Adaptive robust control with extended disturbance observer for motion control of DC motors

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This Letter proposes an extended disturbance observer (EDO) based adaptive robust control method for motion control of DC motors. An adaptive part for structured uncertainties, an EDO for unstructured uncertainties and a feedback robust law for global robustness are designed in this method. The theoretical analysis guarantees a prescribed tracking performance under various uncertainties and comparative numerical simulation shows the excellent high-precision performance of proposed method on DC motor motion control.

Introduction: High-accuracy motion control of DC motors has attracted much attention for its wide application both in manufacturing and industry [1, 2]. However, the complex actual working conditions make design of the high-accuracy motion system quite difficult. Disturbance and modelling uncertainties [1, 3–5] will lead to a poor system accuracy or even system instability.

The adaptive robust control (ARC) guarantees prescribed output tracking transient performance and final tracking accuracy in general while achieving asymptotic output tracking in the absence of unstructured uncertainties [5]. However, demand of high-gain feedback for high accuracy may limit its wide practical implementation.

Combining control law with uncertainties and disturbances estimates is an novel attractive proposition, which can deal with larger unstructured uncertainties. Extended disturbance observer (EDO) proposed by Ginoya [6], which is used for sliding mode control (SMC), shows more excellent performance than conventional disturbance observer case. While, its chattering problem from SMC is still hard to solved thoroughly.

This Letter, we borrow the ideas of EDO [6] and ARC approach [1], and combine them via the robust control action with some assumptions, a new ARC with EDO is proposed for high-accuracy dc motor motion control. The proposed design appears an outstanding tracking performance in the presence of both structured and unstructured uncertainties from comparative numerical results.

Problem formulation and dynamic models: The motor investigated in this Letter is a current-controlled permanent-magnet dc motor with a servo electrical driver directly driving an inertia load. The control goal is to make the inertia load track any specified smooth motion trajectory x_{1d} with a high precision. In the derivation of the model, the current dynamics is neglected in comparison with our interest frequency range due to the much faster electric response. The dynamics of the inertia load is:

$$M\ddot{q} = K_i u - B\dot{q} - f(q, \dot{q}, t) \tag{1}$$

where q and M represents the angular displacement and inertia load. K_i is the torque constant, u is the control input, B is the viscous friction coefficient, and f represents the composite un-modelled disturbances, such as nonlinear frictions, external disturbances and un-modelled dynamics. Rewrite (1) in a state-space form as follows:

$$\dot{x}_1 = x_2 \tag{2}$$

$$\dot{x}_2 = \theta_1 u - \theta_2 x_2 + d(x, t) \tag{3}$$

where $\boldsymbol{x} = [x_1, x_2]^{\mathrm{T}} = [q, \dot{q}]^{\mathrm{T}}$ represents the state vector of the position and velocity, parameter set $\theta = [\theta_1, \theta_2]^{\mathrm{T}}$ in which $\theta_1 = K_i/M$ and $\theta_2 = B/M$, and $d(x, t) = f(x, t)/M$ represents the lumped disturbance.

In general, the system is subjected to structured uncertainties due to large variations in system parameters M, K_i and B. In addition, $d(x, t)$ is clearly the system unmodelled unstructured uncertainty.

Extend disturbance observer: Let $\hat{\theta}$ and $\tilde{\theta}$ denote the estimate of θ and estimation error (i.e. $\tilde{\theta} = \hat{\theta} - \theta$). By defining $d_e(x, t) = -\tilde{\theta}_1 u + \tilde{\theta}_2 x_2 + d(x, t)$ as the equivalent effect of estimation error and disturbance, rewrite system (2)–(3) with estimate of θ_1 and θ_2, we have:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \hat{\theta}_1 u - \hat{\theta}_2 x_2 + d_e(x, t) \end{aligned} \tag{4}$$

The second order EDO for system (4) is expressed as follows:

$$\hat{d}_e = p_1 + l_1 x_2 \tag{5}$$

$$\dot{p}_1 = -l_1\left(\hat{\theta}_1 u - \hat{\theta}_2 x_2 + \hat{d}_e\right) + \hat{\dot{d}}_e \tag{6}$$

$$\hat{\dot{d}}_e = p_2 + l_2 x_2 \tag{7}$$

$$\dot{p}_2 = -l_2\left(\hat{\theta}_1 u - \hat{\theta}_2 x_2 + \hat{d}_e\right) \tag{8}$$

where $\hat{d}_e$ and $\hat{\dot{d}}_e$ are estimates of $d_e(t)$ and $\dot{d}_e(t)$ respectively, p_1 and p_2 are auxiliary variables, and l_1 and l_2 are design constants. Define estimation error as

$$\tilde{e} = \left[\tilde{d}_e \tilde{\dot{d}}_e\right]^{\mathrm{T}}, \quad \tilde{d}_e = d_e - \hat{d}_e \, \tilde{\dot{d}}_e = \dot{d}_e - \hat{\dot{d}}_e \tag{9}$$

where $\tilde{d}_e$, $\tilde{\dot{d}}_e$ are the estimate errors of $d_e(t)$ and $\dot{d}_e(t)$.

Assumption 1: The extent of the parametric uncertainties are known, the equivalent disturbance d_e is continuous and satisfies i.e.,

$$\theta \in \Omega_\theta \triangleq \{\theta : \theta_{\min} < \theta < \theta_{\max}\} \tag{10}$$

$$\left|\frac{\mathrm{d}^j d_e(t)}{\mathrm{d}t^j}\right| \le \mu \quad \text{for} \quad j = 1, 2 \tag{11}$$

where $\theta_{\min} = [\theta_{1\min}, \theta_{2\min}]^{\mathrm{T}}$, $\theta_{\max} = [\theta_{1\max}, \theta_{2\max}]^{\mathrm{T}}$, and μ is a positive number, but unnecessary known.

Theorem 1: If Assumption 1 holds, the EDO (5)–(8) for system (4) is stable and after a finite time T_s, its estimate error of equivalent disturbance can be bounded as γ, where γ is involved with μ.

Proof: The proof process is similar to [6]. □

Adaptive robust control law: In the view of Assumption 1, the following parameter adaptation law with projection is used to guarantee that the parameter estimates remain in the known bounded region all the time.

$$\dot{\hat{\theta}} = \mathrm{Proj}_{\hat{\theta}}(\boldsymbol{\Gamma}\varphi e) \tag{12}$$

where $\boldsymbol{\Gamma}$ is a positive definite diagonal matrix of adaptation rates and φ is a regressor for parameter adaptation (i.e. $\varphi = [u, -x_2]^{\mathrm{T}}$), e is the relevant tracking error of desired state (i.e. $e = z_2$).

$$\mathrm{Proj}_{\hat{\theta}}(\bullet_i) = \begin{cases} 0 & \text{if } \hat{\theta}_i = \hat{\theta}_{i\max} \text{ and } \bullet_i > 0 \\ 0 & \text{if } \hat{\theta}_i = \hat{\theta}_{i\min} \text{ and } \bullet_i < 0 \\ \bullet_i & \text{otherwise} \end{cases} \tag{13}$$

where $i = 1,2$, and $\bullet_i$ represents the ith component of the vector $\bullet$. Such a parameter adaptation law with projection can guarantee [7]:

$$\text{Term 1}: \hat{\theta} \in \Omega_{\hat{\theta}} \triangleq \left\{\hat{\theta} : \theta_{\min} \le \hat{\theta} \le \theta_{\max}\right\} \tag{14}$$

$$\text{Term 2}: \tilde{\theta}^{\mathrm{T}}\left[\boldsymbol{\Gamma}^{-1}\mathrm{Proj}_{\hat{\theta}}(\boldsymbol{\Gamma}\tau) - \tau\right] \le 0 \tag{15}$$

The ARC law can be expressed as:

$$\begin{aligned} u &= (u_a + u_s)/\hat{\theta}_1 \\ u_a &= \dot{\alpha} + \hat{\theta}_2 x_2 \\ u_s &= u_{s1} + u_{s2}, u_{s1} = -k_2 z_2 \end{aligned} \tag{16}$$

where u_a is adaptive control part, u_s is determined robust control part, $\hat{\theta}_1$, $\hat{\theta}_2$ are updated by (12), $\alpha = \dot{x}_{1d} - k_1 z_1$ is a virtual input, $z_2 = x_2 - \alpha$, k_1, k_2 are the feedback gains. Noting Assumption 1 and Term 1 in (14), if there exist u_{s2} satisfy the following two conditions [7]:

$$z_2 u_{s2} \le 0 \tag{17}$$

$$z_2\left[u_{s2} - \theta^{\mathrm{T}}\varphi - d(x, t)\right] \le \varepsilon \tag{18}$$

where ε is a positive tunable parameter.

An example is given as follows (refer to [7]): Let g be any smooth function satisfying

$$g \ge ||\theta_M||||\varphi|| + \delta_d \tag{19}$$

where $\theta_M = \theta_{max} - \theta_{min}$ and δ_d is the upper bound of $d(x, t)$ (ARC demands δ_d is known). Then, one smooth example of u_{s2} satisfying (17)–(18) is given by

$$u_{s2} = -g^2 z_2/(4\varepsilon) \quad (20)$$

Adaptive robust control with extended disturbance observer: To dominate the disturbance from various uncertainties without high-gain feedback, we propose an ARC with EDO control scheme as following:

$$\begin{aligned} u &= (u_a + u_s)/\hat{\theta}_1 \\ u_a &= \dot{\alpha}_1 + \hat{\theta}_2 x_2 - \hat{d}_e \\ u_s &= u_{s1} + u_{s2}, u_{s1} = -k_2 z_2 \end{aligned} \quad (21)$$

where d_e is observed by (5)–(8) based on system (4).

By applying the resulting control law (21) into (4), we have

$$\dot{z}_2 = -k_2 z_2 + u_{s2} + \tilde{d}_e \quad (22)$$

From Theorem 1, there exist a positive constant γ and a finite time $T_s > 0$:

$$|\tilde{d}_e| \le \gamma, \quad t > T_s \quad (23)$$

Due to the exact bound of γ is unknown, the level of accuracy such as (18) cannot be prespecified and thus we can choose the robust control function u_{s2} satisfying a more relaxed condition as

$$z_2[u_{s2} + \tilde{d}_e] \le \varepsilon\gamma^2 \quad (24)$$

As an example, we can choose g in (19) as $g = \gamma$, we get u_{s2} as

$$u_{s2} = -z_2/(4\varepsilon) \quad (25)$$

Theorem 2: With adaptation law (12), u_{s2} in (25) and $\hat{d}_e$ in (5)–(8), the control law in (21) guarantees that all signals are bounded. Furthermore, after a finite time T_s, the positive definite function V_s is bounded by

$$V_s(t) \le \exp(-\lambda T)V_s(T_s) + \frac{\varepsilon\gamma^2}{\lambda}[1 - \exp(-\lambda T)], \quad \forall t \ge T_s \quad (26)$$

where $T = t - T_s$.

Proof: For $t < T_s$, from (22) and (25), we have

$$\dot{z}_2 = -\left(k_2 + \frac{1}{4\varepsilon}\right)z_2 + \tilde{d}_e \quad (27)$$

Noting Theorem 1, the equivalent disturbance estimation error is always bounded. Thus, z_2 is bounded before T_s. For $t > T_s$, (23) and (25) make (24) be true. Thus, the derivative of V_s with (21) satisfies

$$\dot{V}_s + \lambda V_s \le \varepsilon\gamma^2 \quad (28)$$

Integrating (28) from T_s to t,

$$V_s(t)\,e^{\lambda t} - V_s(T_s)\,e^{\lambda T_s} \le \frac{\varepsilon\gamma^2}{\lambda}(e^{\lambda t} - e^{\lambda T_s}) \quad (29)$$

Moreover, it is easy to ensure that the control input u is bounded. □

Comparative numerical results: The parameters for simulation are showed in Table 1.

Table 1: Parameters setting

k_1, k_2	240, 300	Γ	diag{1500, 1000}
l_1, l_2	1000, 300	θ_{min}	$[250, 1]^T$
θ_{max}	$[700, 900]^T$	$\hat{\theta}(0)$	$[600, 6]^T$

The controllers are tested for motion trajectory x_{1d} as (30).

$$x_{1d} = \frac{\pi}{18}[1 - \cos(3.14t)][1 - \exp(-t)] \quad (30)$$

We set disturbance as $d(t) = 0.2\sin(200/\pi t)$. In Fig. 1, ARC-EDO has better performance than ARC in terms of both transient performance and final tracking errors. Combination with equivalent disturbance compensation, ARC can deal with more unstructured uncertainty. The tracking error of ARCEDO is reduced to almost about 1.5×10^{-6} rad, while ARC's error is about 6×10^{-6} rad. Parameters adaption and disturbance estimation show that ARCEDO has a learning process to attenuate structured uncertainty. Besides, it can estimate the equivalent disturbance of remained structured and unstructured uncertainty.

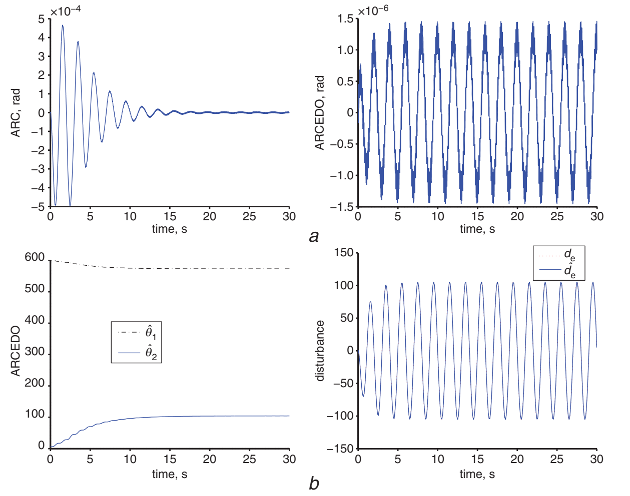

Fig. 1 *Performances of two controllers*

a Tracking errors of two controllers.
b Parameter adaption and disturbance estimation of designed method.

Conclusion: In this Letter, an ARC with EDO was proposed for a high-accuracy motion system driven by a dc motor. The stabilities and estimation accuracy is analysed via a Lyapunov method. Numerical results show that excellent tracking accuracy can be achieved by the proposed ARC-EDO controller. Besides, the proposed combination, in which δ_d can be unknown, can allow more relax demand of unstructured uncertainties part such as d than ARC. For further study, saturation problem will be considered for ARC-EDO, and the influence between parameters adaptation and equivalent disturbance estimate on θ estimation can also be considered.

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One or more of the Figures in this Letter are available in colour online.

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