



## Quantum transmissivity of one-dimensional mirror structure photonic crystals



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### ABSTRACT

In this paper, we have studied the transmission characteristic of one-dimensional mirror structure photonic crystals (MSPCs)  $(AB)^N D_1 D_2 D_3 (BA)^N$  with the quantum theory method. We find there are some sharp peaks (quantum transmissivity  $T=1$ ) in the PBG of MSPCs, and the number of sharp peaks is added with the increasing of thickness, refractive index and numbers of defect layer, which is beneficial to design the optic filter of multiple channel.

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## 1. Introduction

Photonic crystals (PCs) originate from (a) theoretical work of Yablonovitch, and (b) John's experimental work; both of which were published almost simultaneously in 1987 [1,2]. PCs are artificial dielectric or metallic structures in which the refractive index changes periodically in space. Such materials are unique in that their dielectric permittivity can change periodically in one, two or three dimensions at a spatial scale comparable with the light wavelength. This kind of periodic structure affects the propagation of electromagnetic waves in the similar way as the periodic potential in a semiconductor crystal affects the electron motion by defining allowed and forbidden electronic energy bands. Whether or not photons propagate through PC structures depends on their frequency. Frequencies that are allowed to travel are known as modes, and groups of allowed modes form bands. Disallowed bands of frequencies are called photonic band gaps (PBGs) [3,4]. The properties of PBGs in one-dimensional (1D) PCs have been proven to play an important role in some potential applications such as photonic devices, optical filters, resonance cavities, laser applications, high reflecting omnidirectional mirrors, and the optoelectronic circuits [5–10].

In Refs. [11,12], we have studied the quantum transmission property of 1D PCs with quantum theory method. In this paper, we have optimized the quantum theory method, and obtained the simplified quantum transfer matrix, but the calculation results were unchanged. We use the improved quantum theory method to research the MSPCs quantum transmissivity, and obtain some valuable results for MSPCs, which are beneficial to design the optic filter of multiple channel.

## 2. The quantum wave equation and probability current density of photon

The quantum wave equations of free and non-free photon have been obtained in Refs. [13,14], they are

$$i\hbar \frac{\partial}{\partial t} \vec{\psi}(\vec{r}, t) = c\hbar\nabla \times \vec{\psi}(\vec{r}, t), \quad (1)$$

and

$$i\hbar \frac{\partial}{\partial t} \vec{\psi}(\vec{r}, t) = c\hbar\nabla \times \vec{\psi}(\vec{r}, t) + V\vec{\psi}(\vec{r}, t), \quad (2)$$

where  $\vec{\psi}(\vec{r}, t)$  is the vector wave function of photon, and  $V$  is the potential energy of photon in medium. In the medium of refractive index  $n$ , the photon's potential energy  $V$  is [13,14]

$$V = \hbar\omega(1 - n). \quad (3)$$

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The conjugate of Eq. (2) is

$$-i\hbar \frac{\partial}{\partial t} \vec{\psi}^*(\vec{r}, t) = c\hbar \nabla \times \vec{\psi}^*(\vec{r}, t) + V \vec{\psi}^*(\vec{r}, t). \quad (4)$$

Multiplying the Eq. (2) by  $\vec{\psi}^*$ , the Eq. (4) by  $\vec{\psi}$ , and taking the difference, we get

$$i\hbar \frac{\partial}{\partial t} (\vec{\psi}^* \cdot \vec{\psi}) = c\hbar (\vec{\psi}^* \cdot \nabla \times \vec{\psi} - \vec{\psi} \cdot \nabla \times \vec{\psi}^*) = c\hbar \nabla \cdot (\vec{\psi} \times \vec{\psi}^*), \quad (5)$$

i.e.,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0, \quad (6)$$

where

$$\rho = \vec{\psi}^* \cdot \vec{\psi}, \quad (7)$$

and

$$J = i\epsilon \vec{\psi} \times \vec{\psi}^*, \quad (8)$$

are the probability density and probability current density, respectively.

By the method of separation variable

$$\vec{\psi}(\vec{r}, t) = \vec{\psi}(\vec{r})f(t) = \vec{\psi}(\vec{r})e^{-i\omega t}, \quad (9)$$

the time-dependent Eq. (2) becomes the time-independent equation

$$c\hbar \nabla \times \vec{\psi}(\vec{r}) + V \vec{\psi}(\vec{r}) = E \vec{\psi}(\vec{r}), \quad (10)$$

where  $E$  and  $V$  are the energy and potential energy of photon in medium, respectively.

### 3. The quantum transmissivity and reflectivity

In Refs. [11,12], we have taken curl in Eq. (10), and obtain second-order differential equation (13). In this paper, we should not take curl in Eq. (10), and directly solve the Eq. (10) in one-dimensional Photonic crystals. Taking Eq. (3) into Eq. (10), we have

$$c\hbar \nabla \times \vec{\psi}(\vec{r}) = (E - V) \vec{\psi}(\vec{r}) = \hbar \omega n \vec{\psi}(\vec{r}), \quad (11)$$

i.e.,

$$\nabla \times \vec{\psi}(\vec{r}) = \frac{\omega}{c} n \vec{\psi}(\vec{r}), \quad (12)$$

and

$$\nabla \times \vec{\psi}(\vec{r}) = \frac{\omega}{c} \vec{\psi}(\vec{r}). \quad (13)$$

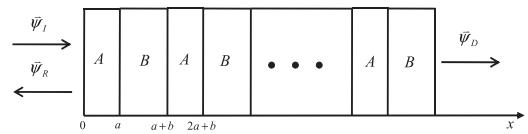
The Eqs. (12) and (13) are the quantum wave equations of photon in medium and vacuum, respectively. Eq. (12) can be written as

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \psi_x & \psi_y & \psi_z \end{vmatrix} = \frac{\omega}{c} n (\psi_x \vec{i} + \psi_y \vec{j} + \psi_z \vec{k}) \quad (14)$$

i.e.,

$$\begin{cases} \frac{\partial \psi_z}{\partial y} - \frac{\partial \psi_y}{\partial z} = \frac{\omega}{c} n \psi_x \\ \frac{\partial \psi_x}{\partial z} - \frac{\partial \psi_z}{\partial x} = \frac{\omega}{c} n \psi_y \\ \frac{\partial \psi_y}{\partial x} - \frac{\partial \psi_x}{\partial y} = \frac{\omega}{c} n \psi_z \end{cases} \quad (15)$$

we consider the photon travels along with the  $x$ -axis in one-dimensional Photonic crystals, which is shown in Fig. 1. The wave



**Fig. 1.** The structure of one-dimensional photonic crystals.

vector  $k_y = k_z = 0$  and  $k_x \neq 0$ . Since the photon wave is transverse wave, we have

$$\begin{cases} \psi_x = 0 \\ \psi_y = \psi_{0y} e^{ikx}, \\ \psi_z = \psi_{0z} e^{ikx} \end{cases} \quad (16)$$

where  $k = (\omega/c)n$  is the wave vector of photon in medium. Substituting Eq. (16) into Eq. (15), we get

$$\begin{cases} -ik\psi_z = \frac{\omega}{c} n \psi_y = k\psi_y \\ ik\psi_y = \frac{\omega}{c} n \psi_z = k\psi_z \end{cases}, \quad (17)$$

i.e.,

$$\psi_y = -i\psi_z, \quad \psi_z = i\psi_y, \quad (18)$$

by Eq. (15), we get

$$\frac{d\psi_y}{\psi_y} = i \frac{\omega}{c} n dx \quad (19)$$

by integration, we obtain  $\psi_y(x)$

$$\psi_y(x) = \psi_{0y} e^{i \frac{\omega}{c} n x} = \psi_{0y} e^{ikx} \quad (20)$$

and the  $\psi_z(x)$  is

$$\psi_z(x) = \psi_{0z} e^{ikx} = i\psi_{0y} e^{ikx} \quad (21)$$

the total including time wave function of photon in medium is

$$\vec{\psi}(x, t) = \psi_y(x, t) \vec{j} + \psi_z(x, t) \vec{k} = \psi_{0y} e^{i(kx-\omega t)} \vec{j} + \psi_{0z} e^{i(kx-\omega t)} \vec{k}. \quad (22)$$

the total including time wave function of photon in vacuum is

$$\vec{\psi}(x, t) = \psi_y(x, t) \vec{j} + \psi_z(x, t) \vec{k} = \psi_{0y} e^{i(Kx-\omega t)} \vec{j} + \psi_{0z} e^{i(Kx-\omega t)} \vec{k}. \quad (23)$$

where  $K = (\omega/c)$  is the wave vector of photon in vacuum.

For one-dimensional Photonic crystals, we should define and calculate its quantum transmissivity and quantum reflectivity. The one-dimensional PCs structure is shown in Fig. 1.

In Fig. 1,  $\vec{\psi}_I$ ,  $\vec{\psi}_R$ ,  $\vec{\psi}_D$  are the wave functions of incident, reflection and transmission photon, respectively, they can be written as

$$\vec{\psi}_I = F_y e^{i(\vec{k} \cdot \vec{r} - \omega t)} \vec{j} + F_z e^{i(\vec{k} \cdot \vec{r} - \omega t)} \vec{k}, \quad (24)$$

$$\vec{\psi}_R = F'_y e^{i(\vec{k} \cdot \vec{r} - \omega t)} \vec{j} + F'_z e^{i(\vec{k} \cdot \vec{r} - \omega t)} \vec{k}, \quad (25)$$

$$\vec{\psi}_D = D_y e^{i(\vec{k} \cdot \vec{r} - \omega t)} \vec{j} + D_z e^{i(\vec{k} \cdot \vec{r} - \omega t)} \vec{k}, \quad (26)$$

where  $F_y$ ,  $F_z$ ,  $F'_y$ ,  $F'_z$ ,  $D_y$ , and  $D_z$  are incident, reflection and transmission amplitudes of  $y$  and  $z$  components. We obtain the solutions (22)–(26) are the same as the solutions (20)–(22) in Refs. [11,12].

By Eq. (22), the probability current density can be written as

$$J = i\epsilon \vec{\psi} \times \vec{\psi}^* = 2c|\psi_z|^2 \vec{i} = 2c|\psi_{0z}|^2 \vec{i}, \quad (27)$$

where

$$\psi_z = \psi_{0z} e^{i(\vec{k} \cdot \vec{r} - \omega t)}, \quad (28)$$

the  $\psi_{0z}$  is  $\psi_z$  amplitude.

For the incident, reflection and transmission photon, their probability current density  $J_I$ ,  $J_R$ ,  $J_T$  are

$$J_I = 2c|F_z|^2, \quad (29)$$

$$J_R = 2c|F'_z|^2, \quad (30)$$

$$J_T = 2c|D_z|^2, \quad (31)$$

We can define quantum transmissivity  $T$  and quantum reflectivity  $R$  as

$$T = \frac{J_T}{J_I} = \left| \frac{D_z}{F_z} \right|^2, \quad (32)$$

$$R = \frac{J_R}{J_I} = \left| \frac{F'_z}{F_z} \right|^2. \quad (33)$$

By the amplitudes of  $z$  component  $F_z$ ,  $F'_z$  and  $D_z$ , we can calculate the quantum transmissivity and quantum reflectivity.

#### 4. The photon wave function and quantum transfer matrix in one-dimensional photonic crystals

In the following, we should calculate the incident, reflection and transmission amplitudes  $F_z$ ,  $F'_z$  and  $D_z$  with quantum theory approach. In Eqs. (22) and (23), we give the plane wave solutions of photon in medium and vacuum. In one-dimensional photonic crystals, the photon total wave function is the superposition of incident and reflection wave, then we can give the wave functions of every medium. In Fig. 2, we give the simplification form of wave function in every medium, such as symbols  $A_{k_A}^1$  and  $A_{-k_A}^1$  express simplifying wave function of medium  $A$  in the first period, it express wave function

$$\psi_A^1(x) = A_{k_A}^1 e^{ik_A x} + A_{-k_A}^1 e^{ik_A a + ik_A(a-x)}, \quad (34)$$

in medium  $B$  of first period, the symbols  $B_{k_B}^1$  and  $B_{-k_B}^1$  express wave function

$$\psi_B^1(x) = B_{k_B}^1 e^{ik_A a + ik_B(x-a)} + B_{-k_B}^1 e^{ik_A a + ik_B b + ik_B(a+b-x)}, \quad (35)$$

in medium  $A$  of second period, the symbols  $A_{k_A}^2$  and  $A_{-k_A}^2$  express wave function

$$\psi_A^2(x) = A_{k_A}^2 e^{ik_A a + ik_B b + ik_A(x-a-b)} + A_{-k_A}^2 e^{ik_A 2a + ik_B b + ik_A(2a+b-x)}, \quad (36)$$

in medium  $B$  of second period, the symbols  $B_{k_B}^2$  and  $B_{-k_B}^2$  express wave function

$$\psi_B^2(x) = B_{k_B}^2 e^{ik_A 2a + ik_B b + ik_B(x-2a-b)} + B_{-k_B}^2 e^{ik_A 2a + ik_B 2b + ik_B(2a+2b-x)}, \quad (37)$$

similarly, in medium  $A$  of  $N$ th period, the symbols  $A_{k_A}^N$  and  $A_{-k_A}^N$  express wave function

$$\begin{aligned} \psi_A^N(x) = & A_{k_A}^N e^{ik_A(N-1)a + ik_B(N-1)b + ik_A(x-(N-1)a-(N-1)b)} \\ & + A_{-k_A}^N e^{ik_A Na + ik_B(N-1)b + ik_A(Na+(N-1)b-x)}, \end{aligned} \quad (38)$$

in medium  $B$  of  $N$ th period, the symbols  $B_{k_B}^N$  and  $B_{-k_B}^N$  express wave function

$$\begin{aligned} \psi_B^N(x) = & B_{k_B}^N e^{ik_A Na + ik_B(N-1)b + ik_B(x-Na-(N-1)b)} \\ & + B_{-k_B}^N e^{ik_A Na + ik_B Nb + ik_B(Na+Nb-x)}. \end{aligned} \quad (39)$$

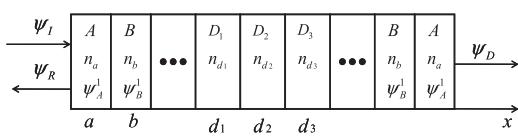


Fig. 2. The quantum structure of one-dimensional photonic crystals.

In the incident area, the total wave function  $\psi_{tot}(x)$  is the superposition of incident and reflection wave function, it is

$$\psi_{tot}(x) = \psi_I(x) + \psi_R(x) = F e^{ikx} + F' e^{-ikx}, \quad (40)$$

where  $K$  is the wave vector of incident, reflection, and transmission photon.

In the following, we should use the condition of wave function and its derivative continuation at interface of two mediums:

(1) At  $x=0$ , by the continuation of  $\psi_{tot}(x)$ ,  $\psi_A^1(x)$  and its derivative, we have

$$F + F' = A_{k_A}^1 + A_{-k_A}^1 e^{ik_A 2a}, \quad (41)$$

$$iKF - iKF' = ik_A A_{k_A}^1 - ik_A A_{-k_A}^1 e^{ik_A 2a}, \quad (42)$$

we obtain

$$A_{k_A}^1 = \frac{1}{2} \left[ \left( 1 + \frac{K}{k_A} \right) F + \left( 1 - \frac{K}{k_A} \right) F' \right], \quad (43)$$

$$A_{-k_A}^1 = \frac{1}{2} \left[ \left( 1 - \frac{K}{k_A} \right) e^{-ik_A 2a} F + \left( 1 + \frac{K}{k_A} \right) e^{-ik_A 2a} F' \right], \quad (44)$$

the Eqs. (43) and (44) can be written as matrix form

$$\begin{pmatrix} A_{k_A}^1 \\ A_{-k_A}^1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + K/k_A & 1 - K/k_A \\ (1 - K/k_A)e^{-ik_A 2a} & (1 + K/k_A)e^{-ik_A 2a} \end{pmatrix} \begin{pmatrix} F \\ F' \end{pmatrix} = M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix}, \quad (45)$$

where  $M_A^1$  is the quantum transfer matrix of the first period medium  $A$ , it is

$$M_A^1 = \frac{1}{2} \begin{pmatrix} 1 + K/k_A & 1 - K/k_A \\ (1 - K/k_A)e^{-ik_A 2a} & (1 + K/k_A)e^{-ik_A 2a} \end{pmatrix}, \quad (46)$$

(2) At  $x=a$ , by the continuation of  $\psi_A^1(x)$ ,  $\psi_B^1(x)$  and its derivative, we have

$$A_{k_A}^1 e^{ik_A a} + A_{-k_A}^1 e^{ik_A a} = B_{k_B}^1 e^{ik_A a} + B_{-k_B}^1 e^{ik_A a} e^{ik_B 2b}, \quad (47)$$

$$k_A A_{k_A}^1 e^{ik_A a} - k_A A_{-k_A}^1 e^{ik_A a} = k_B B_{k_B}^1 e^{ik_A a} - k_B B_{-k_B}^1 e^{ik_A a} e^{ik_B 2b}, \quad (48)$$

we get

$$B_{k_B}^1 = \frac{1}{2} \left[ \left( 1 + \frac{k_A}{k_B} \right) A_{k_A}^1 + \left( 1 - \frac{k_A}{k_B} \right) A_{-k_A}^1 \right], \quad (49)$$

$$B_{-k_B}^1 = \frac{1}{2} \left[ \left( 1 - \frac{k_A}{k_B} \right) e^{-ik_B 2b} A_{k_A}^1 + \left( 1 + \frac{k_A}{k_B} \right) e^{-ik_B 2b} A_{-k_A}^1 \right], \quad (50)$$

the Eqs. (49) and (50) can be written as matrix form

$$\begin{pmatrix} B_{k_B}^1 \\ B_{-k_B}^1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + k_A/k_B & 1 - k_A/k_B \\ (1 - k_A/k_B)e^{-ik_B 2b} & (1 + k_A/k_B)e^{-ik_B 2b} \end{pmatrix} \begin{pmatrix} A_{k_A}^1 \\ A_{-k_A}^1 \end{pmatrix} = M_B^1 \begin{pmatrix} A_{k_A}^1 \\ A_{-k_A}^1 \end{pmatrix}, \quad (51)$$

where  $M_B^1$  is the quantum transfer matrix of the first period medium  $B$ , it is

$$M_B^1 = \frac{1}{2} \begin{pmatrix} 1 + k_A/k_B & 1 - k_A/k_B \\ (1 - k_A/k_B)e^{-ik_B 2b} & (1 + k_A/k_B)e^{-ik_B 2b} \end{pmatrix}, \quad (52)$$

(3) At  $x=a+b$ , by the continuation of  $\psi_B^1(x)$ ,  $\psi_A^2(x)$  and its derivative, we have

$$\begin{aligned} & B_{k_B}^1 e^{ik_A a + ik_B b} + B_{-k_B}^1 e^{ik_A a + ik_B b} \\ & = A_{k_A}^2 e^{ik_A a + ik_B b} + A_{-k_A}^2 e^{ik_A 3a + ik_B b}, \end{aligned} \quad (53)$$

$$\begin{aligned} & k_B(B_{k_B}^1 e^{ik_A a + ik_B b} - B_{-k_B}^1 e^{ik_A a + ik_B b}) \\ & = k_A(A_{k_A}^2 e^{ik_A a + ik_B b} - A_{-k_A}^2 e^{ik_A 3a + ik_B b}), \end{aligned} \quad (54)$$

we get

$$A_{k_A}^2 = \frac{1}{2} \left[ \left( 1 + \frac{k_B}{k_A} \right) B_{k_B}^1 + \left( 1 - \frac{k_B}{k_A} \right) B_{-k_B}^1 \right], \quad (55)$$

$$A_{-k_A}^2 = \frac{1}{2} \left[ \left( 1 - \frac{k_B}{k_A} \right) e^{-ik_A 2a} B_{k_B}^1 + \left( 1 + \frac{k_B}{k_A} \right) e^{-ik_A 2a} B_{-k_B}^1 \right], \quad (56)$$

the Eqs. (55) and (56) can be written as matrix form

$$\begin{aligned} & \begin{pmatrix} A_{k_A}^2 \\ A_{-k_A}^2 \end{pmatrix} \\ & = \frac{1}{2} \begin{pmatrix} 1 + k_B/k_A & 1 - k_B/k_A \\ (1 - k_B/k_A)e^{-ik_A 2a} & (1 + k_B/k_A)e^{-ik_A 2a} \end{pmatrix} \begin{pmatrix} B_{k_B}^1 \\ B_{-k_B}^1 \end{pmatrix} \\ & = M_A^2 \begin{pmatrix} B_{k_B}^1 \\ B_{-k_B}^1 \end{pmatrix}, \end{aligned} \quad (57)$$

where  $M_A^2$  is the quantum transfer matrix of the second period medium A, it is

$$M_A^2 = \frac{1}{2} \begin{pmatrix} 1 + k_B/k_A & 1 - k_B/k_A \\ (1 - k_B/k_A)e^{-ik_A 2a} & (1 + k_B/k_A)e^{-ik_A 2a} \end{pmatrix}, \quad (58)$$

(4) at  $x=2a+b$ , by the continuation of  $\psi_A^2(x)$ ,  $\psi_B^2(x)$  and its derivative, we get

$$\begin{aligned} & \begin{pmatrix} B_{k_B}^2 \\ B_{-k_B}^2 \end{pmatrix} \\ & = \frac{1}{2} \begin{pmatrix} 1 + k_A/k_B & 1 - k_A/k_B \\ (1 - k_A/k_B)e^{-ik_B 2b} & (1 + k_A/k_B)e^{-ik_B 2b} \end{pmatrix} \begin{pmatrix} A_{k_A}^2 \\ A_{-k_A}^2 \end{pmatrix} \\ & = M_B^2 \begin{pmatrix} A_{k_A}^2 \\ A_{-k_A}^2 \end{pmatrix}, \end{aligned} \quad (59)$$

where  $M_B^2$  is the quantum transfer matrix of the second period medium B, it is

$$M_B^2 = \frac{1}{2} \begin{pmatrix} 1 + k_A/k_B & 1 - k_A/k_B \\ (1 - k_A/k_B)e^{-ik_B 2b} & (1 + k_A/k_B)e^{-ik_B 2b} \end{pmatrix}. \quad (60)$$

By the above calculation, we can obtain the results of transfer matrixes:

(1) For the transfer matrix  $M_A^1$  of the first period medium A is independent form.

(2) For the transfer matrixes  $M_A^N$  of the Nth period ( $N \geq 2$ ), they can be written as

$$M_A^N = M_A = \frac{1}{2} \begin{pmatrix} 1 + k_B/k_A & 1 - k_B/k_A \\ (1 - k_B/k_A)e^{-ik_A 2a} & (1 + k_B/k_A)e^{-ik_A 2a} \end{pmatrix}, \quad (61)$$

(3) For the transfer matrixes  $M_B^N$  of the Nth period ( $N \geq 1$ ), they can be written as

$$M_B^N = M_B = \frac{1}{2} \begin{pmatrix} 1 + k_A/k_B & 1 - k_A/k_B \\ (1 - k_A/k_B)e^{-ik_B 2b} & (1 + k_A/k_B)e^{-ik_B 2b} \end{pmatrix}. \quad (62)$$

We can find the quantum transfer matrices  $M_A^1$ ,  $M_B^1$ ,  $M_A^2$ ,  $M_B^2$ ... $M_A^N$  and  $M_B^N$  are simplified, which are different from the quantum transfer matrices in Refs. [11,12], but their calculation results of quantum transmissivity are identical.

From Eqs. (46)–(62), we can obtain the quantum transfer matrix of jth layer medium, it is

$$M_j = \frac{1}{2} \begin{pmatrix} 1 + \frac{k_{j-1}}{k_j} & 1 - \frac{k_{j-1}}{k_j} \\ \left( 1 - \frac{k_{j-1}}{k_j} \right) e^{-2ik_j d_j} & \left( 1 + \frac{k_{j-1}}{k_j} \right) e^{-2ik_j d_j} \end{pmatrix}, \quad (63)$$

where  $k_j$  and  $d_j$  are the wave vector and thickness of photon in the travelling layer  $j$ , and  $k_{j-1}$  is the wave vector of photon in the travelling layer  $j-1$ . By the general form of  $M_j$ , we can directly give out the quantum transfer matrix of arbitrary medium layer.

By the quantum transfer matrixes, we can give their relations:

(1) The representation of the first period quantum transfer matrixes are

$$\begin{pmatrix} A_{k_A}^1 \\ A_{-k_A}^1 \end{pmatrix} = M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix}, \quad (64)$$

$$\begin{pmatrix} B_{k_B}^1 \\ B_{-k_B}^1 \end{pmatrix} = M_B \begin{pmatrix} A_{k_A}^1 \\ A_{-k_A}^1 \end{pmatrix} = M_B M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix}. \quad (65)$$

(2) The representation of the second period quantum transfer matrixes are

$$\begin{pmatrix} A_{k_A}^2 \\ A_{-k_A}^2 \end{pmatrix} = M_A \begin{pmatrix} B_{k_B}^1 \\ B_{-k_B}^1 \end{pmatrix} = M_A M_B M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix}, \quad (66)$$

$$\begin{pmatrix} B_{k_B}^2 \\ B_{-k_B}^2 \end{pmatrix} = M_B \begin{pmatrix} A_{k_A}^2 \\ A_{-k_A}^2 \end{pmatrix} = M_B M_A M_B M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix}. \quad (67)$$

(3) Similarly, the representation of the Nth period quantum transfer matrixes in medium B can be written as

$$\begin{pmatrix} B_{k_B}^N \\ B_{-k_B}^N \end{pmatrix} = (M_B M_A)^{N-1} M_B M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix} = M \begin{pmatrix} F \\ F' \end{pmatrix}, \quad (68)$$

where  $M = (M_B M_A)^{N-1} M_B M_A^1 = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}$  is the total quantum transfer matrix of N period.

By Eq. (67), we can give the wave function of Nth period in medium B, it is

$$\begin{aligned}\psi_B^N(x) &= B_{k_B}^N e^{ik_A Na + ik_B(N-1)b + ik_B(x-Na-(N-1)b)} \\ &\quad + B_{-k_B}^N e^{ik_A Na + ik_B Nb + ik_B(Na+Nb-x)} \\ &= (m_1 F + m_2 F') e^{ik_A Na + ik_B(N-1)b + ik_B(x-Na-(N-1)b)} \\ &\quad + (m_3 F + m_4 F') e^{ik_A Na + ik_B Nb + ik_B(Na+Nb-x)}. \end{aligned}\quad (69)$$

In Fig. 2, the transmission wave function is

$$\psi_D(x) = D \cdot e^{ik_A Na + ik_B Nb + K(x-Na-Nb)}. \quad (70)$$

At  $x=N(a+b)$ , by the continuation of wave function and its derivative, we have

$$m_1 F + m_2 F' + m_3 F + m_4 F' = D, \quad (71)$$

and

$$k_B(m_1 F + m_2 F') - k_B(m_3 F + m_4 F') = KD, \quad (72)$$

we can obtain

$$r = \frac{F'}{F} = \frac{m_1(k_B - K) - m_3(k_B + K)}{m_2(K - k_B) + m_4(K + k_B)}, \quad (73)$$

By Eqs. (71)–(73), we have

$$t = \frac{D}{F} = (m_1 + m_2 r) + (m_3 + m_4 r), \quad (74)$$

and the quantum transmissivity  $T$  is

$$T = |t|^2. \quad (75)$$

## 5. The quantum transmissivity of mirror symmetrical one-dimensional photonic crystals

We study the structure of one-dimensional photonic crystals is  $(AB)^N(D_1D_2D_3)(BA)^N$ , i.e., we insert the layers  $D_1, D_2$  and  $D_3$  into the mirror symmetrical structure  $(AB)^N(BA)^N$ . By the quantum transfer matrix  $M_j$  of  $j$ th layer medium (63), we can give the total quantum transfer matrix of  $(AB)^N(D_1D_2D_3)(BA)^N$ , it is

$$M = (M_A M_B)^N (M_{D_3} M_{D_2} M_{D_1}) (M_B M_A)^N = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}. \quad (76)$$

By Eq. (63), we can give arbitrary quantum transfer matrix in Eq. (76), such as  $M_{D_1}$ ,  $M_{D_2}$  and  $M_{D_3}$ , they are

$$M_{D_1} = \frac{1}{2} \begin{pmatrix} 1 + \frac{k_b}{k_{d_1}} & 1 - \frac{k_b}{k_{d_1}} \\ \left(1 - \frac{k_b}{k_{d_1}}\right) e^{-2ik_{d_1}d_1} & \left(1 + \frac{k_b}{k_{d_1}}\right) e^{-2ik_{d_1}d_1} \end{pmatrix}, \quad (77)$$

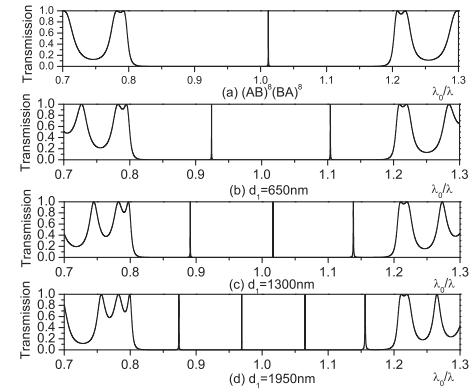
$$M_{D_2} = \frac{1}{2} \begin{pmatrix} 1 + \frac{k_{d_1}}{k_{d_2}} & 1 - \frac{k_{d_1}}{k_{d_2}} \\ \left(1 - \frac{k_{d_1}}{k_{d_2}}\right) e^{-2ik_{d_2}d_2} & \left(1 + \frac{k_{d_1}}{k_{d_2}}\right) e^{-2ik_{d_2}d_2} \end{pmatrix}, \quad (78)$$

$$M_{D_3} = \frac{1}{2} \begin{pmatrix} 1 + \frac{k_{d_2}}{k_{d_3}} & 1 - \frac{k_{d_2}}{k_{d_3}} \\ \left(1 - \frac{k_{d_2}}{k_{d_3}}\right) e^{-2ik_{d_3}d_3} & \left(1 + \frac{k_{d_2}}{k_{d_3}}\right) e^{-2ik_{d_3}d_3} \end{pmatrix}. \quad (79)$$

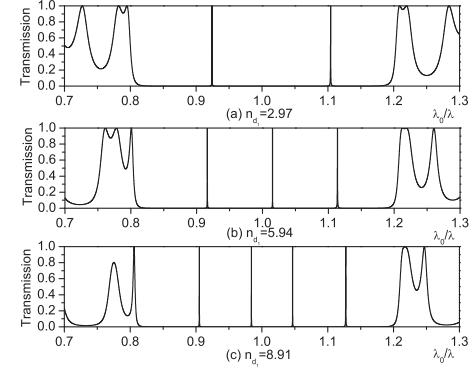
We consider the photon travels along with the  $x$ -axis, the MSPCs and wave function distribution are shown in Fig. 2. The thicknesses and refractive indexes of layers  $A$ ,  $B$ ,  $D_1$ ,  $D_2$  and  $D_3$  are  $a$ ,  $b$ ,  $d_1$ ,  $d_2$ ,  $d_3$  and  $n_a$ ,  $n_b$ ,  $n_{d_1}$ ,  $n_{d_2}$ ,  $n_{d_3}$ , respectively. By the Eqs. (73)–(75), we can calculate the quantum transmissivity of MSPCs structure  $(AB)^N(D_1D_2D_3)(BA)^N$ .

## 6. Numerical result

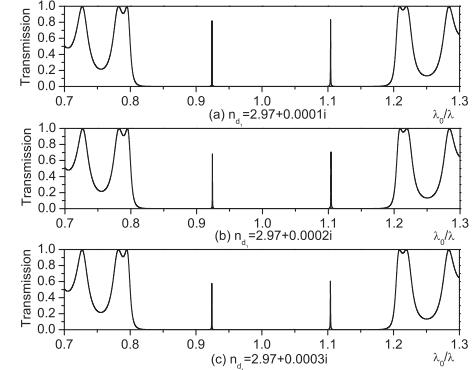
In this section, we report our numerical results of quantum transmissivity. Firstly, we study the MSPCs structure  $(AB)^8D_1(BA)^8$ , its main parameters are: The medium  $A$  is  $MgF_2$ , refractive indexes  $n_a = 1.38$ , and thickness  $a = 298$  nm, the medium  $B$  is  $ZnS$ , refractive indexes  $n_b = 2.35$ , thickness  $b = 160$  nm, and the defect layer  $D_1$  is  $AlAs$ , refractive indexes  $n_{d_1} = 2.97$ , thickness  $d_1 = 650$  nm. The central wavelength is  $\lambda_0 = 2(n_a a + n_b b)$ . With Eqs. (73)–(75), we can calculate the quantum transmissivity of the MSPCs. In numerical calculation, we have calculated the influence of defect layer parameters on quantum transmissivity, which are shown in Figs. 3–6. From the figures we find the quantum transmissivity of MSPCs



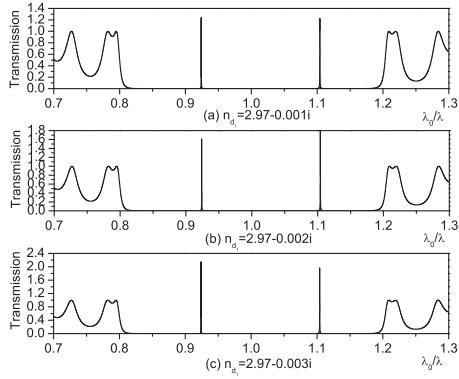
**Fig. 3.** Influence of the defect layer thickness on quantum transmissivity for MSPCs  $(AB)^8D_1(BA)^8$ .



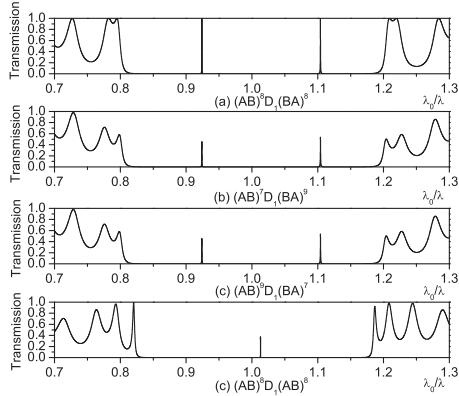
**Fig. 4.** Influence of the defect layer refractive index on quantum transmissivity for MSPCs  $(AB)^8D_1(BA)^8$ .



**Fig. 5.** Influence of the positive imag of defect layer refractive index on quantum transmissivity for MSPCs  $(AB)^8D_1(BA)^8$ .

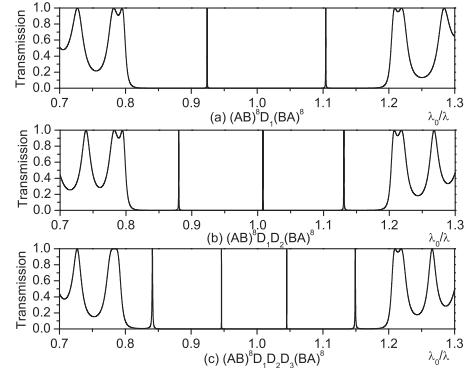


**Fig. 6.** Influence of the negative imag of defect layer refractive index on quantum transmissivity for MSPCs  $(AB)^8D_1(BA)^8$ .



**Fig. 7.** Influence of the structure of PCs on quantum transmissivity.

has appeared as the sharp peaks (the  $T=1$ ) in the PBG ( $\lambda_0/\lambda$  is in the range of 0.8–1.2), and the number of sharp peaks is added with the increasing of defect layer thickness and refractive index, which are shown in Figs. 3 and 4. The MSPCs can be designed the optic filter of multiple channel. In Fig. 5, the defect layer  $D_1$  is absorbing medium, i.e., its refractive index imaginary part is positive, we can find the sharp peaks become weaken (the  $T < 1$ ), with the imaginary part increasing, it makes light weaken at sharp peaks position. In Fig. 6, the defect layer  $D_1$  is active medium, i.e., its imaginary part of refractive index is positive, we can find the sharp peaks become strong (the  $T > 1$ ), with the imaginary part decreasing, it makes light strong at sharp peaks position. In Fig. 7, we compare the quantum transmissivity of mirror symmetrical with unsymmetrical PCs, Fig. 7(a) is mirror symmetrical structure  $(AB)^8D_1(BA)^8$ , and Fig. 7(b)–(d) are unsymmetrical structure  $(AB)^7D_1(BA)^9$ ,  $(AB)^9D_1(BA)^7$  and  $(AB)^8D_1(AB)^8$ , respectively. We can find quantum transmissivity of sharp peaks is equal to 1 in mirror symmetrical structure, and the quantum transmissivity of sharp peaks is less than 1 in unsymmetrical structure. In order to design the optic filter of multiple channel, we should choose the MSPCs. In Fig. 8, we consider the influence of the number of defect layer on quantum transmissivity of MSPCs. The structure of Fig. 8(a)–(c) is  $(AB)^8D_1(BA)^8$ ,  $(AB)^8D_1D_2(BA)^8$  and  $(AB)^8D_1D_2D_3(BA)^8$ , respectively. main parameters of  $D_2$  and  $D_3$  are:  $n_{d2} = 2.58$ ,  $d_2 = 770$  nm,



**Fig. 8.** Influence of the number of defect layer on quantum transmissivity.

$n_{d3} = 2.80$ ,  $d_3 = 450$  nm. From Fig. 8(a)–(c), we can find the number of sharp peaks increasing with the defect layer number increasing.

## 7. Conclusion

In summary, we have studied the quantum transmissivity of MSPCs with quantum method. By calculation, we obtain some results for MSPCs: (1) In the PBG ( $\lambda_0/\lambda$  is in the range of 0.8–1.2), there are sharp peaks ( $T=1$ ) in the quantum transmissivity spectrum of MSPCs. (2) The number of sharp peaks are added with the increasing of defect layer thickness, refractive index and numbers, so it is beneficial to design the optic filter of multiple channel. (3) When the defect layer is absorbing (active) medium, the sharp peaks become weaken (strong). (4) The quantum transmissivity of sharp peaks is equal to 1 in the mirror symmetrical structure PCs, but it is less than 1 in the unsymmetrical structure PCs. So, we design the optic filter of multiple channel should be used the MSPCs.

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