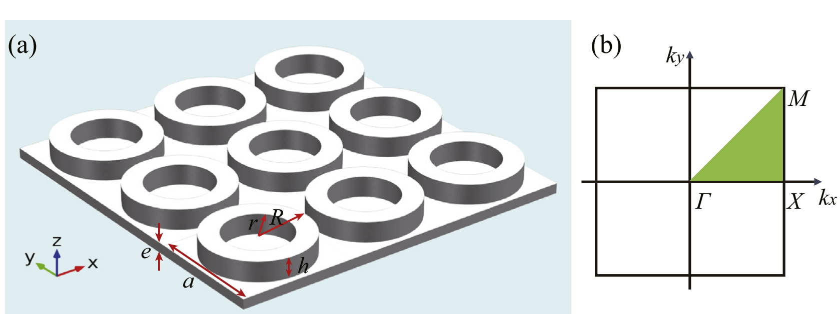

Fig. 1. (a) Schematic of the PC constituted by a square lattice of tubular pillars arranged on a plate. The lattice constant is a, the thickness of the plate is e. The height, inner radius and outer radius of tubular pillar are h, r, and R, respectively. (b) The corresponding first irreducible Brillouin zone (green region) and the high-symmetry points Γ, X, and M.

absolute PBG spanning in the $f\ a$ = 4121–4490 m/s range. The width of the band gap increases by a factor of more than 2 from 174 m/s to 369 m/s, compared to that in Fig. 2(a). In contrast, as shown in Fig. 3(b), there exists no band gap in a solid pillar case with the same filling factor and the pillar height. The significant enlargement of gap suggests that tubular pillars could help to enhance the acoustic scattering and give rise to the PBG opening in PC plate with low height of tubular pillars.

3.2. Effect of resonant mode on band gap

Generally, a resonant mode corresponds a flat dispersion branch while a plate mode corresponds a sharp branch. When resonant mode couples to plate mode, a partly flat dispersion curve is observed in band structure [16]. For single tubular pillar, the coupling effect is strong and has a significant influence on the band gap. The band structures of the model in Fig. 1(a) are calculated with graduated parameters r = 0.26 a, 0.24 a and 0.22 a. Other parameters for all three cases are as follows, h = 0.25 a, e = 0.1 a and R = 0.45 a. The results are shown in Fig. 4(a)–(c), respectively. It could be found that the frequency of the resonant mode, represented by the mode C at point Γ here, decreases as r increases. When the frequency of mode C is less than that of mode A, as shown in Fig. 4(a), the branch bends upward and mode A determines the lower edge of the PBG. When it is great than mode A's, the branch bends downward, as shown in Fig. 4(b). The width of band gap decreases as the resonant mode's frequency increases. The PBG is closed when its frequency is equal or great than the frequency of mode B, as shown in Fig. 4(c).

Fig. 5(a) and (b) represent band structures of double-pillar PC with different parameters, for the former R_1 = 0.45 a, r_1 = 0.32 a, R_2 = 0.18 a and r_2 = 0.1 a, and for the later R_1 = 0.42 a, r_1 = 0.3 a, R_2 = 0.22 a and r_2 = 0.11 a. The filling factor is fixed to 0.385 and h = 0.2 a. As can be seen in Fig. 5(a), it exists two PBGs, the first one spanning in the $f\ a$ = 3955–4095 m/s range and the second one in the $f\ a$ = 4178–4324 m/s range with a single branch separating the two PBGs. The displacement field of mode D on this branch at point X is shown in the Fig. 5(c). The band structure in Fig. 5(b) is similar to that in Fig. 5(a). The difference between them is the location of the single branch in the band gap. the Band structures in Figs. 2, 5(a) and (b) suggest that the coupling between the resonant mode and the plate mode in double tubular pillar PC is much weaker than that in single one PC. Thus, a much flatter dispersion branch is obtained even around the edge of the band gap.

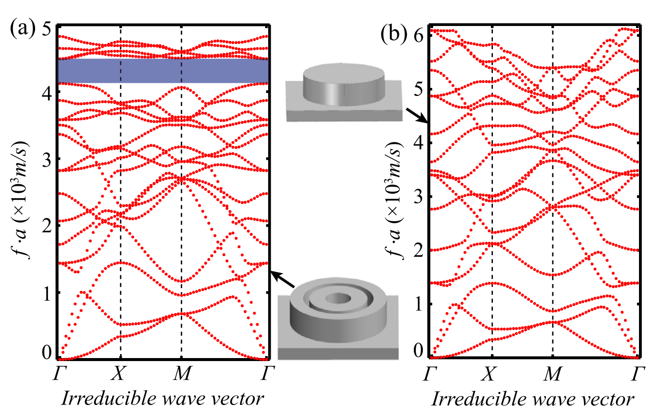

Fig. 3. (a) The band structure of a PC structure with double tubular pillars different in size stacked together (one within the other). The geometrical model is shown aside. The shaded rectangular region represents the complete band gap. (b) The band structure of a PC structure with solid pillars with geometrical model aside.

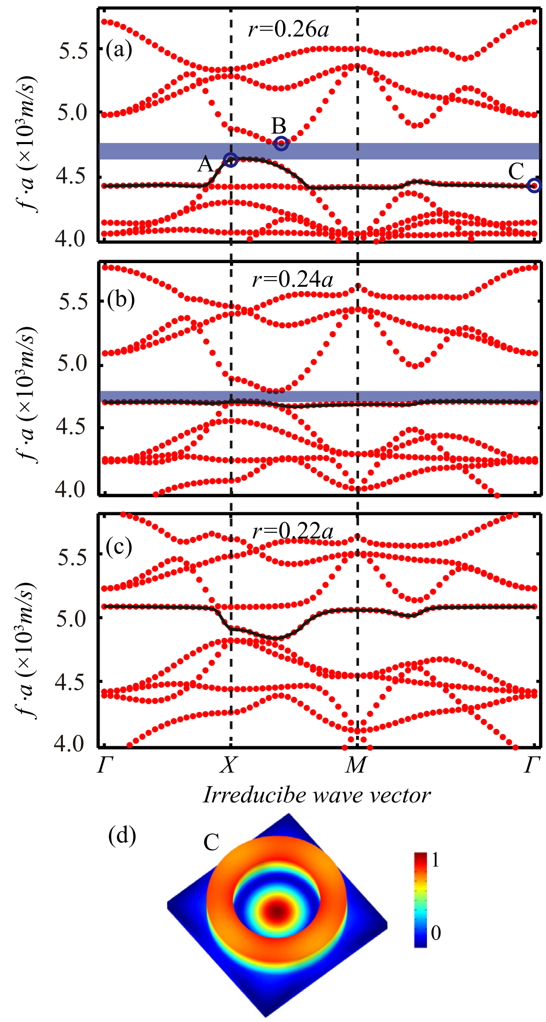

Fig. 4. The coupling effect of the resonant mode and the plate mode for cases (a) r = 0.26 a, (b) r = 0.24 a, and (c) r = 0.22 a. C denotes a resonant mode at point Γ. (d) The displacement profile of mode C. The color legend denotes the normalized displacement.

3.3. Effect of geometrical parameters

Since the coupling effect between the resonant mode and the plate mode, the position and extent of the PBG in tubular PC show highly dependency on the geometrical parameters. In order to investigate this effect, the evolution of the PBG with respect to the outer radius of the pillar is calculated. For the case of single tubular pillar, the parameters are as follows, e = 0.1 a, h = 0.2 a, β = 0.385 and R ranges from 0.425 a to 0.49 a. Simulation result is illustrated in Fig. 6(a). It can be found that the band gap starts to open at R = 0.43 a. The width of the PBG increases gradually as the increase of R and reaches its maximal value at approximately R = 0.46 a. Then it turns to decrease and close at R = 0.485 a. The central normalized frequency of the PBG decreases as the increase of R. The evolution of the PBG shows that, at a given filling factor, the radius of the tubular pillar can significantly influence the width of the PBG. When the outer radius of the pillar is less than 0.43 a, the room in tubular pillar is too small to efficiently enhance the acoustic multiple scattering. While, when R is greater than 0.46 a, the small gaps between the pillars result in the coupling of elastic energy between them, forming travelling waves in the structure, and thus, the PBG disappears. For the case of double tubular pillars, e = 0.1 a, h = 0.2 a, β = 0.385. The parameters of inner pillar are fixed (R_2 = 0.24 a and r_2 = 0.1 a) while these of outer

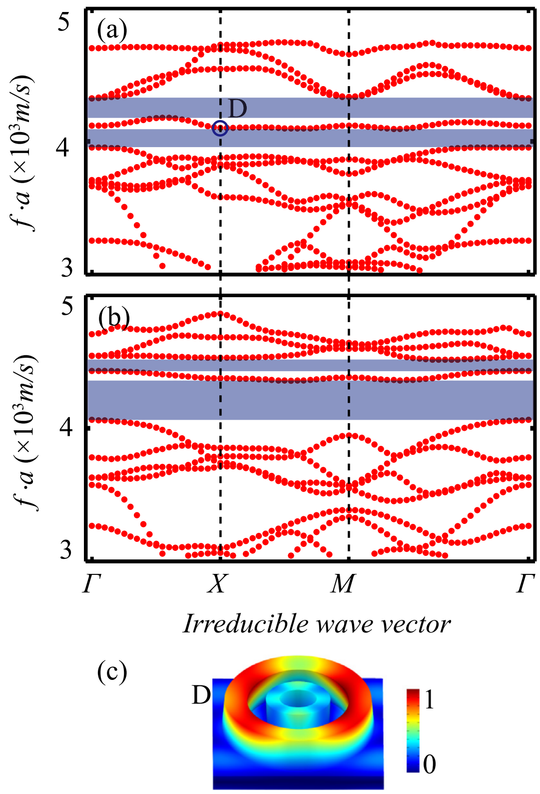

Fig. 5. Locations of the resonant mode in the band structures of double-pillar PC with different parameters, (a) $R_1 = 0.45\ a$, $r_1 = 0.32\ a$, $R_2 = 0.18\ a$, $r_2 = 0.1\ a$, and (b) $R_1 = 0.42\ a$, $r_1 = 0.3\ a$, $R_2 = 0.22\ a$, $r_2 = 0.11\ a$. The shaded rectangular regions represent the complete band gaps. (c) The displacement profile of mode D at point X. The color legend denotes the normalized displacement.

pillar vary (R_1 ranges from 0.39 a to 0.45 a). As can be seen in Fig. 6(c), it exists two shaded regions, showing the evolution of two PBGs which are separated by a single branch (see Fig. 5). Numerical results also show that the sizes of the shaded regions are related to the parameters of inner pillar and the results given in Fig. 6(c) is a typical one. The total extension of the PBGs decreases gradually as the increase of R_1 and PBG is closed at approximately $R = 0.45\ a$.

The evolution of the PBG with respect to the height of the pillar is characterized as well. For the case of single tubular pillar, the parameters are as follows, $e = 0.1\ a$, $R = 0.45\ a$, $\beta = 0.385$ and h ranges from 0.425 a to 0.49 a. For the case of double tubular pillars, $R_1 = 0.39\ a$, $r_1 = 0.278\ a$, $R_2 = 0.24\ a$ and $r_2 = 0.1\ a$. The results are illustrated in Fig. 6(b) and (d). For the single pillar case, it can be found that the band gap starts to open at $h = 0.155\ a$. The width of the PBG increases gradually as the increase of h and reaches its maximal value at approximately $h = 0.325\ a$ and then it turns to decrease slowly. However, for the double pillars case, the gap starts to open at $h = 0.17$ a and closes at $h = 0.26\ a$. Compared with the results in previous work [13], the evolution of the band gap behaves differently in position and extension. It is due to that pillar modes with different geometrical sizes and vibrational types behave differently and open band gap at different frequency ranges. The position of the PBG in single tubular pillar PC is about 2 to 3 times great than that in classic cylinder pillar PC.

Previous researches have reported that a tall height of pillar contributes to the opening and enlargement of band gaps [10,13]. In this paper, we show that a tubular pillar design causes the similar results. Therefore, only low height of tubular pillar is required to generate a band gap. However, the two effects seem to do not occur simultaneously with the present parameters. As can be seen in Fig. 6(b), the width of the band gap does not enlarge significantly as the height of pillar increases. It may be a challenging but promising task to design a structure combining such two effects for band gap enlargement.

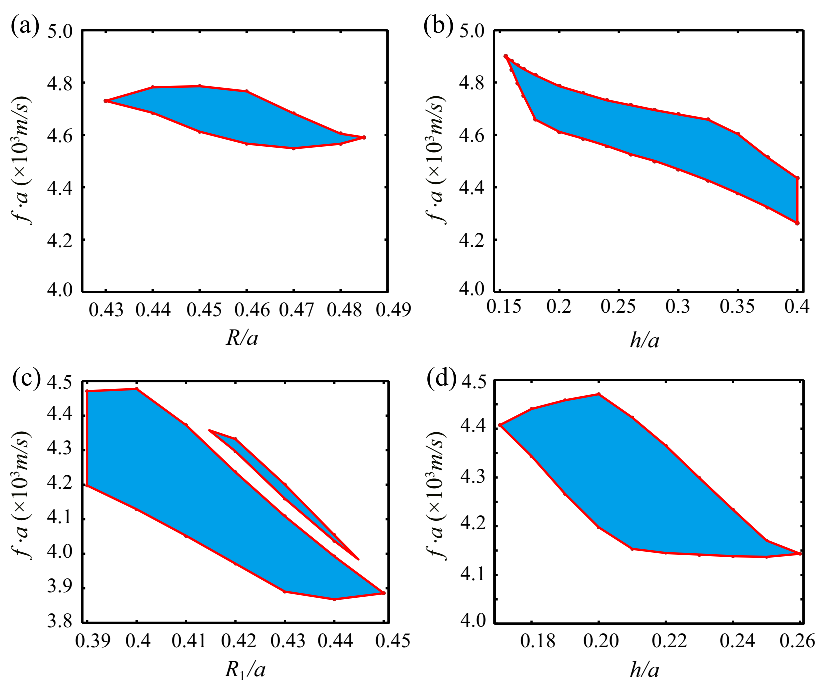

Fig. 6. Evolution of the PBG in tubular pillar PC plate with respect to (a) the normalized outer radius (R/a) for single pillar case, (b) the normalized height (h/a) for single pillar case, (c) the normalized outer radius (R_1/a) for double pillars case, (d) the normalized height (h/a) for double pillars case. The extension of the PBGs is shown by the shaded region (blue).

A double tubular pillars PC plate supports a much wider band gap than a single one PC at same filling factor. However, Structures with more tubular pillars PC plate (e.g. a triple tubular pillars PC plate), which is believed to be of great interest for band gap enlargement without material adding, is not considered in this paper.

4. Conclusion

In this paper, a phononic crystal (PC) plate with tubular pillars is presented and investigated. The band structures and mode displacement profiles are calculated by using finite element method. The results show that a complete band gap opens when the ratio of the pillar height to the plate thickness is about 1.6. However, for classic cylinder pillar structures, a band gap opens when the ratio is equal or greater than 3. Acoustic multiple scattering enhanced by the tubular pillar design gives rise to the opening of the band gap. In order to verify it, a PC structure with double tubular pillars different in size (one within the other) is introduced and a more than 2 times band gap enlargement is observed. Furthermore, coupling between the resonant mode and the plate modes around the band gap is characterized. The coupling effect in single tubular pillar PC plate is strong and the band gap width shows highly dependency on the geometrical sizes of the tubular pillar. In contrast, the coupling effect in double tubular pillars PC plate is much weaker. Therefore, a flat band branch is observed and its position in the band gap shows highly dependency on the geometrical sizes of the outer tubular pillar. The tubular pillar design could be utilized to complete a pillar PC plate with stronger structural stability and less fabrication cost. In addition, these works also suggest a method of band gap enlargement.

Acknowledgment

This work was supported by the National High Technology Research and Development Program (No. 2015AA042603).

References

[1] M. Sigalas, E. Economou, Elastic and acoustic wave band structure, J. Sound Vib. 158 (1992) 377–382.
[2] M.S. Kushwaha, P. Halevi, L. Dobrzynski, B. Djafari-Rouhani, Acoustic band structure of periodic elastic composites, Phys. Rev. Lett. 71 (1993) 2022–2025.
[3] Z. Liu, X. Zhang, Y. Mao, Y.Y. Zhu, Z. Yang, C.T. Chan, P. Sheng, Locally resonant sonic materials, Science 289 (2000) 1734–1736.
[4] M. Oudich, M. Senesi, M.B. Assouar, M. Ruzenne, J.-H. Sun, B. Vincent, Z. Hou, T.-T. Wu, Experimental evidence of locally resonant sonic band gap in two-dimensional phononic stubbed plates, Phys. Rev. B 84 (2011).
[5] S. Mohammadi, A.A. Eftekhar, W.D. Hunt, A. Adibi, High-Q micromechanical resonators in a two-dimensional phononic crystal slab, Appl. Phys. Lett. 94 (2009) 051906.
[6] N. Wang, F.-L. Hsiao, M. Palaniapan, C. Lee, Silicon two-dimensional phononic crystal resonators using alternate defects, Appl. Phys. Lett. 99 (2011) 234102.
[7] J. Liu, F. Li, Y. Wu, The slow zero order antisymmetric Lamb mode in phononic crystal plates, Ultrasonics 53 (2013) 849–852.
[8] D. Feng, D. Xu, B. Xiong, Y. Wang, Continuous tuning of line defect modes in silicon two-dimensional phononic crystal, J. Phys. D Appl. Phys. 48 (2015) 225102.
[9] P. Li, F. Li, Y. Liu, F. Shu, J. Wu, Y. Wu, Temperature insensitive mass sensing of mode selected phononic crystal cavity, J. Micromech. Microeng. 25 (2015) 125027.
[10] T.-T. Wu, Z.-G. Huang, T.-C. Tsai, T.-C. Wu, Evidence of complete band gap and resonances in a plate with periodic stubbed surface, Appl. Phys. Lett. 93 (2008) 111902.
[11] Y. Pennec, B. Djafari-Rouhani, H. Larabi, J.O. Vasseur, A.C. Hladky-Hennion, Low-frequency gaps in a phononic crystal constituted of cylindrical dots deposited on a thin homogeneous plate, Phys. Rev. B 78 (2008).
[12] Y. Pennec, B. Djafari Rouhani, H. Larabi, A. Akjouj, J.N. Gillet, J.O. Vasseur, G. Thabet, Phonon transport and waveguiding in a phononic crystal made up of cylindrical dots on a thin homogeneous plate, Phys. Rev. B 80 (2009).
[13] T.-C. Wu, T.-T. Wu, J.-C. Hsu, Waveguiding and frequency selection of Lamb waves in a plate with a periodic stubbed surface, Phys. Rev. B 79 (2009).
[14] S. Mohammadi, A.A. Eftekhar, A. Khelif, W.D. Hunt, A. Adibi, Evidence of large high frequency complete phononic band gaps in silicon phononic crystal plates, Appl. Phys. Lett. 92 (2008) 221905.
[15] R. Pourabolghasem, A. Khelif, S. Mohammadi, A.A. Eftekhar, A. Adibi, Physics of band-gap formation and its evolution in the pillar-based phononic crystal structures, J. Appl. Phys. 116 (2014) 013514.
[16] M. Oudich, Y. Li, B.M. Assouar, Z. Hou, A sonic band gap based on the locally resonant phononic plates with stubs, New J. Phys. 12 (2010) 083049.
[17] M. Badreddine Assouar, M. Oudich, Enlargement of a locally resonant sonic band gap by using double-sides stubbed phononic plates, Appl. Phys. Lett. 100 (2012) 123506.
[18] Y. Li, T. Chen, X. Wang, Y. Xi, Q. Liang, Enlargement of locally resonant sonic band gap by using composite plate-type acoustic metamaterial, Phys. Lett. A 379 (2015) 412–416.
[19] M. Badreddine Assouar, J.H. Sun, F.S. Lin, J.C. Hsu, Hybrid phononic crystal plates for lowering and widening acoustic band gaps, Ultrasonics 54 (2014) 2159–2164.
[20] J.-C. Hsu, Local resonances-induced low-frequency band gaps in two-dimensional phononic crystal slabs with periodic stepped resonators, J. Phys. D Appl. Phys. 44 (2011) 055401.
[21] C. Ma, J. Guo, Y. Liu, Extending and lowing band gaps in one-dimensional phononic crystal strip with pillars and holes, J. Phys. Chem. Solids 87 (2015) 95–103.
[22] O.R. Bilal, M.I. Hussein, Trampoline metamaterial: local resonance enhancement by springboards, Appl. Phys. Lett. 103 (2013) 111901.
[23] R. Pourabolghasem, S. Mohammadi, A.A. Eftekhar, A. Khelif, A. Adibi, Experimental evidence of high-frequency complete elastic bandgap in pillar-based phononic slabs, Appl. Phys. Lett. 105 (2014) 231908.