Spectral broadband anastigmatic Wadsworth imaging spectrometer

Lei Yu,[*] Shu-rong Wang, Guan-yu Lin, and Dian-sheng Cao

Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun 130033, China

[*]*top1god@mail.ustc.edu.cn*

Abstract: A new advanced optical design based on the Wadsworth mounting for a broadband stigmatic, coma-free practical spectrometer with high imaging quality is presented. By the addition of an inclined cylindrical lens with a wedge angle, the stigmatic imaging conditions in a broad waveband have been obtained by our analysis. An example which presents excellent optical performances over a spectral broadband of 380nm centered at 570nm has been designed to certify the analysis.

OCIS codes: (120.4570) Optical design of instruments; (120.6200) Spectrometers and spectroscopic instrumentation; (300.6190) Spectrometers.

References and links

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1. Introduction

Many applications in frequency-domain optical coherence tomography (OCT), remote sensing, and spatially resolved ultrashort pulse measurement have enabled the rapid development of aberration-corrected imaging spectrometer (e.g. high-quality imaging) over a broadband and a low NA [1–4]. These mountings usually sensitive to power efficiency are required to utilize high speed CCD or CMOS cameras where the width of the detector area is limited to maximize signal to noise ratio. Moreover, the slit of the optical system would make the ray in the meridian and sagittal orientations to focus in different locations and introduce astigmatism. Therefore, the uncorrected astigmatism does result in degraded performance.

Attempts have been provided to eliminate or reduce the astigmatism in grating imaging spectrometers. The single grating Rowland spectrometer, Czerny-Turner spectrometer and Wadsworth spectrometer, will make one single wavelength stigmatic [1–3]. Offner and Dyson spectrometers which have been applied widely at present will make two wavelengths

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anastigmatic [5,6]. By the application of special gratings such as type III holographic concave grating or toroidal varied line-space grating in Rowland mounting, three stigmatic points in the waveband can be obtained [7]. As usual, the Czerny-Turner spectrometer is commonly used in previous scientific areas involved by its definition. However, the Wadsworth spectrometer with simpler composition could not be suitable for these applications with its shortcomings such as curved imaging plane and short working waveband. At present, some modifications such as tandem gratings and paraboloidal grating for Wadsworth mounting have been investigated to improve the performances [2,8,9]. Yu [10] has provided an advanced form of tandem gratings spectrometer based on Bartoe and Brueckner's study [3], which will make each wavelength astigmatic and supply excellent performances over a wide spectral range. Here we have proposed a novel solution to a modified Wadsworth geometry using only an inclined cylindrical lens having a wedge angle without requiring restrictive conditions. When the cylindrical lens with a wedge angle is located in an adequate location between the grating and the imaging plane and rotated by an adequate angle, this solution makes the advanced Wadsworth spectrometer own following characteristics, coma-free, low NA, astigmatism-corrected over a broadband width and a small volume by definition, only one off-axis parabolic mirror, a concave grating and a cylindrical lens. Our method enables low cost construction using simple optics in place of components such a toroidal mirror or a cylindrical grating, both more complex elements to fabricate. We present how the novel design reduces the astigmatism over the broad waveband and an example of the broadband astigmatism-corrected Wadsworth spectrometer with excellent results.

2. Stigmatic imaging conditions analysis of the advanced Wadsworth spectrometer

The new advanced Wadsworth imaging spectrometer includes a collimating mirror, a concave grating and a cylindrical lens with a wedge angle in our design.

2.1 The study on best location of cylindrical lens

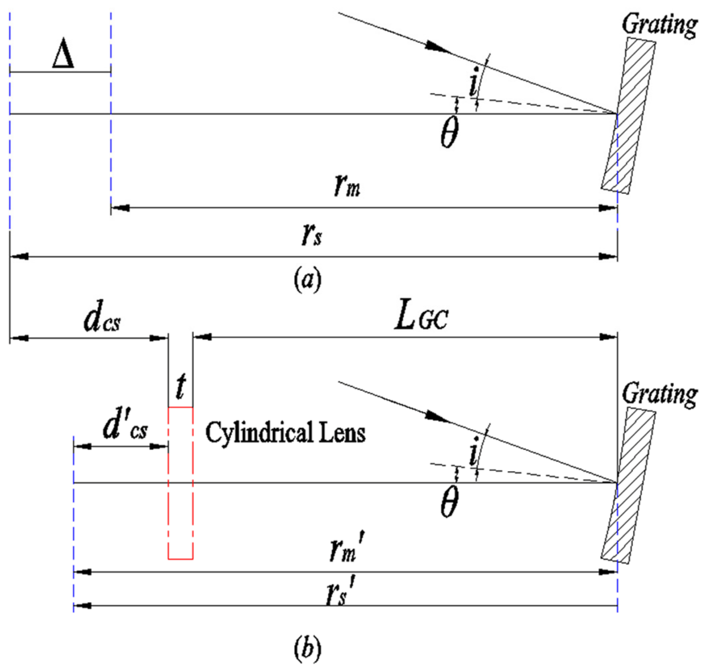

Fig. 1. Two Wadsworth mountings. (*a*) traditional Wadsworth mounting. *i* and *θ* are the incidence and diffraction angles for the grating with radius R. (*b*) advanced mounting with a cylindrical lens. The subscript *m* stands for meridian and *s* stands for sagittal.

According to the grating theory given by Beutler [11], the meridian and the sagittal focal distances in Fig. 1(*a*) are given by

$$\begin{cases} r_m = \left[\dfrac{\cos i + \cos\theta}{R} - \dfrac{\cos^2 i}{r} \right]^{-1} \cos^2\theta \\ r_s = \left[\dfrac{\cos i + \cos\theta}{R} - \dfrac{1}{r} \right]^{-1} \end{cases} \tag{1}$$

As the incidence light on the grating is collimated by the collimating mirror, we consider the source distance $r \to \infty$. Therefore, the meridian and the sagittal focal distances of the traditional Wadsworth mounting at the imaging plane are

$$\begin{cases} r_m = R\cos^2\theta(\cos i + \cos\theta)^{-1} \\ r_s = R(\cos i + \cos\theta)^{-1} \end{cases} \tag{2}$$

The distance Δ is

$$\Delta = r_s - r_m = R\sin^2\theta(\cos i + \cos\theta)^{-1} \tag{3}$$

The adding cylindrical lens which locates in the light path between the grating and the imaging plane will help the astigmatism elimination. By the sagittal lens equation of the cylindrical lens $d'_{cs}{}^{-1}$ equal $f_{cs}{}^{-1} + s_{cs}{}^{-1}$, the change of sagittal focus d_{cs} - d'_{cs} is

$$d_{cs} - d'_{cs} = d_{cs} - \left[d_{cs} f_{cs} / \left(d_{cs} + f_{cs} \right) \right] \tag{4}$$

where f_{cs} is the sagittal focal length of cylindrical lens. This change also equals to the change of the sagittal focal distances r_s - r'_s. The change of the meridian focus introduced by the cylindrical lens (plane parallel plates in the meridian view) could be expressed as

$$r'_m - r_m = \frac{n-1}{n} t \tag{5}$$

where n is the refractive index of material of the cylindrical lens and t is the thickness of the cylindrical lens in the meridian plane. Thus the meridian and sagittal focal distances in the new mounting are

$$\begin{cases} r'_m = r_m + \dfrac{n-1}{n} t \\ r'_s = r_s - \left[d_{cs} - \dfrac{d_{cs} f_{cs}}{d_{cs} + f_{cs}} \right] \end{cases} \tag{6}$$

As in Fig. 1(*b*), r'_m equals to r'_s. Therefore, the following equation is obtained.

$$d_{cs}^2 - P d_{cs} - P f_{cs} = 0 \quad (P = \Delta - \frac{n-1}{n} t) \tag{7}$$

The solving of d_{cs} yields

$$d_{cs} = \frac{P + \sqrt{P^2 + 4 P f_{cs}}}{2} \tag{8}$$

With the Eqs. (4) and (8), the distance d'_{cs} could be calculated. As in Fig. 1, the following expression

$$L_{GC} = r_s - d_{cs} - t = r'_s - d'_{cs} - t \tag{9}$$

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would determine the location of the cylindrical lens. To compensate the astigmatism, the optimal tilt angle and wedge angle of cylindrical lens will be studied in the following discussion.

2.2 The approach on optimal inclined angle and wedge angle of the cylindrical lens

The study of optimal inclined angle and wedge angle of the cylindrical lens would be presented in two steps. The inclined angle of cylindrical lens would be first calculated. In this process of the calculation, the cylindrical lens has no wedge angle. By the characteristics of cylindrical lens, it is considered as a parallel plate in the meridian plane.

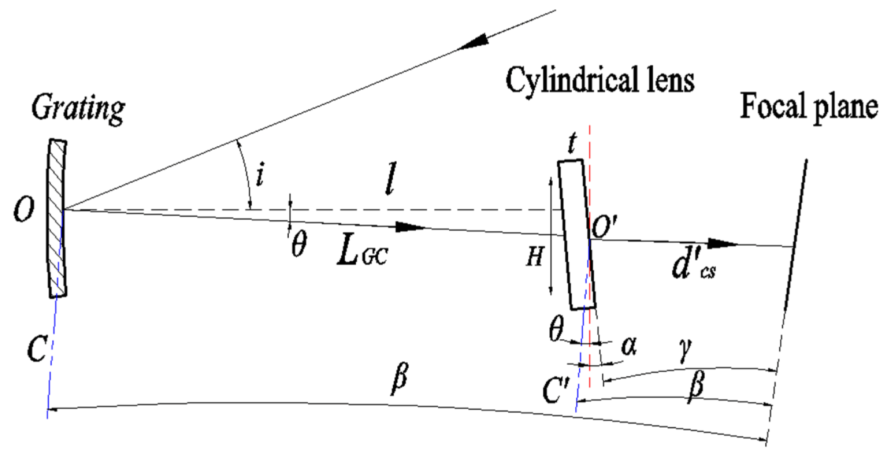

Fig. 2. Ray-tracing for an arbitrary 1st order diffraction wavelength in advanced Wadsworth spectrometer by a tilted cylindrical lens in meridian view, *i* is the incident angle to the grating.

In Fig. 2, the ray incident on the grating is the central light from the collimating mirror. The local normal of grating is *l*. An arbitrary diffraction wavelength is along the optical path L_{GC} and d_{cs}. *OC* is perpendicular to L_{GC}. The red imaginary line crossing the point *O*' which is the emergent point of the arbitrary wavelength on the back surface of cylindrical lens is perpendicular to *l*. The angle β is the included angle of the focal plane and *OC*. It is changing with the variation of wavelength. We also have a blue imaginary line *O*'*C*' paralleling to *OC*. The tilt angle α of cylindrical lens is the included angle of the red imaginary line and the back surface of cylindrical lens. The angle γ is the included angle of the focal plane and the back surface of cylindrical lens. *H* is the incidence height of light on the cylindrical lens.

In the Wadsworth mounting, the diffraction angle θ of central wavelength is zero (the central diffraction wavelength is along *l*). The diffraction angles of shorter waves in the band are negative and the angles of longer waves are positive. The variation of θ results in a variation of astigmatism Δ across the detector. The astigmatism could be compensated effectively if the cylindrical lens is rotated by an optimized angle α. Without loss of generality, our following discussion will focus on an arbitrary wavelength. For the geometric relationship in Fig. 2, it can be easily obtained that

$$\alpha = \gamma - \beta - \theta \tag{10}$$

For the angle γ, we have the relationship

$$\gamma = \tan^{-1}\left(\frac{dd'_{cs}}{dH}\right) = \tan^{-1}\left(\frac{dd'_{cs}}{d\Delta}\frac{d\Delta}{d\lambda}\frac{d\lambda}{dH}\right) = \tan^{-1}\left(\frac{f_{cs}^2}{(d_{cs}+f_{cs})^2}\frac{dd_{cs}}{d\Delta}\frac{d\Delta}{d\lambda}\frac{d\lambda}{dH}\right) \tag{11}$$

by Eq. (4). The first differential term $dd_{cs}/d\Delta$ on the right side is derived from Eq. (8).

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$$\frac{dd_{cs}}{d\Delta}=\frac{1}{2}+\frac{P+2f_{cs}}{2\sqrt{P^2+4Pf_{cs}}} \tag{12}$$

The second differential term $d\Delta/d\lambda$ on the right side in Eq. (11) is derived as

$$\frac{d\Delta}{d\lambda}=\frac{d\Delta}{d\theta}\frac{d\theta}{d\lambda}=R\frac{\sin\theta}{\cos i+\cos\theta}(2\cos\theta+\frac{\sin^2\theta}{\cos i+\cos\theta})\frac{g}{\cos\theta} \tag{13}$$

The first term $d\Delta/d\theta$ in Eq. (13) is calculated from Eq. (3), the second term $d\theta/d\lambda$ is the angular dispersion of the grating, and g is the groove density of grating.

The infinitesimal dH is

$$dH=d\theta\cdot L_{GC} \tag{14}$$

Therefore, the last term $d\lambda/dH$ on the right side in Eq. (11) is expressed as

$$\frac{d\lambda}{dH}=\frac{d\lambda}{d\theta\cdot L_{GC}}=\frac{\cos\theta}{g(r_s-d_{cs}-t)} \tag{15}$$

Therefore, the angle γ would be obtained.

According to Fig. 2, the angle β for an arbitrary diffraction wavelength is

$$\beta=\tan^{-1}(\frac{dr'_m}{dH})=\tan^{-1}(\frac{dr'_s}{dH}) \tag{16}$$

The term dr'_s/dH is presented as

$$\frac{dr'_s}{dH}=\frac{dr'_s}{d\theta}\frac{d\theta}{d\lambda}\frac{d\lambda}{dH}=\left[\frac{dr_s}{d\theta}+(\frac{f_{cs}}{d_{cs}+f_{cs}}-\frac{d_{cs}f_{cs}}{\left(d_{cs}+f_{cs}\right)^2}-1)\frac{dd_{cs}}{d\Delta}\frac{d\Delta}{d\theta}\right]\frac{d\theta}{d\lambda}\frac{d\lambda}{dH} \tag{17}$$

with Eq. (6). The first term $dr_s/d\theta$ on the right side in Eq. (17) is

$$\frac{dr_s}{d\theta}=R\frac{\sin\theta}{(\cos i+\cos\theta)^2} \tag{18}$$

Therefore, the angle β will be obtained by the above discussion. And the tilted angle of cylindrical lens, α, would also be calculated by the analysis.

For the near-linear dispersion of the grating, the tilted angle is near linear decreasing with the linear increasing of wavelengths (from positive to negative). Both of the diffracted angle and the corresponding tilt angle α are equal to zero under the central wavelength. And the absolute value of tilt angle would be maximal at marginal wavelength (minimal and maximal). When the cylindrical lens rotates in counter-clockwise, the rotation will accord with the variation. And the astigmatism could be reduced by the rotation.

An important point should be noticed that the variation of inclined angle would not influent the stigmatic focusing of central wavelength for the basic Wadsworth condition. To simplify the optimization, the marginal wavelength (the longest or shortest wavelength) could be chosen to calculate the initial inclined angle of cylindrical lens. The varying trend determines the direction of rotation of the cylindrical lens for the symmetric linear variation of the inclined angle. If the direction of rotation angle is counter clockwise, the astigmatism would decrease to the minimal value with the increase of rotation angle of the cylindrical lens. However, when the rotation angle exceeds the extreme value, the astigmatism would increase. The absolute value of the angle is the value of the counter-clockwise rotation angle of the cylindrical lens. This selecting principle is proved appropriately and definitely in the following design example.

The final focal plane is usually curved in the meridian orientation in the Wadsworth mounting. However, the detector is usually a plane in applications. In other words, the imaging quality would not be the best only by the optimal tilted angle. To compensate the effects introduced by the defocus and residual astigmatism, the optimal wedge angle of cylindrical lens is needed. Therefore, the next step is to calculate an adequate wedge angle of the cylindrical lens.

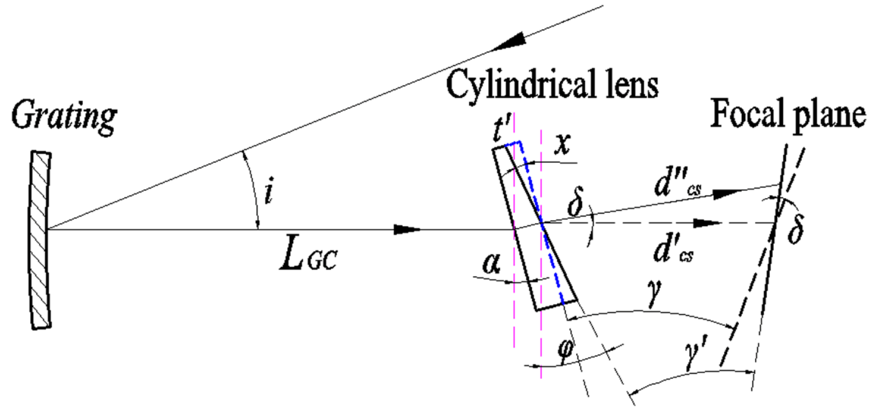

Fig. 3. The 1st order center diffraction wavelength tracing for the wedge angle calculation.

The calculation is under the central diffraction wavelength. According to Fig. 3, the two pink imaginary lines are parallel and they are perpendicular to the central wavelength (e.g. L_{GC} which is also the local normal of the grating). *t'* is the thickness of the cylindrical lens in the meridian plane. The angle α is the included angle of the front surface of cylindrical lens and the pink imaginary line, which is the optimal inclined angle of cylindrical lens in the previous optimization. The angle φ is the included angle of the back surface of cylindrical lens and the pink imaginary line. The blue imaginary line stands for the back surface of cylindrical lens which parallels the front surface. At this time, when the arbitrary wavelength is chosen as the central wavelength in Fig. 2, the imaginary line L_{GC}-d'_{cs} in Fig. 3 is the same with the optical path of the previous optical system in Fig. 2 which owns the cylindrical lens without a wedge angle. The imaginary black line entered by d'_{cs} in Fig. 3 stands for the previous focal plane in Fig. 2. The included angle γ of the back surface and the previous focal plane in Fig. 3 is the same as it in Fig. 2. When the cylindrical lens has a wedge angle (the back surface is a black line in Fig. 3 which doesn't parallel the front surface), the new optical path between the cylindrical lens and the imaging plane is changed as L_{GC}-d''_{cs} in Fig. 3. The line entered by d''_{cs} stands for the present focal plane. The new included angle γ' is of the back surface and the present focal plane in the new optical path in Fig. 3. As the geometric relationship shown in Fig. 3, the wedge angle of cylindrical lens is φ-α = x. By the wedge angle, the emergent wave would be rotated by an angle δ. It is the included angle of d'_{cs} and d''_{cs} which is also considered as the included angle of two focal planes.

It is evidently seen that *t'* is equal to *t* when they stand for the central thickness of the cylindrical lens. Therefore, the wedge angle does not influent the best focal distance r'_s for the central wavelength across the central thickness. Thus the focal distance in Fig. 3 at the central diffraction wavelength with the cylindrical lens having a wedge angle would be equal to r'_s in Eq. (9). Namely

$$r_s' = L_{GC} + d'_{cs} + t = L_{GC} + d''_{cs} + t' \tag{19}$$

With the differential calculus of Eq. (19), we have

$$\frac{dr'_s}{dH} = \frac{dL_{GC}}{dH} + \frac{dd'_{cs}}{dH} + \frac{dt}{dH} = \frac{dL_{GC}}{dH} + \frac{dd''_{cs}}{dH} + \frac{dt'}{dH} \tag{20}$$

The equation could be derived as

$$\frac{dd''_{cs}}{dH} - \frac{dd'_{cs}}{dH} + \frac{dt'}{dH} - \frac{dt}{dH} = \tan\gamma' - \tan\gamma - \tan x = 0 \tag{21}$$

By the Snell's law, we have

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$$\delta = \alpha + x - \sin^{-1}\left(n \cdot \sin\left(\sin^{-1}\left(\frac{\sin\alpha}{n}\right) + x\right)\right) \tag{22}$$

With the geometric relationship in Fig. 3, the Eq. (21) is changed into

$$\tan\gamma' = \tan\gamma + \tan x = \tan(\gamma + x - \delta) \tag{23}$$

Substituting each value in previous calculation into Eq. (23), the wedge angle of cylindrical lens x would be estimated by the computer program. To make the calculation easier, we could estimate some values of the angle x and substitute them into the above equations to obtain an initial interval. With Matlab program, a suitable value would be quickly obtained from this initial interval.

2.3 Residual coma in the advanced Wadsworth mounting

The coma of the advanced Wadsworth mounting mainly comes from the collimating mirror and the concave grating. As M. Seya and T. Namioka's research [12], the coma of Wadsworth mounting using a spherical collimating mirror is given by

$$\Delta p = \frac{3W^2}{8R}\cos\alpha\left[\tan\theta_0 \frac{\cos\theta(\cos i + \cos\theta_0)}{\cos\theta_0(\cos i + \cos\theta)}\right] \times \left[1 + \tan^2\theta\left(1 + \frac{\cos i}{\cos i + \cos\theta}\right)\right]^{1/2} \tag{24}$$

where Δp is the linear size of the coma, W is the grating width, R is the grating radius, i and θ are the incidence and diffraction angles for the grating, and θ_0 is the angle of diffraction at which the coma is zero. When $\theta_0 = 0$, the object source would be on the normal of collimating mirror and the coma computed from the equation for Δp is due to the grating only. The form of collimating mirror could be chosen as the off-axis parabolic mirror to satisfy this coma-free condition.

In our design, the grating in the Wadsworth mounting is used in the parallel illumination and the image is located on the grating normal. Here we consider the cylindrical lens and the grating as an integrated "grating". Therefore, there is only a residual, very small, coma effect which is derived from Eq. (24).

$$\varphi_c = \frac{3}{8(F\#)^2}\frac{(r'_m)^2}{Rr'_s}\cos i \bullet \tan\theta \times \left[1 + \tan^2\theta\left(1 + \frac{\cos i}{\cos i + \cos\theta}\right)\right]^{1/2} \tag{25}$$

In the expression, $F\# = r'_m/W$ is the F number of the system, and W is the grating width. According to our design, then we have

$$\begin{cases} \sin i = \lambda_c g \\ \sin i + \sin\theta = \lambda g \end{cases} \tag{26}$$

where λ_c is the 1st order central diffraction wavelength, g is the ruling density of grating. Thus, the diffraction angle at the grating is

$$\sin\theta = (\lambda - \lambda_c)g \tag{27}$$

By combining Eqs. (25)-(27), we could calculate the residual coma of our advanced Wadsworth arrangement as a function of the wavelength. In the following example, it would be found that the advanced design realize coma-free.

3. Design procedure for an example of the new Wadsworth spectrometer

An example in a wide bandwidth spanning 380nm to 760nm has been simulated by ZEMAX on the previous strategy discussed. The numerical aperture is chosen as 0.05. The basic parameters of the detector are set that the length of the detector is 25.6 mm with a pixel size of 25μm. To determine the ruling density of grating, we can analysis the spectral resolution first. According to Yu's study [7], the resolution of the system will be expressed as

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$$d\lambda = \frac{\cos i}{mgf_1} \cdot b\cos\zeta \tag{28}$$

where b is the width of the slit and ζ is the inclined angle of imaging plane. Then we can approximate the ruling density of grating by Eq. (28). In the design, the spectral resolution is decided as 5nm. The refractive index of cylindrical lens is 1.62. All the fixed parameters of design in the spectrometer are shown in Tab. 1.

Table 1. Fixed Parameters of Advanced Mounting

Fixed parameter	R_1	R_2	g	f_{cs}	t	$\lambda_{central}$
Value	200 mm	250 mm	300 l/mm	100 mm	4 mm	570nm

The residual coma in our design is shown in Fig. 4. It is considered that the mounting realizes coma-free.

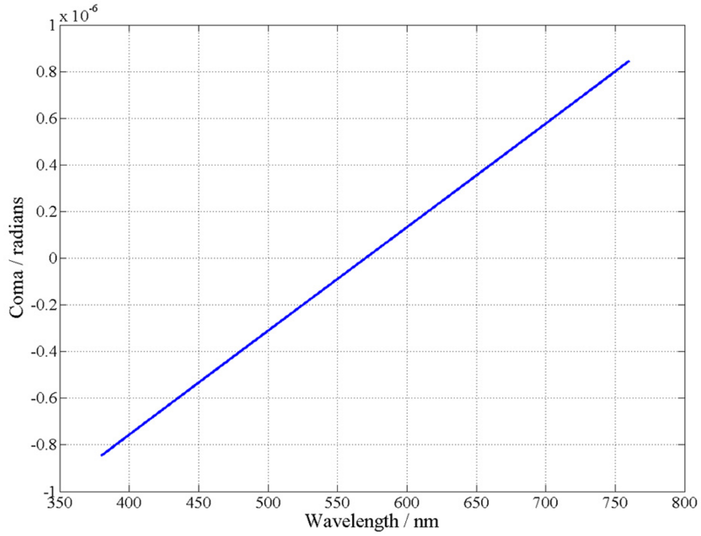

Fig. 4. Residual coma in the design.

We calculated the initial parameters of advanced Wadsworth spectrometer and optimized them using the ray-tracing software Zemax to obtain the final parameters as shown in Table 2.

Table 2. Optimized Parameters of Advanced Wadsworth Mounting

Parameter	ω (off-axis angle)	i	α	L_{GC}	x
Initial value	6°	9°	28.44°	113.41mm	5.3°
Optimized value	5.72°	9.85°	27.67°	112.5mm	3.77°

The optimized layout is shown in Fig. 5(a).

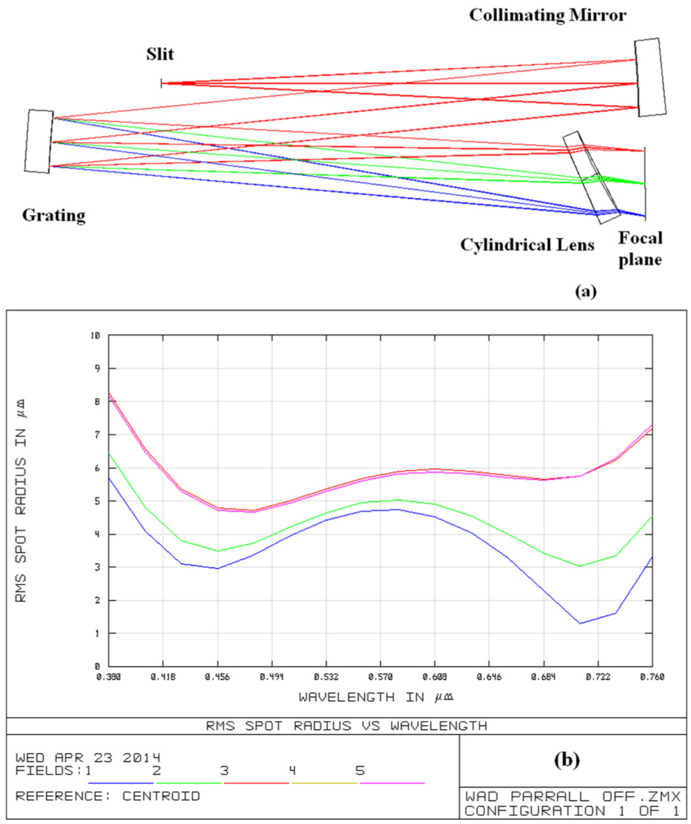

Fig. 5. (a) Optimized layout of the broadband advanced Wadsworth spectrometer. (b) RMS spots radii versus wavelengths.

As shown in Fig. 5(b), all spots are completely enclosed in a square pixel of side 25μm of all waveband. The advanced Wadsworth spectrometer presents excellent imaging performances.

4. Conclusions

To summarize, we present a reasonable and detailed analysis of the new broadband astigmatism-corrected Wadsworth spectrometer using an inclined cylindrical lens with a wedge angle. The optimal parameters of the arrangement, which give excellent imaging conditions, have been calculated by the geometric method. The solution is effective for the spectrometer working in a broad spectral range and does not require restrictions in 1st order parameter selection for aberration compensation. An example in 380nm~760nm confirms the optical performances of our design. The new design provides excellent optical quality in a spectral broadband, with a simple and compact device.