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Abstract. Convex aspheric surface is tested by a circular amplitude computer-generated hologram (CGH) fabricated with our equipment and techniques, and much research work has been done simultaneously. However, the analysis of the detailed characters of the CGH used in the test system has not been systematically given in detail, including the correct phase, amplitude, and filter condition of the CGH. The calculation equation of the proper duty circle and the phase of the CGH are deduced, the frequency filter condition of the different diffracted orders of the CGH is demonstrated, and the deduction results are validated by the related experiment. The conclusion can help us to determine the radius ratio of the uncontrolled area over the full aperture of the aspheric surface during the process of optical system design, and it also points out that the radius ratio can be reduced by adjusting the radius of curvature of the reference surface and the distance between the reference surface and the convex aspheric surface. The work can assist us in designing the test system efficiently and correctly with CGH.
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Key words: computer-generated hologram; optical testing; aspheric.

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1 Introduction

A circular amplitude computer-generated hologram (CGH) is a recording of many microscopic concentric circles that amounts to an encoded recording of a certain rotationally symmetrical wavefront of light. It consists of hundreds to thousands of circular gratings recorded on the substrate surfaces, such as glass, silicon, and PMMA. The circular CGH used for testing aspheric surfaces could be found back in 1972,[1] but it was used to test large-aperture concave surfaces. Then many liner CGHs were used to test convex aspheric surfaces.[2,3] Burge proposed to use circular CGHs to test convex surfaces with a special optical setup,[4–7] and the CGH was fabricated with its own laser writer. In 2000, a laser direct writer system was built by our team, and we have fabricated a large CGH on a concave lens surface with precise alignment using the laser direct writer.[8–11] This technology allows precise alignment, superior linear profile, and high resolution of the gratings that compose the CGH. So we have done much work concerning aspheric tests with circular CGHs.[12–15] Although some conclusions were given in these papers, they were not derived systematically, and the analysis of the characters of the CGH used in the test system has not been given in detail and systematically.

In this paper, the characters of the CGH for the aspheric test are analyzed based on our previous works. The calculation equation of the proper duty circle and the phase of the CGH is deduced, as well as the frequency filtering conditions of different diffracted orders of the CGH, and the related experimental results validate the analytical results very well. The conclusions are very useful for a designer to determine the radius ratio of the uncontrolled area over the full aperture of the aspheric surface during the process of optical system design. They also tell us that the radius ratio can be reduced by adjusting the radius of curvature of the reference surface and the distance between the reference surface with a CGH and the convex aspheric surface. The work can assist us to design an optical test system with CGH efficiently and correctly.

2 Phase and Amplitude of the Computer-Generated Hologram in Test System

Generally, one rotationally symmetrical wavefront at the $z = 0$ plane can be expressed as a complex amplitude distribution

$$u(r) = A \exp[i\phi(r)], \tag{1}$$

where A is the amplitude of the wavefront and $\phi(r)$ is the phase of the wavefront. The following equation is used to qualify the phase and then to get r; the position value of the transmitted parts $(T = 1)$ or nontransmitted parts $(T = 0)$ of the circular grating

$$\begin{aligned} T &= 1, \qquad i\lambda \geq \phi(r) > i\lambda - D\lambda \\ T &= 0, \qquad i\lambda - D\lambda \geq \phi(r) > (i+1)\lambda, \end{aligned} \tag{2}$$

where D is the duty cycle of the grating, i is the nature number, and the CGH on a substrate surface can be fabricated with all the values of r. The reconstructed wavefront of different reflected diffractive orders of the CGH can be expressed as

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$$
\begin{aligned}
H[W(r)] &= \sum_m A_{\mathrm{CGH\_rm}} \exp(im\phi_{\mathrm{CGH\_rm}}(r)) \\
\phi_{\mathrm{CGH\_rm}}(r) &= m\pi + m\sum_{i=1}^{N}\left[(i-1)+\frac{r}{\sum_{j=1}^{i} s_j}\right]2\pi \\
&= m(\pi + b_1 r^2 + b_2 r^4 + b_3 r^6 + \cdots) = m\pi + m\phi_{\mathrm{CGH}}(r), \\
A_{\mathrm{CGH\_rm}} &= \begin{cases} \{a_0 + [a_1 \cos(\varphi_r) - a_0]D\} + i\{a_1 \sin(\varphi_r)D\} & m=0 \\ \{[a_1 \cos(\varphi_r) - a_0]D \sin c(mD)\} + i\{a_1 \sin(\varphi_r)D \sin c(mD)\} & m=\pm 1, \pm 2, \cdots \end{cases},
\end{aligned} \tag{3}
$$

where $\phi_{\mathrm{CGH}}(r)$ is the phase of the CGH; m is the diffraction order of CGH; $b_1, b_2, b_3\ldots$ are the coefficients of different terms of the phase of the CGH; a_0 is the reflective coefficient of the nontransmitted part of the CGH; and a_1 is the transmittance coefficient of the transmitted part of the CGH. φ_r represents the phase shift of the reflective light, and its value is usually π.

The reconstructed wavefront of different transmitted diffractive orders of the CGH can be expressed as

$$
\begin{aligned}
H[W(r)] &= \sum_m A_{\mathrm{CGH\_tm}} \exp(im\phi_{\mathrm{CGH\_tm}}(r)) \\
\phi_{\mathrm{CGH\_tm}}(r) &= m\sum_{i=1}^{N}\left[(i-1)+\frac{r}{\sum_{j=1}^{i} s_j}\right]2\pi \\
&= m(b_1 r^2 + b_2 r^4 + b_3 r^6 + \cdots) = m\phi_{\mathrm{CGH}}(r), \\
A_{\mathrm{CGH\_}tm} &= \begin{cases} \{a_0 + [a_1 \cos(\varphi_t) - a_0]D\} + i\{a_1 \sin(\varphi_t)D\} & m=0 \\ \{[a_1 \cos(\varphi_t) - a_0]D \sin c(mD)\} + i\{a_1 \sin(\varphi_t)D \sin c(mD)\} & m=\pm 1, \pm 2, \cdots \end{cases}
\end{aligned} \tag{4}
$$

where φ_t represents the phase shift of the transmitted light, and its value is usually 0.

Figure 1 shows the schematic diagram of our test system, which is optimized at the wavelength of 632.8 nm. The laser is focused by a microscope to form a point source of light, a beam splitter is placed after the point source, and two lenses are used for the illumination. A CGH is fabricated on the concave spherical surface of the test lens. A wavefront, $W_{\mathrm{source}}(r)$, from the laser and microscope passes the illumination lens and enters the test plate. The wavefront of the first order of reflected diffraction of the CGH on the sphere's reference surface, called $r1$, passes the test plate and the illumination lens and forms the reference wavefront. The wavefront of the zero order of transmitted diffraction of the CGH, called $t0$, impinges perpendicularly onto the aspheric surface and is reflected to the CGH again. The wavefront of the zero order of transmitted diffraction, called $t0'$, transmits to the test plate and the illumination lens and forms the test wavefront. Although the CGH is fabricated on a curved reference surface, it can be just regarded as being on plane P. The

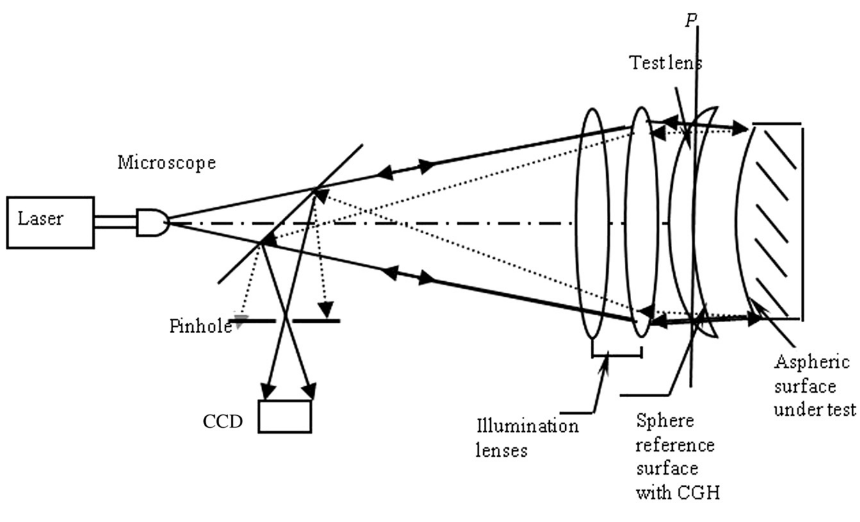


Fig. 1 Layout for measuring convex surface with diffractive optical element.

interference between the reference and the test wavefront is used in this system to get the error of the aspheric surface.

The reference wavefront is expressed as $W_{\text{ref}}(r)$

$$W_{\text{ref}}(r) = A_{\text{ref}}\ \exp(i\phi_{\text{ref}}(r)),$$
$$\phi_{\text{ref}}(r) = \phi_{\text{system}}(r) - 2n\phi_{\text{ref}}(r) + \phi_{\text{CGH\_}r1}(r),$$
$$A_{\text{ref}} = A_{\text{system}}A_{\text{CGH\_}r1}, \tag{5}$$

where $\phi_{\text{system}}(r)$ and A_{system} are, respectively, the phase and amplitude of the wavefront $W_{\text{system}}(r)$ on plane P formed by the system, $\phi_{\text{CGH\_}r1}(r)$ and $A_{\text{CGH\_}r1}$ are, respectively, the phase and amplitude of the wavefront $W_{\text{CGH\_}r1}(r)$ formed by the reflected first diffraction order of CGH, and n is the refraction index of the test plate. The test wavefront is expressed as $W_{\text{test}}(r)$

$$W_{\text{test}}(r) = A_{\text{test}}\ \exp(i\phi_{\text{test}}(r)),$$
$$\phi_{\text{test}}(r) = \phi_{\text{system}}(r) - 2n\phi_{\text{ref}}(r) - 2\phi_{\text{asph}+d}(r) + 2\phi_{\text{CGH\_}t0} + 2\phi_{\text{ref}}(r) + \pi$$
$$A_{\text{test}} = A_{\text{system}}A_{\text{asph}+d}A^2_{\text{CGH\_}t0}, \qquad A_{\text{asph}+d} = (R)^{0.5}, \tag{6}$$

where $\phi_{\text{asph}+d}(r)$ and $A_{\text{asph}+d}$ are, respectively, the phase and amplitude of the wavefront $W_{\text{asph}+d}(r)$ on plane P formed by the aspheric surface under test, d is the distance between the CGH and the aspheric surface along axis, R is the reflectivity of the aspheric surface, and $\phi_{\text{CGH\_}t0}(r)$ and $A_{\text{CGH\_}t0}$ are, respectively, the phase and amplitude of the wavefront $W_{\text{CGH\_}t0}(r)$ formed by the transmitted zero diffraction order of the CGH. In order to achieve the interference figure with a high contrast, the phase and amplitude of both the reference and test wavefronts should be consistent. According to Eqs. (5) and (6), we can get the following equations:

$$\phi_{\text{CGH\_}r1}(r) = \pi + \phi_{\text{CGH}}(r), \qquad \phi_{\text{CGH\_}t0}(r) = 0 \tag{7}$$

$$(A_{\text{asph}+d}A^2_{\text{CGH\_}t0})^2 = (A_{\text{CGH\_}r1})^2. \tag{8}$$

The duty circle D can be deduced with Eq. (8) as the following equation:

$$(R^2(a_0 + (a_1 - a_0)D)^2 = (a_0 + a_1)^2 D^2\ \sin\ c^2(D).$$

By making the phase of the reference wavefront $\phi_{\text{ref}}(r)$ coincident with the test one $\phi_{\text{test}}(r)$, the phase of the CGH can be deduced as

$$\phi_{\text{CGH}}(r) = 2(\phi_{\text{asph}+d}(r) - \phi_{\text{ref}}(r))$$
$$= b_1 r^2 + b_2 r^4 + b_3 r^6 + \cdots$$
$$b_1 \cong -\frac{2\pi}{\lambda}\left(\frac{1}{R_{\text{ref}}} - \frac{1}{R_{\text{asph}} + d}\right), \tag{9}$$

where R_{ref} is the radius of curvature of the reference surface with CGH, and R_{asph} is the vertex radius of curvature of the convex aspheric surface under test. It can be found that the phase of the CGH relates to the radius of the reference and aspheric surfaces, as well as to the distance of d. When the aspheric surface under test is determined, the radius of the reference surface and d can be chosen correctly to control the proper phase of the CGH. The larger the phase of the CGH is, the easier the unwanted diffractive orders are filtered, but the harder the CGH is fabricated. In fact, the radius of the reference surface tends to be equal to the vertex radius of the aspheric surface.

The CGH has several reflected diffraction orders and transmitted orders, but only the combination of $r1$, $t0$, and $t0'$ is desired, and the other combinations are spurious and must be filtered out. In the test system, a filtering hole is placed at the focus of the correct orders as shown in Fig. 2, and its size is just large enough to pass both the reference beam and the test beam. The spurious orders are out of focus and can be filtered. However, the central region of the spurious orders at the filtering plane can pass through the pinhole, resulting in the noncontrolled area of the surface under test.

3 Frequency Filter Condition of Different Diffractive Orders of Computer-Generated Hologram

Although the filtering condition has been given in Ref. 10, we deduce it here in more detail. As shown in Fig. 2, we still use frequency coordinates at the filtering plane to establish the relation of r', r_1, v_c, $v_{rm}(r)$, and $v_{tmm'}(r)$. Here, the radius of the noncontrolled area of the surface under test is noted as r_1, the cut-off frequency of the filtering hole with radius of r' is noted as v_c, and the frequency functions of the reflected and transmitted diffraction orders on the filtering plane are noted as $v_{rm}(r)$ and $v_{tmm'}(r)$, respectively. Then we have

$$v_c = \frac{r'}{\lambda l}, \tag{10}$$

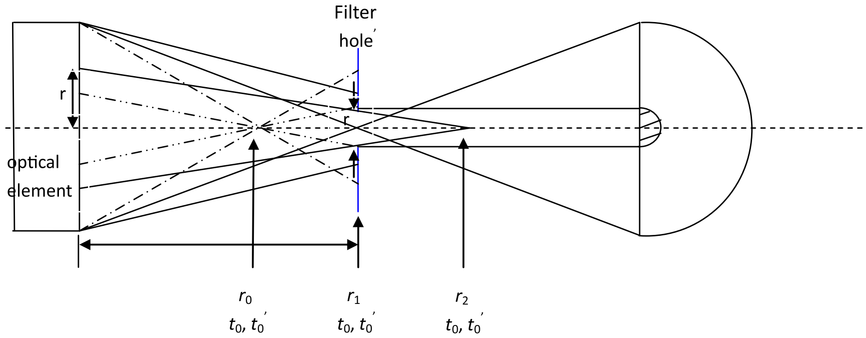


Fig. 2 Filtering conditions of different diffractive orders in the filter plane.

$$v_{rm}(r) = \frac{1}{2\pi}\frac{\partial[(m-1)\phi_{\text{CGH}}(r) + \Delta\phi_{\text{system}}(r)]}{\partial r}, \\ = (m-1)v_{\text{CGH}}(r) + v_{\text{system}}(r) \tag{11}$$

$$v_{tmm'}(r) \\ = \frac{1}{2\pi}\frac{\partial[m\phi_{\text{CGH}}(r) + m'\phi_{\text{CGH}}(r) + \Delta\phi_{\text{system}}(r) + \Delta\phi_{\text{asp}}(r)]}{\partial r} \\ = mv_{\text{CGH}}(r) + m'v_{\text{CGH}}(r) + v_{\text{system}}(r) + v_{\text{asp}}(r), \tag{12}$$

where $v(r_{\text{system}})$ represents the frequency function with respect to the errors of the system on the filtering plane, $v(r_{\text{asp}})$ represents that with respect to the aspheric shape, and $v(r_{\text{CGH}})$ represents that with respect to the CGH. We have

$$v_{\text{CGH}}(r) = \frac{1}{2\pi}\frac{\partial\phi_{\text{CGH}}(r)}{\partial r} \\ = \frac{1}{2\pi}(2b_1r + 4b_2r^3 + 6b_3r^5 + \cdots) \cong \frac{b_1r}{\pi}, \tag{13}$$

where the coefficients of b_2, b_3, and so on are usually small enough to be neglected compared with b_1. In fact, when the reflected order of diffraction is 1, we have

$$v_{r1}(r) = \frac{1}{2\pi}\frac{\partial[\Delta\phi_{\text{system}}(r)]}{\partial r} = v_{\text{system}}(r). \tag{14}$$

The reflected first diffraction order from the CGH focuses at the filtering plane including the information about the errors of the system. When the transmitted order of diffraction is 0, we have

$$v_{t00}(r) = \frac{1}{2\pi}\frac{\partial[\Delta\phi_{\text{system}}(r) - \Delta\phi_{\text{asp}}(r)]}{\partial r} \\ = v_{\text{system}}(r) + v_{\text{asp}}(r). \tag{15}$$

This transmitted zero diffraction order from the CGH contains the information about the shape of the aspheric surface and the errors of the system. For a test system with a large error in the illumination system, the value of $v(r_{\text{asp}})$ is small enough to be neglected compared with $v(r_{\text{system}})$. So

$$v_{t00}(r) \cong v_{\text{system}}(r). \tag{16}$$

The filtering hole is a low-pass filter and will suppress the information above its cut-off frequency of v_c. In order to enable the testing wavefront and reference wavefront to pass without being truncated, the size of the filter should be no less than the maximum height of a ray of the transmitted zero-order diffraction on the filtering plane

$$v_c = v_{t00}(r)|_{\max}. \tag{17}$$

Thus, we can deduce the size of the filter

$$r' = \lambda l v_{t00}(r)|_{\max} \quad = \lambda l \nu_{\text{system}}(r)|_{\max}. \tag{18}$$

By using this filter at the focus of the desired order, the central part of the other spurious diffraction order will pass, and the outer part will be truncated. Since the values of $v(r_{\text{system}})$ and $v(r_{\text{asp}})$ are small enough to be neglected compared to $v(r_{\text{CGH}})$, the frequency function of other spurious orders on the filter plane can be expressed as

$$v_{rm}(r) = (m-1)v_{\text{CGH}}(r) \\ \cong (m-1)\frac{b_1r}{\pi} \\ v_{tmm'}(r) = (m+m')v_{\text{CGH}}(r) \\ \cong (m+m')\frac{b_1r}{\pi}. \tag{19}$$

If a noncontrolled area of the aspheric surface with a radius of r_1 is permitted, all the spurious rays that would blur the interferogram outside r_1 must be blocked by the pinhole. In fact, in all of the reflected diffraction orders, the zero order and the second order have the strongest intensity for influencing the interferogram, and the effect of other reflected high-order diffractions is small enough to be neglected. All the transmitted diffraction orders have a negligible effect. Mathematically, this condition can be written as

$$v_c \geq |v_{r0}(r)| \quad \text{and} \quad v_c \geq |v_{r2}(r)| \quad \text{for all } r \geq r_1. \tag{20}$$

Therefore, we have

$$b_1r \geq \frac{\pi r'}{\lambda l} \quad r' = \lambda l \nu_{\text{system}}(r)|_{\max}. \tag{21}$$

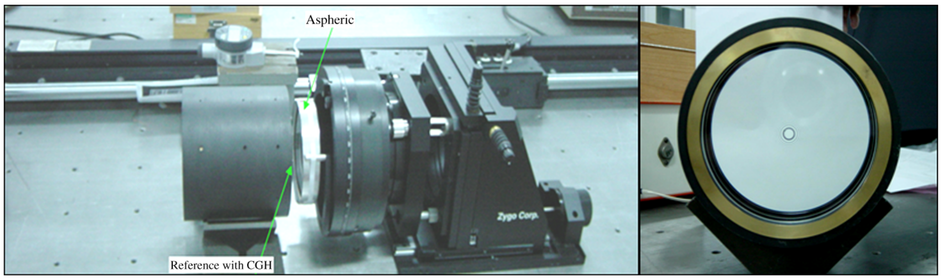
When a hole with a radius of r' is used as the filter, there is a bright disc with a radius of r_1 in the center of the interferogram in the experiment. The bright disc will be smaller when the b_1 of the phase of the CGH is larger, but the fabricaton of the CGH is more difficult for the smaller minimum period of the CGH. In fact, two methods can be used to balance the contradiction. One is testing the aspheric surface two times with different holes.[14] The smaller hole is used to test the center part of the aspheric, and the larger hole is used to test the marginal part. The size of the two holes can be calculated with Eq. (21). Another method is testing the aspheric surface two times with a special CGH,[15] which is composed of two parts. The phase of the center part is calculated with the larger d, and the phase of the marginal part is calculated with the smaller d.

4 Experiment

The convex surface under test is an elliptic mirror with a 100-mm diameter and a vertex radius of curvature of 500 mm. So, we choose a reference surface with a 100 mm diameter and in radius of curvature of 500 mm . The distance of d is determined to be 10 mm. According to Eq. (9), b_1 is about 0.392. In fact, the value of b_1 is 0.38988 after optimization with optical design software Zemax. Then the phase of the CGH in our test system is given as follows:

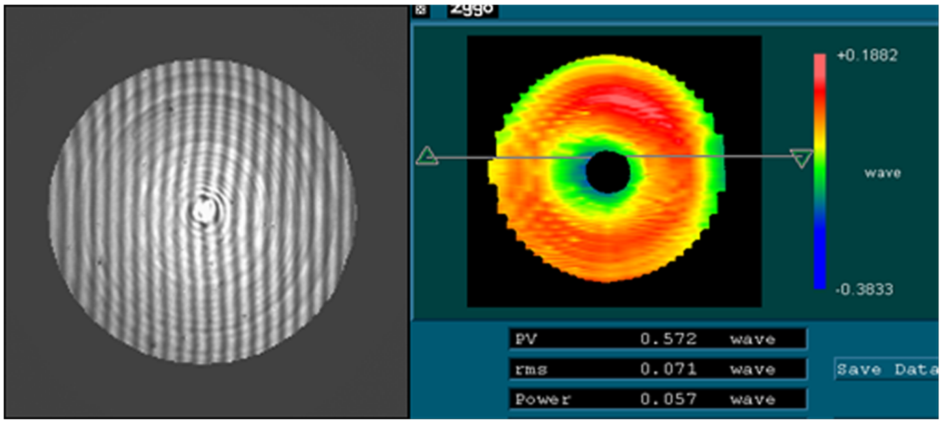
$$\phi_{\text{CGH}}(r) = -0.38988r^2 + 1.8321 \times 10^{-5}r^4.$$

The minimum period of the CGH is about 50 μm, and it is very easy to fabricate with high accuracy. The CGH is fabricated with the laser direct writer in our lab by following five procedures: photoresist coating, laser exposing, chrome



(a) (b)

Fig. 3 (a) Optical test system and (b) the reference surface with CACGH.



(a) (b)

Fig. 4 (a) Interferogram with bright disc in center, and (b) the surface profile of the convex aspheric under test with the noncontrolled area in center.

developing, depositing, and photoresist stripping. The test system is set up as shown in Fig. 3(a) where the laser source and the CCD camera are not displayed. The reference surface with the CGH is shown in Fig. 3(b).

The frequency function curves have been given in our previous research paper of Ref. 14. Figure 3(a) in Ref. 14 shows the frequency function curve, $v_{t00}(r)$, with respect to the error in the illumination system. As shown in the figure, the maximal value is 1.05 mm^{-1}, and it is the cut-off frequency of the test system. Figure 3(b) in Ref. 14 shows the frequency function curve, $v_{r0}(r)$, with respect to the information of the CGH. $v_{r2}(r)$ has the opposite value of $v_{r0}(r)$ according to Eq. (20), so it has not been shown. According to Eqs. (20) and (21), the radius of the noncontrolled area is 11 mm when using a filtering hole with a cut-off frequency of 1.05 mm^{-1}, and the corresponding interferogram is shown in Fig. 4(a). The surface profile of the tested convex aspheric surface obtained with Zygo static interferogram analysis software is shown in Fig. 4(b), and the wavefront error is 0.572λ in peak value (PV) and 0.071λ in root-mean-square. The measurement result includes four parts: the figure error from the aspheric surface under test, which is the one we want to get; the figure error from the concave spherical reference surface; the error from the hologram; and the adjustment error from misalignment. If the reference surface and the CGH are fabricated with high accuracy, the wavefront error from them can be controlled to be less than $1/30\lambda$ in PV. The error from misalignment can be controlled to be very small if we adjust the system carefully, because it is a common-path interferometer.

From the interferogram of Fig. 4(a), the central part is blued by the light of the other diffractive orders. The diameter of the noncontrolled area is about one-fifth of the whole test area. It is clear that the experimental result matches well with our analysis.

5 Conclusions

By analyzing the characters of the CACGH for an aspheric test, the proper duty circle and the phase of the CGH are deduced. The frequency filter conditions of different diffracted orders of the CGH are also demonstrated. The experimental results confirm that the filter condition is correct. The conclusions are very useful for a designer to determine the radius ratio of the uncontrolled area over the full aperture of the aspheric surface during the process of optical system design. This can efficiently and correctly lead us to design the test system with CGH.

Acknowledgments

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