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# A new star pattern identification technique using an improved triangle algorithm 

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#### Abstract

Triangle identification algorithm has been widely applied in star pattern identification of star sensor. However, its lower characteristic dimensions lead to matching redundancy and identification errors. To improve the identification accuracy, the identification process of the triangle identification algorithm is improved. First, the method of "quasi uniform distribution" of space solid angles is proposed. That is, based on the size of space solid angle a certain zone at the celestial equator corresponds to, the whole celestial sphere is partitioned into a number of sub-blocks. Then, guide stars are selected uniformly according to sub-blocks, obtaining a guide star catalogue with fewer stars, better distribution uniformity and higher completeness. Finally, an approach is proposed to further optimize primarily selected observation triangle groups for the second time for matching identification. The constraint relationship between the sides and included angles of the observation triangle is converted onto the image plane. The simulation experiment result shows that the star pattern identification technique based on the improved triangle algorithm has strong anti-interference performance against star point position noises, magnitude (brightness) noises and "missing stars" and it is superior to traditional algorithms in identification rate.


## Keywords

Star sensor, star pattern identification, triangle map, space solid angle, observation triangle

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## Introduction

Star sensor is a kind of space attitude measurement instrument with high accuracy and high reliability. It determines the spacecraft attitude through the observation stars, which is widely used in the aerospace field. ${ }^{1-2}$ Star pattern identification is an important step for star sensors to complete the spacecraft attitude measurement, so whether this algorithm is advantageous or not has a direct impact on the performance of star sensor. Star pattern identification refers that stars in the current viewing field of star sensor are matched with reference stars in the guide star catalogue to complete the task of identifying stars in the viewing field. It plays an essential role in accurately determining the space attitude and position of the spacecraft. Star angular distance and magnitude are viewed as essential characteristics information of star pattern. As the angular distance information has higher accuracy and rotational invariance, it plays an important role in the star pattern identification.

The star pattern identification algorithms in China and abroad roughly fall into two kinds: sub-graph isomorphism algorithm and pattern identification algorithm. The sub-graph isomorphism algorithm refers that, the observation star map is viewed as the sub-graph of all-sky star map, taking the angular distances between stars as the sides and the star as the vertex. In fact, most of traditional star pattern identification algorithms belong to this category, such as

[^0]polygonal angular distance method, ${ }^{3}$ matched group method ${ }^{4}$ and triangle algorithm. ${ }^{5,6}$ The pattern identification algorithm is different-each star is endowed with a unique feature, that is, "star pattern". Typically, this pattern is constituted by geometric distribution characteristics of other stars in a certain neighborhood. This kind of algorithm is mostly represented by grid algorithm, ${ }^{7}$ genetic algorithm, ${ }^{8}$ and singular value decomposition (SVD) algorithm. ${ }^{9}$

Currently, the triangle star pattern identification algorithm has been widely used in practical engineering, such as Denmark's spaceborne star sensor of Oereted small satellite and America's DIGISTAR mini star tracker. ${ }^{10}$ It has many variants, such as sixfeature algorithm proposed by Scholl et al. ${ }^{11}$ and pyramid algorithm proposed by Mortari et al. ${ }^{12}$ The traditional triangle algorithm is simple in structure and easy in use. The basic principle is that all the guide triangles constituted by guide stars are stored for retrieval matching, taking the angular distances between three stars as the characteristic parameters. If some guide triangle can match with the observation triangle, the identification is achieved successfully. But as guide triangles are too large in number, this becomes a major problem challenging the triangle algorithm and causes such serious problems as lower identification success rate, larger guide star catalogue storage space for requirement, and longer identification time.

To remedy these problems, the triangle algorithm is improved in this paper. First, a method is proposed to partition the star catalogue. By properly selecting guide stars, the capacity of guide star catalogue is reduced, further reducing the matching time. At the same time, the observation triangles are optimized and selected to further improve the matching efficiency. In the process of identification, the redundant matching is reduced through taking the star angular distances as major identification features. In addition, an improved triangle star identification algorithm is proposed based on the study on star pair creation and storage. According to the simulation results, this algorithm still has higher identification rates in a shorter identification time under these disadvantageous conditions: louder noises at the star position and louder magnitude noises and "star missing".

## Star catalogue partition

Star catalogue partition is significant in improving star pattern identification and star tracking efficiency. The retrieval speed of guide star has a direct impact on the speed, reliability, and robustness of star pattern identification. In order to improve the retrieval speed, it is necessary to partition the star catalogue properly. The star catalogue partitioning approaches mainly include the declination belt method, ${ }^{13}$ circular cone method, ${ }^{10}$ sphere rectangle method ${ }^{14}$ and inscribed cubes method. ${ }^{15}$ Relatively speaking, the inscribed cubes method can make the partition of star catalogue
have better uniformity, but it actually partitions the surface of inscribed cubes uniformly with no overlap. After the surface is being partitioned, the space solid angle ${ }^{16,17}$ corresponding to each sub-block is different; especially when the optical axis of star sensor respectively points to the vicinity of celestial pole and the nearby of eight apexes of inscribed cube, the range of variation in space solid angle reaches almost 5.2 times in difference. Therefore, this method cannot be used to partition the whole celestial sphere uniformly. On the whole, the above-mentioned methods for partitioning the celestial sphere have one basic principle: first, the surface of celestial sphere is expanded into a rectangular plane, and then the right ascension and declination of it are partitioned according to a fixed value, or the surface of celestial sphere is first projected onto a plane, and then this plane is partitioned without overlapping. At work, the optical axis pointing of star sensor rotates in an unfixed way, resulting in instability in sizes of projection of the celestial sphere onto a plane. Moreover, the coordinates of right ascension and declination are not uniform in space by nature. Obviously, if the traditional geometrical method is used to partition the celestial sphere, the space solid angles of sub-blocks in the guide star catalogue will be distributed unevenly. However, in the practical work, the space solid angles of the objects captured by the space camera of star sensor are fixed, so based on this, the method of "quasi uniform distribution" of space solid angles is proposed to partition the celestial sphere. The partition process is as follow.

The whole celestial sphere is sequentially partitioned into a number of sub-blocks according to the sizes of space solid angles which the zones with equal right ascension and declination ( $\alpha$ for right ascension, $\beta$ for declination) at the equator correspond to, hence obtaining a more uniformly distributed guide star catalogue (see Figure 1). The surface of celestial


Figure I. Diagrammatic sketch of partitioning the celestial sphere according to the space solid angle.
sphere is equally partitioned into $n$ parts from the equator to the North Pole or South Pole according to the value of $\beta$. The zone from Part 1 to $\operatorname{Part} n-1$ is called "spherical ring", and the $n$th part is called "spherical crown". The spherical ring from Part 1 to Part $n-1$ is partitioned in sequence along the right ascension direction according to the value of $\alpha_{n}$. The variation rule is as follow

$$
\begin{equation*}
\alpha_{n}=\beta / \cos (n \cdot \beta) \tag{1}
\end{equation*}
$$

After the partition, the space solid angle in each spherical ring is

$$
\begin{equation*}
\Omega_{n}=\int_{\alpha_{n}} \mathrm{~d} \alpha_{n} \int_{\beta} \sin \beta \mathrm{d} \beta \tag{2}
\end{equation*}
$$

The spherical crowns at the South and North Poles are equally partitioned as a whole according to the specific values of $(\alpha, \beta)$.

Take the space solid angle $15^{\circ} \times 15^{\circ}$ at the equator corresponds to as an example. The whole celestial sphere is partitioned into 202 sub-blocks in sequence. Notably, the sphere at the crown of the North Pole or South Pole is equally partitioned into four parts. Table 1 shows the sizes of space solid angles which standard sub-blocks between the equator and the North Pole correspond to.

It can be seen from Table 1 that the difference between the space solid angle of the maximum subblock and that of the minimal sub-block is $22.86 \%$. Obviously, in terms of degree of uniformity, this method is far better than the inscribed cubes method, achieving the "quasi uniformly distributed" partition of the whole celestial sphere and laying foundation for verifying the rapid generation of simulated star patterns and selecting guide stars in the process of star pattern identification.

## Guide stars selection

Guide star selection has a direct influence on the performance and reliability of star pattern recognition and attitude determination, which plays an important role in the modern star sensor design. The original

Table I. Sizes of space solid angles of standard sub-blocks between the equator and the North Pole.

| Space solid angle | Steradian (sr) |
| :--- | :--- |
| $\Omega_{1}$ | 0.0678 |
| $\Omega_{2}$ | 0.0659 |
| $\Omega_{3}$ | 0.062 |
| $\Omega_{4}$ | 0.0587 |
| $\Omega_{5}$ | 0.0523 |
| $\Omega_{6}$ | 0.0535 |

star catalogue contains a large number of stars, but not all the stars can be used for celestial navigation and satellite attitude determination. The traditional method for selecting the guide star is visual magnitude threshold filtering method. ${ }^{18}$ That is, the star whose brightness is higher than or equal to that of a certain star in the original star catalogue is taken as the guide star. As the stars are unevenly distributed on the celestial sphere, the completeness of star catalogue will be reduced (A large number of holes will appear) if the magnitude threshold is set too low. This will lead to reduction in the success rate of star pattern identification and accuracy of attitude determination. However, if the magnitude threshold increases, the number of stars will increase exponentially. As a result, the capacity of guide star catalogue is too large, causing the redundancy. Accordingly, the accuracy of star pattern identification will be reduced.

Considering the defects of the visual magnitude threshold filtering method, the method of "quasi uniform distribution" of space solid angles is adopted in this paper. According to the sizes of space solid angles $4^{\circ} \times 4^{\circ}$ at the equator corresponds to, the whole celestial sphere is partitioned into 2664 sub-blocks in sequence. At the spherical crown of the South Pole and North Pole, the $2^{\circ}$-latitude band is taken as the sub-block as a whole. The original star catalogue is taken from Smithsonian Astrophysical Observatory (SAO). A total of 5103 stars with the brightness higher than (or equal to) 6 Mv are selected as guide stars. Table 2 shows the distribution of guide stars in the sub-blocks.

It can be seen from Table 2 that there are 466 subblocks in which there is no guide star. These 466 subblocks are taken as the center. Then let us count the number of guide stars contained in its neighboring eight sub-blocks. It can be found that there is no "hole phenomenon" in the sub-block of $3 \times 3$ : at worst, 4 zones containing 3 guide stars and 17 zones containing 4 guide stars. It means that, at worst, at least three guide stars can be involved in star pattern identification. Obviously, this partition method enables the guide star catalogue to have a higher level of completeness.

Table 2. Distribution of guide stars from SAO in the sub-blocks.

| Guide stars | Sub-blocks | Guide stars | Sub-blocks |
| :--- | :--- | :--- | :--- |
| 0 | 466 | 7 | 21 |
| 1 | 820 | 8 | 11 |
| 2 | 604 | 9 | 8 |
| 3 | 395 | 10 | 1 |
| 4 | 203 | 11 | 3 |
| 5 | 95 | 12 | 2 |
| 6 | 34 | 13 | 1 |

On this basis, the guide stars with magnitude of less than 5.5 Mv in each sub-block are left. If the brightest star has the magnitude of greater than 5.5 Mv , it will be left and the rest are ruled out. Although the magnitude threshold is set 5.5 Mv , the guide star catalogue still has the completeness equal to 6 Mv . The total number of guide stars is reduced from 5103 to 3406 (by $33.25 \%$ ). As the storage space for guide stars is reduced, the rate of star pattern identification increases. If the guide star catalogue is partitioned using the inscribed cubes method, the number of selected guide stars is $3360 .{ }^{15}$ In total, 412 guide stars are ruled out as they have the magnitude of less than 5.5 Mv . But in this paper, the number of selected guide stars is 3406 , and all the guide stars with the magnitude of less than 5.5 Mv are included. As the stars with high brightness can provide more reliable information, they are preferably selected for star pattern recognition. Therefore, in this paper, as 3406 guide stars are selected, the success rate of star pattern identification is higher than the method described in Zhang et al. ${ }^{15}$

In order to verify the performance of the generated guide star catalogue, the method of "quasi uniform distribution" of space solid angles is used. According to the sizes of space solid angles $3^{\circ} \times 3^{\circ}$ at the equator corresponds to, the whole celestial sphere is partitioned into 4694 sub-blocks in sequence. The spherical crown of the South Pole and North Pole are taken as the sub-block as a whole. The medial axis pointing of each sub-block is scanned in sequence. Table 3 shows the number of viewing fields where there are five or less than five guide stars in the viewing field of $12^{\circ} \times 12^{\circ}$.

To ensure the high success rate of star pattern identification, there must be at least five observation stars appearing in the viewing field of star sensor. According to Table 3, in the viewing field of $12^{\circ} \times 12^{\circ}$, after the method of "quasi uniform distribution" of space solid angles and the inscribed cube method are used to partition the guide catalogue, respectively, the viewing fields in which five or less

Table 3. Comparison of two kinds of guide star catalogues in performance.

| Performance of the guide star catalogue obtained using the inscribed cubes method |  | Performance of the guide star catalogue obtained using the space solid angle method |  |
| :---: | :---: | :---: | :---: |
| Guide stars appearing in the field of view | Field of view | Guide stars appearing in the field of view | Field of view |
| 2 | 4 | 2 | 1 |
| 3 | 18 | 3 | 6 |
| 4 | 52 | 4 | 39 |
| 5 | 121 | 5 | 108 |

than five stars appear are different in number; specifically, their emergence probabilities are $3.3 \%$ and $4.2 \%$, respectively. This shows that, in the case that guide stars are basically equal in number, the method of "quasi uniform distribution" of space solid angles is superior to the inscribed cube method in terms of the partition of star catalogue because it can make the guide star catalogue have better distribution uniformity.

The selected guide stars are scanned. If the angular distance between two guide stars is less than $d$, the angular distance and the serial numbers of two stars shall be recorded. " $d$ " is the diagonal distance of viewing fields for star sensor. For example, for the viewing field of $12^{\circ} \times 12^{\circ}, d=12 \sqrt{2}^{\circ}$. After the generated diagonal distances are arranged in an ascending order, they are stored with an interval of $0.05^{\circ}$ as guide star characteristic catalogue.

In fact, the selection method developed in this paper is not sensitive to the original star catalogue. That is to say, almost the same results can be obtained when different original star catalogues are utilized. For example, we tried to adopt SKY2000 as the original star catalogue to extract 5066 stars whose brightness is no less than 6 Mv as the guide stars. After the celestial sphere is divided into 2664 subblocks in sequence according to the space solid angles, the distribution of guide stars is shown in Table 4. It can be found that the total number of guide stars is reduced from 5066 to 2916 using our proposed approach. From Table 4, we can see that these data are basically consistent with those in Table 2.

## Optimization of observation triangles selection

Optimizing the selection of observation triangles can improve the success rates of the triangle algorithmbased star pattern identification, and simplify the computation in the matching process, further improving the performance of real-time output attitude of star sensor. Quine ${ }^{19}$ and Kruijff et al. ${ }^{20}$ put forward proposals on how to select proper observation stars to

Table 4. Distribution of guide stars from SKY2000 in the sub-blocks.

| Guide stars | Sub-blocks | Guide stars | Sub-blocks |
| :--- | :--- | :--- | :--- |
| 0 | 497 | 7 | 12 |
| 1 | 823 | 8 | 11 |
| 2 | 593 | 9 | 2 |
| 3 | 393 | 10 | 4 |
| 4 | 193 | 11 | 5 |
| 5 | 97 | 12 | 4 |
| 6 | 48 | 13 | 0 |

form a triangle for star pattern identification mainly based on magnitude information and locations of observation stars in the viewing field. According to the proposal in Zhang et al., ${ }^{15}$ the brightest $N_{B}$ observation stars in the viewing field shall be preferably selected; after they are selected, they can form observation triangles randomly for identification. But in practice, the selected $N_{B}$ observation stars may be double stars or three stars may form a straight line. This will prolong the matching time, resulting in the failure in star pattern identification or the output of attitude with larger error. After the advantages and disadvantages of existing approaches for observation triangle selection are carefully analyzed, a new method is proposed: the primarily selected observation triangle group is further optimized for the second time and then matched for identification.

## Constraints on observation triangles

In the actual star patter identification process, the calculated amount, which the star pattern matching requires, is vastly greater than that required in selecting an optimal observation triangle in the viewing field. Moreover, in some cases, even if the star pattern identification is successful, the normal attitude cannot be output as the observation stars in the viewing field may be located almost in a straight line. At this time, it is necessary to re-select observation triangles for matching and identification. Therefore, the optimized selection of observation triangles for star pattern identification can dramatically accelerate the star sensor attitude output, which is significant in improving the performance of star sensor and information on realtime output attitude.

To perform a successful matching and attitude calculation, the observation triangles have to satisfy some conditions. To improve computational efficiency, the authors of this paper propose to further optimize the primarily selected observation triangles and then match with star pattern identification algorithm. The optimization of the observation triangle is subject to two constraints: right matching and highaccuracy attitude calculation. In order to improve the on-orbit calculation efficiency, it is necessary to transform them through the in-depth analysis into specific direct constraints of image plane which is easy to calculate.

Double stars have great influence on matching identification results. Double stars refer to two stars that seem close to each other in the direction of sight (The actual distance between them may be considerably long); moreover, their star points on the image plane for star sensor cannot be distinguished from each other. ${ }^{21}$ The common star pattern identification algorithms are incapable of correctly identifying double stars. It is reported in $\mathrm{Zhang}^{22}$ that, when the angular distance between two stars is less than $0.047^{\circ}$, they shall be treated as double stars.

Therefore, the side length of the selected observation triangle shall be greater than the double stars threshold (4 pixels). At the same time, in order to improve the localization accuracy at the star point, the offfocus technique is generally adopted. Let us take the image point of $5 \times 5$ pixels as an example. To completely separate the centroids of two observation stars, the side length shall be at least greater than 10 pixels. The above factor is taken as "Constraint 1" of optimizing the selection of observation triangle for the second time, which determines the threshold for minimum side length ( $d_{\text {min }}$ ) of the observation triangle.

The relative positions of three stars that constitute the observation triangle are critical to the final output attitude of star sensor. For example, if these three observation stars are positioned almost in a straight line, it will lead to the failure of star sensor in attitude determination. According to the reference vector array $\left(\boldsymbol{V}_{3 \times \mathrm{k}}=\left(\boldsymbol{V}_{1} \boldsymbol{V}_{2} \ldots \boldsymbol{V}_{\mathrm{k}}\right)\right)$, during the calculation process of matrix inversion, $\boldsymbol{V}^{\mathrm{T}} \boldsymbol{V}$ closely approximates to a singular matrix when the positions of three stars in an observation triangle are highly correlated.

The constraints between the sides of the observation triangle and its included angles are converted to the image plane for analysis, as is shown in Figure 2. $O$ is the origin of the image space coordinate system for star sensor; Star 1, Star 2 and Star 3 are the projections of stars on the image plane; $f$ is the focal length of star sensor; $h$ is the vertical line from Star 2 to the bottom side $c ; \theta$ is the smallest included angle of the triangle, that is, the included angle between Side $a$ and Side $c ; \beta$ is the included angle Side $h$ corresponds to.

From the triangular relationship in Figure 2, this formula can be obtained:

$$
\begin{equation*}
h=a \cdot \sin \theta \tag{3}
\end{equation*}
$$

In order to prevent double stars from disturbing star pattern identification while reducing the position


Figure 2. Diagrammatic sketch of image space coordinate system for star sensor.
correlation of three observation stars on the image plane, that is, allow three observation stars not to be in a straight line, $h$ shall be at least greater than double stars threshold, analyzed from the QUEST ${ }^{23}$ attitude determination principle

$$
\begin{equation*}
h=f \cdot \tan \beta>\text { Double Star Threshold } \tag{4}
\end{equation*}
$$

Equations (3) and (4) are taken as "Constraint 2" of optimizing the selection of observation triangle for the second time, which determines the threshold for minimum included angle $\left(\theta_{\text {min }}\right)$ of the observation triangle.

In this paper, the authors propose to optimize the primarily selected observation triangle for the second time. In the primary selection, the brightest $N$ ( $N=5 \sim 6$ ) stars in the viewing field are first selected to form the observation triangles randomly, and these formed observation triangles constitute a collection of observation triangle to be selected. The observation triangles are optimized for the second time based on the following constraints:

> Constraint 1: On the image plane, the shortest side of observation triangle shall be greater than $d_{\text {min }}$ pixel. Constraint 2 : On the image plane, the smallest included angle of the observation triangle shall be greater than $\theta_{\text {min }}$.

## Optimization approach of the observation triangle selection

Figure 3 shows the workflow of the observation triangle optimization, where $m$ is the number of observation stars in the viewing field, $d_{\text {min }}$ is the side length threshold of observation triangle, $\theta_{\text {min }}$ is the included angle threshold of observation triangle, $N$ is the serial number of observation star, the initial value of which is 1 , and $n$ is the serial number of the fourth brightest star except the selected three ones.

Step 1: Arrange the observation stars in the viewing field of star sensor in an ascending order of magnitudes (The lower the magnitude is, the higher will be the degree of brightness).
Step 2: Using the $N$ th bright star ( $N$ is initialized to 1 ) as the baseline, select the $N+1$ th and $N+2$ th bright stars in sequence to form the triangle, and then assign the value of $N+3$ to $n$.
Step 3: Calculate the length of three sides of the formed triangle, and check whether the shortest side is greater than $d_{\text {min }}$. If it is, follow Step 4. If not, output "There are double stars in the observation triangle", and then jump to Step 7.
Step 4: Check whether the smallest included angle in the triangle is greater than $\theta_{\text {min }}$. If it is, record these three bright stars into the observation triangle candidate group, remove the dimmest one
from the selected three stars, introduce the $n$th star to form a new triangle, and then follow Step 5. If not, output "Three observation stars are positioned almost in a straight line", and then jump to Step 10.
Step 5: Check whether the $n$th bright star is the last bright star in the viewing field. If it is, assign the value of $N+1$ to $N$, and then follow Step 6 ; otherwise, assign the value of $n+1$ to $n$, and then jump to Step 3.
Step 6: Check whether the $N$ th bright star is the penultimate one in the viewing field. If it is, the selection of observation triangle candidate group is completed; otherwise, jump to Step 2.
Step 7: Check whether the $n$th bright star is the last one in the viewing field. If it is, output "The brightest star is a star in a certain nebula", assign the value of $N+1$ to $N$, and then follow Step 8; otherwise, jump to Step 9.
Step 8: Check whether the Nth bright star is the penultimate one in the viewing field. If it is, the star selection fails; otherwise, jump to Step 2.
Step 9: Remove the second brightest star with the shortest sides, select the $n$th bright star to form the triangle, assign the value of $n+1$ to $n$, and then follow Step 3;
Step 10: Check whether the $n$th bright star is the last bright one in the viewing field. If it is, assign the value of $N+1$ to $N$ and jump to Step 8; otherwise, follow Step 11.
Step 11: Remove the dimmest one from the selected three stars, introduce the $n$th star to form a new triangle, assign the value of $n+1$ to $n$, and then jump to Step 3;

## Simulation experiment and results analysis

The angular distance method ${ }^{15}$ is used for matching identification. When the QUEST attitude determination algorithm is employed, in the case of $d_{\min }=10$ and $\theta_{\text {min }}=15^{\circ}$, the errors of the reference vector array $\boldsymbol{V}_{3 \times \mathrm{k}}=\left(\boldsymbol{V}_{1} \boldsymbol{V}_{2} \ldots \boldsymbol{V}_{\mathrm{k}}\right)$ can be reduced in the matrix inversion calculation process, satisfying the practical requirements in engineering. To evaluate the performance of the improved triangle algorithm, the simulation experiments are performed to investigate the impacts of star position noises, magnitudes (brightness) noises and the "missing stars" phenomenon on the identification. These three kinds of noises or interference are major factors of influencing star pattern identification in the generation process of star pattern.

## Impact of star point position noises on identification

Star point position noises reflect the localization error of star point centroid, which are caused by many


Figure 3. Flowchart of the method for optimizing observation triangles.
factors such as image noises, optical system aberration, sampling quantization truncation errors and the centroid localization algorithm itself. In order to investigate how the identification algorithm causes
the star point centroid localization errors, louder position noises are often introduced.

In the experiments, for the star pattern images generated through the simulation, the noises (with the
mean of 0 and the standard deviation $\sigma$ ranging from 0 to 2 pixels) that follow the Gaussian distribution are added at the star point positions. After 1000 star patterns are randomly selected from the whole celestial sphere, the position noises are added in this way. The identification results are plotted in Figure 4.

From Figure 4, it can be seen that, under the condition of lower noise ( $\sigma<0.5$ ), the identification rate based on the improved triangle algorithm is approximately $100 \%$, much higher than Liebe triangle algorithm (the rate of $94.6 \%$ ). For the Liebe triangle algorithm in the case of $\sigma=2$ pixes, the identification rate rapidly drops to $70 \%$ or so. Comparatively, in this case, the identification rate based on the algorithm in Zhang et al. ${ }^{15}$ drops to below $97 \%$, but the identification rate based on improved triangle algorithm is still high up to above $98 \%$.

## Impact of magnitude noises on identification

The magnitude noises reflect the accuracy of photoelectric detector in the measurement of star brightness, which are influenced by such factors as star spectral properties, star brightness variation characteristics, image sensor, and optical system. As the brightness information, more or less, is involved in the optimization of the observation triangle selection, it is necessary to evaluate the impact of magnitude noises on identification.

In the experimental process of star pattern simulation, the noises (with the mean 0 and the standard deviation $\sigma$ ranging from 0 to 1 Mv ) that follow the

Gaussian distribution are added to the magnitude, which is reflected in the star pattern as the star point image grayscale amplitude noises. A total of 1000 star patterns are randomly selected from the whole celestial sphere for identification statistics. The identification results are plotted in Figure 5.

From Figure 5, it can be seen that the improved triangle algorithm has strong robustness on magnitude noises. When the magnitude noise has a standard deviation of 1 Mv , the identification rate based on the algorithm in Zhang et al. ${ }^{15}$ drops to $93 \%$ or so; comparatively, the identification rate based on the improved triangle algorithm is still high up to above $98 \%$. When the standard deviation of the magnitude noises is below 0.5 Mv , the identification rate based on it approaches $100 \%$.

## Impact of "missing stars" on identification

The term "missing stars" refers to the stars that should have been captured but fail to appear in the viewing field for some reason. The impact of "missing stars" on the identification rate is studied through removing a certain number of observation stars (1-2) randomly from the observation star pattern. The magnitudes of the deleted observation stars range from 3 Mv to 6 Mv , and the standard deviation of magnitude noises is 0.2 Mv . The impact of "missing stars" on identification is plotted in Figure 6.

From Figure 6 it can be seen that the "missing stars" have little impact on identification accuracy.


Figure 4. Impact of star point position noises on identification.


Figure 5. Impact of magnitude noises on identification.


Figure 6. Impact of missing stars on identification.

In the case of two missing stars ( 3 Mv ), the identification rate based on the improved triangle algorithm improves by nearly $1 \%$, compared to the algorithm in Zhang et al. ${ }^{15}$

## Conclusions

Star pattern identification has become a key topic in the software research as it has autonomous celestial
navigation functions. In this paper, in order to solve the practical problems in engineering application, the method of "quasi uniform distribution" of space solid angles is proposed for the partition of celestial sphere based on the analysis and improvement of traditional triangle identification algorithm. This study lays a foundation for verifying the rapid generation of simulated star patterns in the process of star pattern identification and further optimizing the selection of guide stars. In addition, two constraints on the selection of observation triangles are established. The observation triangles optimization approach is proposed to further improve matching efficiency. On this basis, an improved triangle-based star pattern identification algorithm is presented. Under the conditions of louder star point position noises, magnitude noises, and "missing stars", it is superior to the algorithms in Zhang et al. ${ }^{15}$ in the identification rate.

## Conflict of interest

None declared.

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