Optics Communications 374 (2016) 92–96

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Optimized non-integer order phase mask to extend the depth of field of an imaging system

Jiang Liu [a,b,*], Erlong Miao [b], Yongxin Sui [b], Huaijiang Yang [b]

[a] Engineering Research Center of Extreme Precision Optics, Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun 130033, Jilin, China
[b] Graduate University of Chinese Academy of Sciences, Beijing 100049, China

ARTICLE INFO

Article history:
Received 4 September 2015
Received in revised form
25 January 2016
Accepted 30 January 2016
Available online 30 April 2016

OCIS codes:
(110.7348) Wavefront encoding
(110.1758) Computational imaging
(110.4100) Modulation transfer function
(110.4850) Optical transfer function

Keywords:
Wavefront coding
Non-integer phase mask
Depth of field
Imaging system

ABSTRACT

Wavefront coding is an effective optical technique used to extend the depth of field for an incoherent imaging system. Through introducing an optimized phase mask to the pupil plane, the modulated optical transfer function is defocus-invariant. In this paper, we proposed a new form phase mask using non-integer order and signum function to extend the depth of field. The performance of the phase mask is evaluated by comparing defocused modulation transfer function invariant and Fisher information with other phase masks. Defocused imaging simulation is also carried out. The results demonstrate the advantages of non-integer order phase mask and its effectiveness on the depth of field extension.

1. Introduction

Extending the depth of field (DOF) of an incoherent imaging system with wavefront coding phase mask has been an active research topic for many years [1–7]. By introducing a purposely designed phase mask in the pupil plane, the point spread function (PSF) or optical transfer function (OTF) has been made insensitive along a wide range of defocus. Then, the images with large DOF can be restored with just one de-convolution filter from encoding images.

During the past decades, there are many kinds of wavefront coding phase mask have been proposed, such as cubic phase mask [1,2], logarithmic phase mask [3–5], sinusoidal phase mask [6] and tangent phase mask [7]. These phase masks have different characteristics on achieving DOF extension. In order to make a reasonable performance analysis, the parameters of the phase masks need to be optimized first by some merit functions, including MTF invariant [8], Fisher information (FI) [9,10], Hilbert space angle [11] and Strehl ratio (S.R.) [12]. The cubic phase mask is the most original one and has been derived by the stationary phase method. The logarithmic phase mask has been improved many times to effectively stabilize the oscillation of MTF on low spatial frequency. The sinusoidal and tangent phase masks have shown their superiority in extending DOF with more invariable defocused MTF.

In this paper, we propose a new asymmetrical phase mask combining non-integer order and sign function through modifying the cubic phase mask. Then we compare the proposed phase mask with some other phase masks to discuss their defocused MTFs invariant characteristics relatively. Finally, imaging simulation is carried out to demonstrate its validity and effectiveness in DOF extension for an incoherent imaging system.

2. Optimization

The phase function of the new wavefront coding phase mask is presented by:

$$\Psi(x, y) = \alpha(sign(x)\cdot x^{\beta} + sign(y)\cdot y^{\beta}) \quad (1)$$

where, x and y are the pupil plane coordinates in the horizontal and vertical direction, *sign* denotes the signum function, which is defined as 1 for x (or y) >0 and -1 for x (or y) <0, α and β are the coefficients of the phase mask, which will be determined

* Corresponding author at: Engineering Research Center of Extreme Precision Optics, Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun 130033, Jilin, China.
E-mail address: liujiang0521@gmail.com (J. Liu).

http://dx.doi.org/10.1016/j.optcom.2016.01.093
0030-4018/

through the optimization.

Here, the pupil function of a wavefront coding imaging system with only defocus aberration is present by:

$$P(x,y)=\begin{cases}\exp\left\{j\cdot\left[\Psi(x,y)+kW_{20}(x^2+y^2)\right]\right\} & for\ \ x^2+y^2\le 1\\ 0 & \text{otherwise}\end{cases}\tag{2}$$

where $k=2\pi/\lambda$ is an optical constant, W_{20} is the defocus parameter.

In order to optimize these parameters of the phase mask, merit function has been built based on MTF invariant, presented as:

$$\mathrm{MF}=\frac{\sum_{j=1}^{N}\sum_{i=1}^{N} Std\left\{\mathrm{MTF}(u_j,v_i,W_{20,k})\right\}}{\sum_{j=1}^{N}\sum_{i=1}^{N} Mean\left\{\mathrm{MTF}(u_j,v_i,W_{20,k})\right\}}\tag{3}$$

where, u_j and v_i are the normalized spatial frequencies at jth and ith positions in the horizontal and vertical direction. $W_{20,k}$ is the kth defocus parameter. *Std* denotes the standard deviation and *Mean* denotes the average of MTFs.

In addition, a punish function based on S.R. has been brought in the merit function to avoid over optimizing, which could make image restoration process fail due to excessively low MTF, presented as:

$$\mathrm{PF}=\kappa\cdot\frac{\sum_{k=1}^{M}\left\{1-\mathrm{S.R.}(W_{20,k})\right\}}{M}\tag{4}$$

Where, κ is the weight of the punish function.

Therefore, the optimization process can be simply presented as following steps:

$$\begin{aligned}&\text{Iteration:}\\&\text{for}\{(\alpha,\beta)=Coef(i),\ i++\}\\&\qquad\begin{bmatrix}\mathrm{MTF}=abs\left\{FFT(|FFT(P(x,y,W_{20}))|^2)\right\}\\ \text{if PF}\le\text{Threshold}\\ Value\ \ (i)=\mathrm{MF}\\ \text{else}\\ Value\ \ (i)=\mathrm{NaN}\end{bmatrix}\\&\text{end}\\&(\alpha,\beta)_{opt}=Coef(\text{Minimize}(Value))\end{aligned}\tag{5}$$

In the optimization, a series of coefficients of the phase mask are stored in a matrix *Coef* within certain ranges, α is [0, 150] and β is [2, 5]. Next, the MTF values of the optical system are calculated through Fourier transform with different defocus parameters W_{20}, whose range is from 0 to 5λ. Then, if the PF value is no larger than the threshold, continue calculating the MF of this set of coefficients and save the MF value in the *Value*. Repeat this process and the best coefficients of the proposed phase mask will be obtained finally with an optimization algorithm.

Simulated annealing algorithm in Matlab is adopted as the optimization algorithm to quickly find the minimum. Then, the phase and contour map of the surface of the optimized non-integer order phase mask are shown in Fig. 1 with optimization coefficient $\alpha=115.12$ and $\beta=3.66$.

3. Comparison

The phase function of cubic mask, logarithmic mask, sinusoidal mask and tangent mask can respectively be described as:

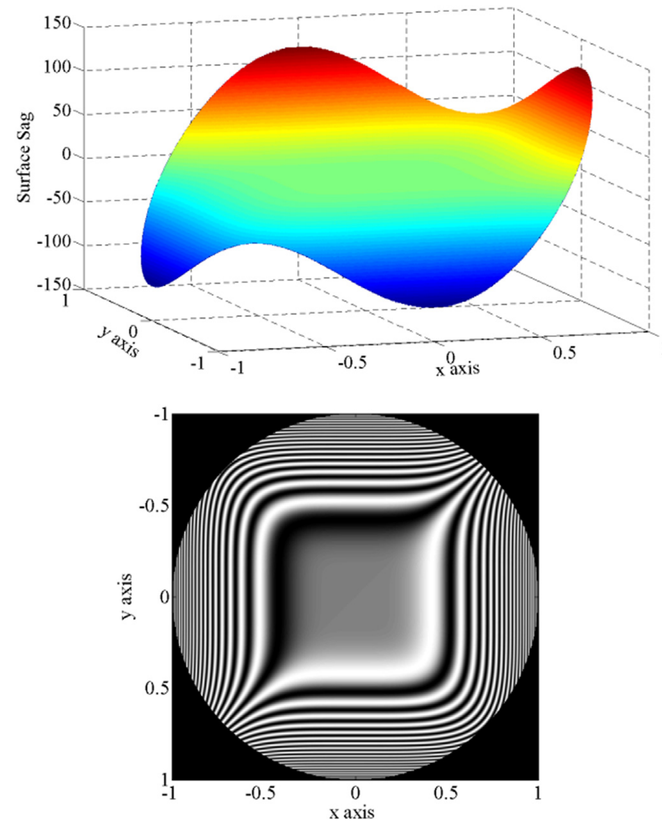


Fig. 1. Phase map and contour map of the surface of the optimized non-integer order phase mask.

Table 1
Coefficient of wavefront coding phase mask.

	α	β
Cubic [7]	74.73	–
Logarithmic [5]	−268.96	−1.52
Sinusoidal [6]	148.76	1.83
Tangent [7]	37.59	1.27

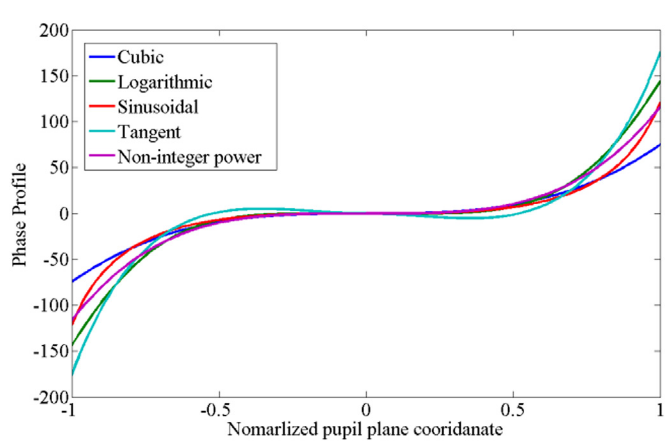


Fig. 2. Profiles of cubic, logarithmic, sinusoidal, tangent and non-integer order phase mask.

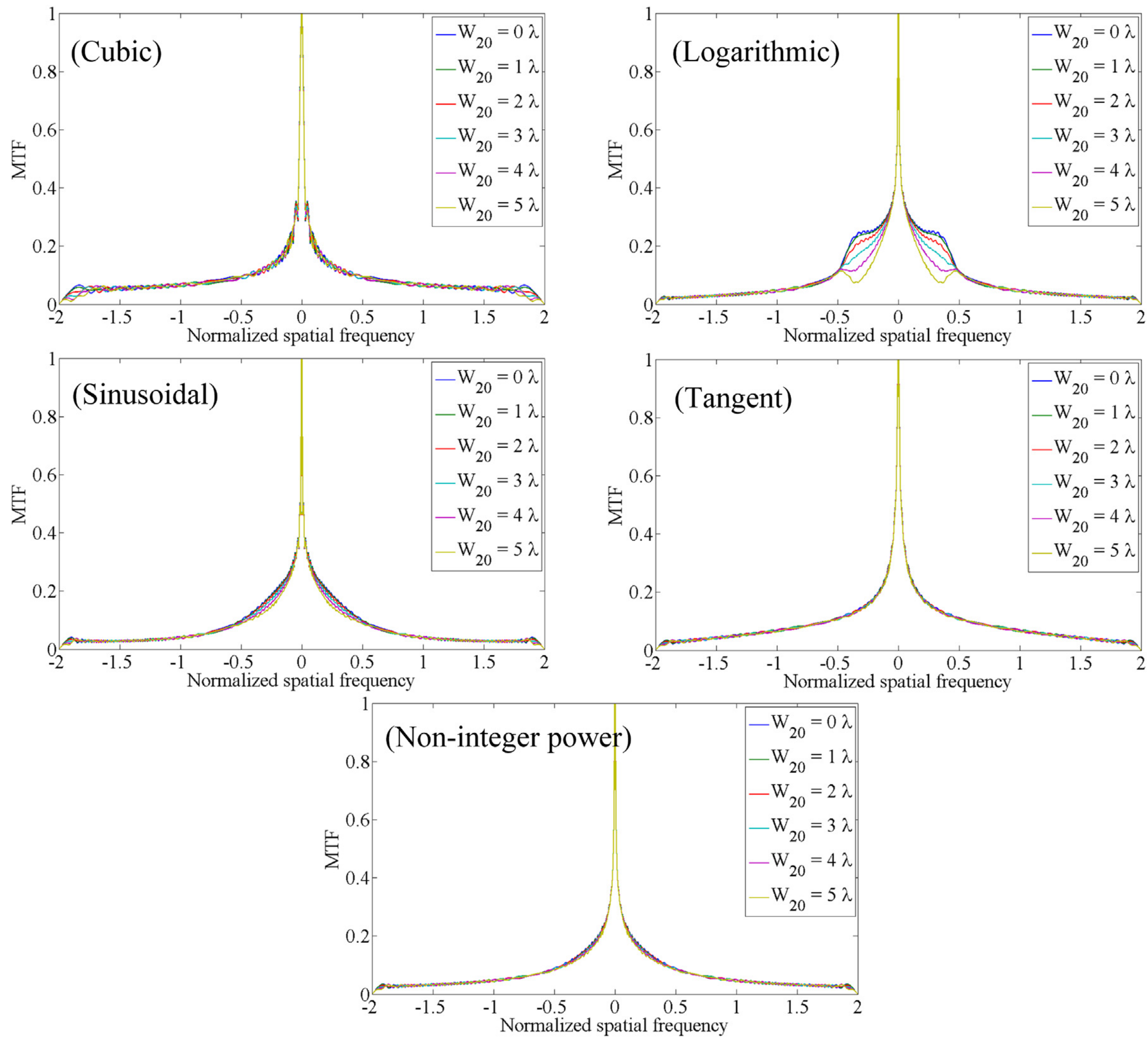


Fig. 3. Defocused MTFs of cubic, logarithmic, sinusoidal, tangent and non-integer order phase mask.

$$\begin{aligned} \Psi_{\text{Cubic}}(x, y) &= \alpha(x^3 + y^3) \\ \Psi_{\text{Logarithmic}}(x, y) &= \alpha \cdot \left[sign(x) \cdot x^2 \ \log(|x| + b|) \right. \\ &\quad \left. + sign(y) \cdot y^2 \ \log(|y| + b|) \right] \\ \Psi_{\text{Sinusoidal}}(x, y) &= \alpha \left[x^4 \ \sin(bx) + y^4 \ \sin(by) \right] \\ \Psi_{\text{Tangent}}(x, y) &= \alpha \left[x^2 \ \tan(bx) + y^2 \ \tan(by) \right] \end{aligned} \tag{6}$$

The optimized coefficients of these phase masks are given in Table 1, and their 2D phase profiles are shown in Fig. 2 compared with the proposed phase mask.

As Fig. 2 shows that slope of proposed phase mask changes gently compared with tangent and logarithmic phase mask, which means it easier to be manufactured with a better surface roughness.

The defocused MTF curves of these phase masks are computed with coefficients in Table 1, the result is shown in Fig. 3. And defocus parameter W_{20} is chosen as $0, 1\lambda, 2\lambda, 3\lambda, 4\lambda, 5\lambda$, respectively.

As shown in Fig. 3, the defocused MTF curves of all five phase masks manifest their low sensitivity to defocus against conventional optical system. However, defocused MTF curve of classical cubic phase mask shows oscillation over the entire spatial frequency range. The logarithmic phase mask and sinusoidal phase mask stabilize the oscillation of MTF curves at high spatial frequency, but at cost of larger oscillation phenomenon at low spatial frequency. Tangent phase mask and non-integer order phase mask present an obvious advantage on eliminating the oscillation, which demonstrates that the defocus invariant characteristics of the two phase masks are better in the DOF extension for an optical imaging system.

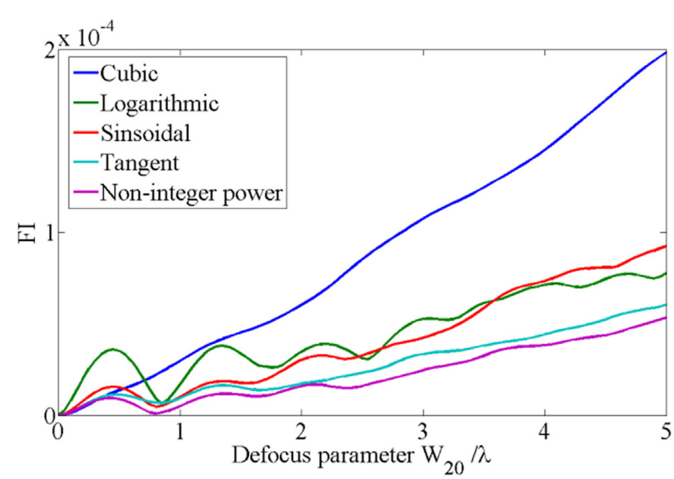


Fig. 4. Fisher information of cubic, logarithmic, sinusoidal, tangent and non-integer order phase mask.

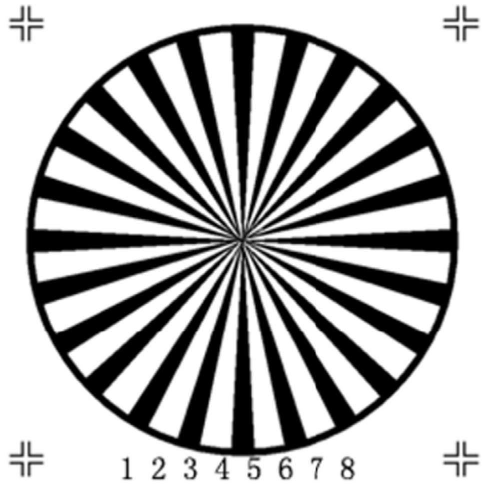


Fig. 5. Original spoke image.

Fisher information (FI) is an effective evaluation criterion to judge the performance of wavefront coding phase mask against defocus. FI is introduced to estimate the similarity of the PSF with different defocus parameters. As the PSF becomes more sensitive to the defocus parameters, the value of FI will be larger. The FI can be expressed as:

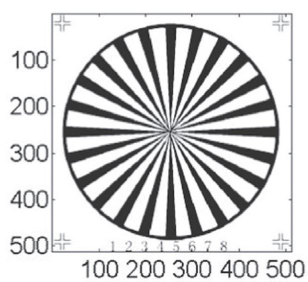
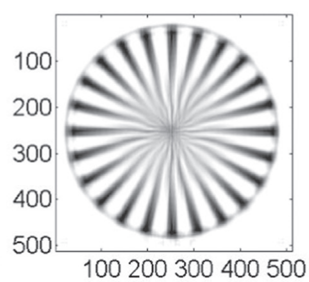
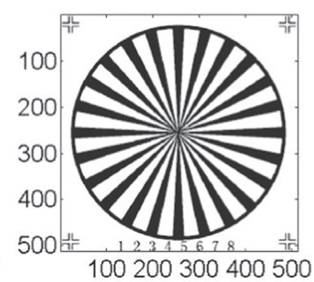
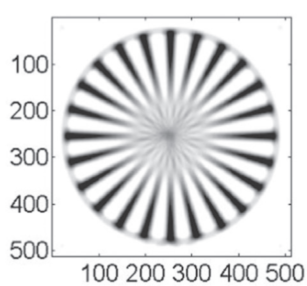
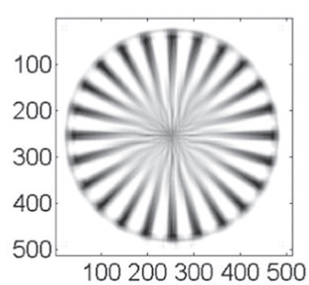
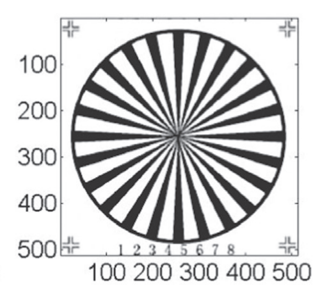
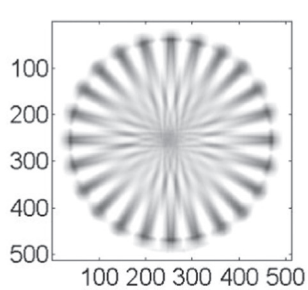
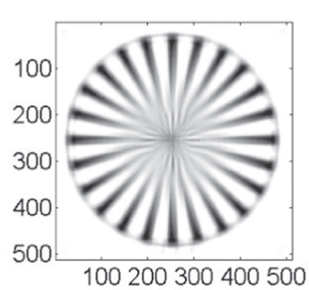
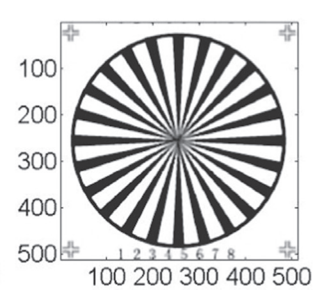
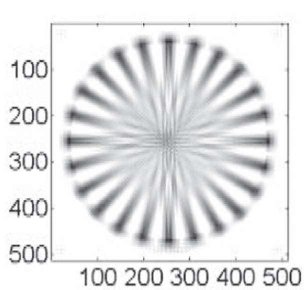
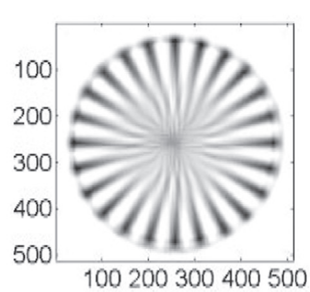
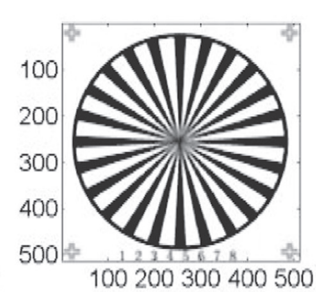
$$\mathrm{FI}(\alpha, \beta, W_{20}) = \iint \frac{1}{h(\alpha, \beta, x', y', W_{20})}\left[\frac{\vartheta \ \ln\ h(\alpha, \beta, x', y', W_{20})}{\vartheta W_{20}}\right]^2 dx'dy' \quad (7)$$

where, x' and y' are the image plane coordinates in the horizontal and vertical direction, h denotes the PSF, which can be obtained through Fourier transformation of pupil function.

The values of FI for cubic phase mask, logarithmic phase mask, sinusoidal phase mask, tangent phase mask and non-integer order phase mask are shown in Fig. 4 with defocus parameter W_{20} from 0 to 5λ.

As Fig. 4 shows, the FI value of non-integer phase mask is the

Table 2
Imaging simulation results.

Defocus parameter W_{20}/λ	Imaging without phase mask	Imaging with non -integer order phase mask	Recovered image
0			
1			
3			
5			

smallest among all five phase masks under any defocus parameters value, which demonstrates the proposed phase mask has better defocus invariant characteristics. Besides, it is clearly seen that FI values of non-integer order phase mask follow a relatively smooth curve against others, which also indicates that the proposed phase mask has better performance on extending DOF.

In conclusion, the wavefront coding phase masks of non-integer and tangent forms have the best performances in DOF extension and MTF invariant. Compared with the tangent one, the proposed phase mask is easier to be manufactured with good surface roughness due to its gentle slope.

4. Simulation

In order to show the performance of proposed phase mask on DOF extension in a more direct way, imaging simulation is carried out. The virtual object used in the simulation is a defocus testing spoke image with resolution of 512pixels × 512pixels, as shown in Fig. 5. In the simulation, the wavelength is set as 0.5 μm.

The final images of imaging system with non-integer phase mask are restored by using a same de-convolution filter, which is presented as:

$$D_{\text{filter}}(u, v) = \frac{OTF_{diff}(u, v)}{OTF_{mask}(u, v, 0)} \tag{8}$$

where, OTF_{diff} denotes the in-focus OTF of the diffraction limited imaging system and OTF_{mask} denotes the OTF of the imaging system with phase mask at defocus parameter equals zero.

The imaging simulation result is shown in Table 2. The left column of images in Table 2 are the imaging results of a conventional imaging system without phase mask over values of defocus parameters from 0 to 5λ. The middle column of images are imaging results with non-integer order phase mask and the right are the images restored from middle column by the same de-convolution filter as above mentioned. It is apparently shown in Table 2 that the imaging results with the proposed phase mask have good resolution and high contrast across a broad range of defocus.

5. Conclusion

In this paper, we proposed a new form phase mask with non-integer order and sign function to realize DOF extension in an incoherent imaging system. Through comparing the defocused MTF curves and values of FI with other phase masks, the results show that the new optimized phase mask has better performance on defocused MTFs invariant characteristics. Finally we make the imaging simulation with and without the proposed phase mask. The comparison of imaging results demonstrate its effectiveness in DOF extension further.

Acknowledgments

This study is funded by the National Science and Technology Major Project of China (Grant 2009ZX02205). The authors give their gratitude to the Engineering Research Center of Extreme Precision Optics for the support.

References

[1] E.R. Dowski, W.T. Cathey, Appl. Opt. 34 (1995) 1859, http://dx.doi.org/10.1364/AO.34.001859.
[2] T.Y. Zhao, F.H. Yu, Opt. Express 20 (2012) 2408, http://dx.doi.org/10.1364/OE.20.002408.
[3] S.S. Sherif, W.T. Cathey, E.R. Dowski, Appl. Opt. 43 (2001) 2709, http://dx.doi.org/10.1364/AO.43.002709.
[4] H. Zhao, Y.C. Li, Opt. Lett. 35 (2010) 2630, http://dx.doi.org/10.1364/OL.35.002630.
[5] H. Zhao, Y.C. Li, Appt. Opt. 49 (2010) 229, http://dx.doi.org/10.1364/AO.49.000229.
[6] H. Zhao, Y.C. Li, Opt. Lett. 35 (2010) 267, http://dx.doi.org/10.1364/OL.35.000267.
[7] V.N. Le, S.Q. Chen, Z.G. Fan, Opt. Lett. 39 (2014) 2171, http://dx.doi.org/10.1364/OL.39.002171.
[8] G. Muyo, A.R. Harvey, Opt. Lett. 30 (2005) 2715, http://dx.doi.org/10.1364/OL.30.002715.
[9] Q.X. Liu, T.Y. Zhao, Y.P. Chen, W.Z. Zhang, F.H. Yu, Chin. Opt. Lett. 8 (2010) 159, http://dx.doi.org/10.3788/col20100802.0159.
[10] Q. Yang, L. Lin, J. Sun, Opt. Commun. 272 (2007) 56, http://dx.doi.org/10.1016/j.optcom.2006.11.021.
[11] S. Prasad, T.C. Torgersen, V.P. Pauca, Int. J. Imaging Syst. Technol. (2004) 67, http://dx.doi.org/10.1002/ima.20009.
[12] S. Prasad, T.C. Torgersen, V.P. Pauca, in AeroSense 2003 (International Society for Optics and Photonics), 2003, p. 1. doi.: http://dx.doi.org/10.1117/12.487572.