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Tunneling induced dark states and the controllable resonance fluorescence spectrum in quantum dot molecules

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Abstract

Optical spectroscopy, a powerful tool for probing and manipulating quantum dots (QDs), has been used to investigate the resonance fluorescence spectrum from linear triple quantum dot molecules controlled by tunneling, using atomic physics methods. Interesting features such as quenching and narrowing of the fluorescence are observed. In such molecules the tunneling between the quantum dots can also induce a dark state. The results are explained by the transition properties of the dressed states generated by the coupling of the laser and the tunneling. Unlike the atomic system, in such quantum dot molecules quantum coherence can be induced using tunneling, requiring no coupling lasers, which will allow tunneling controllable quantum dot molecules to be applied to quantum optics and photonics.

Keywords: dark state, resonance fluorescence, quantum dot molecules, tunneling

(Some figures may appear in colour only in the online journal)

1. Introduction

Over the past few decades, quantum coherence has been shown to be essential for many applications of multilevel atomic systems driven by laser fields and has resulted in a lot of interesting phenomena. One example is dark resonance or coherent population trapping [1, 2]. It is a basis for effects such as electromagnetically induced transparency (EIT) [3, 4], stimulated Raman adiabatic passage (STIRAP) [5, 6], subluminal and superluminal light propagation [7, 8], resonant enhancement of optical nonlinearity [9, 10], spontaneous emission control [11, 12] and lasing without inversion (LWI) [13, 14].

The above investigations are based on atomic systems using coherent control lasers. In semiconductor quantum dots (QDs), confined electrons and holes exhibit atom-like properties, encouraging us to extend the above studies to a solid system. QDs coherently driven by strong electromagnetic fields have been used to investigate quantum coherence phenomena such as Autler-Townes splitting (ATS) and Mollow triplets [15, 16], EIT [17], and resonance fluorescence [18, 19]. Moreover, two or more quantum dots coupled by tunneling can form quantum dot molecules (QDMs). In such molecules, the tunneling of electrons or holes can be controlled by an external electric field and create a multilevel structure of excitonic states. Both vertical [20] and lateral [21] double quantum dots (DQDs) have been realized experimentally. Many studies have been carried out on such DQDs, such as optical spectroscopy [22], excitonic entanglement [23], single photon and spin storage [24], and coherent population trapping (CPT) [25]. On the theoretical side, EIT and slow light [26, 27], entanglement [28-31], optical bistability [32, 33], coherent population transfer [34], and narrowing of the fluorescence spectrum [35] are also investigated. Moreover, triple quantum dots (TQDs), which have been achieved in much experimental progress [36–38], allow one to study new phenomena not present in a single or double QD.



Figure 1. Schematic of the level configuration of a TQD system. (*a*) The band structure without a gate voltage. (*b*) The band structure with a gate voltage. (*c*) The level configuration.

Most studies of the resonance fluorescence spectrum from a multilevel atomic system is controlled by coupling lasers [39, 40], and the effective control of the fluorescence spectrum plays a critical role in bipartite entanglement [41] and accurate spectral measurement [42]. In contrast with previous studies, in the present paper we investigate the resonance fluorescence spectrum from TQDs using tunneling instead of coupling lasers. The fluorescence spectrum can develop quenching, which results from the dark states induced by the tunneling between the dots. And the fluorescence spectrum can also develop narrowing of the central peak due to the slow decay rate of the dressed state generated by the tunneling and the laser field. The advantage of the QDMs proposed here is that we can use tunneling to induce quantum coherence, requiring no coupling lasers.

The rest of this paper is organized as follows. In section 2, we introduce the model and the basic equations. In section 3 we describe the numerical results and explain the corresponding features. Our conclusions are presented in section 4.

2. Triple quantum dot system

We show the schematic of the band structure and level configuration of a TQD system in figure 1. At nanoscale interdot separation, the hole states are localized in the QDs and the electron states are rather delocalized. From figure 1(a), in the absence of a gate voltage, the conduction-band electron levels

are out of resonance and the electron tunneling between the QDs is very weak. In contrast, in the presence of a gate voltage, the conduction-band electron levels come close to resonance and the electron tunneling between the QDs is greatly enhanced, as shown in figure 1(b). In such systems, the tunneling can be controlled by placing a gate electrode between the neighboring dots. And in the latter case hole tunneling can be neglected because of the more off-resonant valence-band energy levels. Thus we can give the schematic of the level configuration of a TQD system, as shown in figure 1(c). Without the excitation of the laser, there are no excitons in any of the QDs, which corresponds to state $| 0 \rangle$. When a laser field is applied, a direct exciton is created inside QD1, and this condition is represented by the state $|1\rangle$. Under the tunneling couplings, the electron can tunnel from QD1 to QD2, and from QD2 to QD3. We denote these indirect excitons as state $|2\rangle$ and state $|3\rangle$.

Under the rotating wave and dipole approximations, the Hamiltonian of this system in the interaction picture is (we use units such that $\hbar = 1$)

$$H = \sum_{j=0}^{3} E_{j} |j\rangle \langle j| + \left[\left(\Omega_{c} e^{-i\omega_{c}t} |0\rangle \langle 1| + T_{1} |2\rangle \langle 1| + T_{2} |3\rangle \langle 2| \right) \right] + \text{H.c.},$$
(1)

where $E_j = \hbar \omega_j$ is the energy of state $|j\rangle$, ω_c is the laser frequency, $\Omega_c = \mu_{01} \cdot \mathbf{e} \cdot E$ is the Rabi frequency of the transition $|0\rangle \rightarrow |1\rangle$, with μ_{01} being the associated dipole transition-matrix element, \mathbf{e} the polarization vector and E the electric field amplitude of the laser. T_1 and T_2 are the tunneling couplings.

From the Liouville equation, we obtain the following equations for the density matrix elements:

$$\dot{\rho}_{01} = i \Big[\Omega_c (\rho_{11} - \rho_{00}) - T_1 \rho_{02} \Big] + (i\delta_1 - \Gamma_{10}) \rho_{01}, \qquad (2a)$$

$$\dot{\rho}_{02} = i \left(\Omega_c \rho_{12} - T_1 \rho_{01} - T_2 \rho_{03} \right) + \frac{1}{2} \left[i \left(\delta_1 + \delta_2 \right) - \Gamma_{20} \right] \rho_{02}, \qquad (2b)$$

$$\dot{\rho}_{03} = i \left(\Omega_c \rho_{13} - T_2 \rho_{02} \right) + \frac{1}{2} \left[i \left(\delta_1 + \delta_3 \right) - \Gamma_{30} \right] \rho_{03}, \quad (2c)$$

$$\dot{\rho}_{11} = i \Big[\Omega_c (\rho_{01} - \rho_{10}) + T_1 (\rho_{21} - \rho_{12}) \Big] - \Gamma_{10} \rho_{11}, \quad (2d)$$

$$\dot{\rho}_{12} = i \left[T_1(\rho_{22} - \rho_{11}) + i\Omega_c \rho_{02} - T_2 \rho_{13} \right] + \frac{1}{2} \left[i \left(\delta_1 - \delta_2 \right) - \Gamma_{10} - \Gamma_{20} \right] \rho_{12}, \qquad (2e)$$

$$\dot{\rho}_{13} = i \Big[\Omega_c \rho_{03} + T_1 \rho_{23} - T_2 \rho_{12} \Big] \\ + \frac{1}{2} \Big[i \big(\delta_1 - \delta_3 \big) - \Gamma_{10} - \Gamma_{30} \Big] \rho_{13}, \qquad (2f)$$

$$\dot{\rho}_{22} = iT_1(\rho_{12} - \rho_{21}) + iT_2(\rho_{32} - \rho_{23}) - \Gamma_{20}\rho_{22}, \qquad (2g)$$

$$\dot{\rho}_{23} = i \left[T_2(\rho_{33} - \rho_{22}) + T_1 \rho_{13} \right] + \frac{1}{2} \left[i \left(\delta_2 - \delta_3 \right) - \Gamma_{20} - \Gamma_{30} \right] \rho_{23}, \qquad (2h)$$

$$\dot{\rho}_{33} = iT_2(\rho_{23} - \rho_{32}) - \Gamma_{30}\rho_{33}, \qquad (2i)$$

$$\dot{\rho}_{ij} = \dot{\rho}_{ji}^{*}, \qquad (2j)$$

$$\rho_{00} + \rho_{11} + \rho_{22} + \rho_{33} = 1. \tag{2k}$$

Here the detunings are defined as $\delta_1 = (\omega_{10} - \omega_c)$, $\delta_2 = \delta_1 + 2\omega_{12}$ and $\delta_3 = \delta_1 + 2\omega_{12} + 2\omega_{23}$, with ω_{mn} being the transition frequency between $|m\rangle$ and $|n\rangle$ states. Γ_{10} , Γ_{20} and Γ_{30} represent the radiative decay rate of populations from $|1\rangle \rightarrow |0\rangle, |2\rangle \rightarrow |0\rangle$ and $|3\rangle \rightarrow |0\rangle$, respectively. In the present paper we consider the low temperature situation and do not include the pure dephasing rates, which come from the coupling with phonons and acoustic phonons.

Following the common method used in reference [43], we calculate the steady state fluorescence spectrum. As is well known, the fluorescence spectrum is proportional to the Fouriertransform of the steady state correlation function $\lim \langle E^{(-)}(r, \tau + t) \cdot E^{(+)}(r, t) \rangle$ [44], where $E^{(\pm)}(r, t)$ are the \dot{pos} stive and negative frequency parts of the radiation field in the far zone, which consists of a free field operator, and a source-field operator that is proportional to the polarization operator. Thus the incoherent fluorescence spectrum $S(\omega)$ can be expressed in terms of the atomic correlation function

$$S(\omega) = \operatorname{Re} \int_{0}^{\infty} \lim_{t \to \infty} \left\langle \Delta D^{+}(\tau + t) \cdot \Delta D^{-}(t) \right\rangle e^{-i\omega t} \mathrm{d}\tau, \quad (3)$$

where Re denotes the real part, and $\Delta D^{\pm}(t) = \Delta D^{\pm}(t) - \langle \Delta D^{\pm}(\infty) \rangle$ is the deviation of the dipole

polarization operator $D^{\pm}(t)$ from its mean steady state value, and

$$D^{+}(t) = \mu_{01} | 0 \rangle \langle 1 |, \qquad (4a)$$

$$D^{-}(t) = \left[D^{+}(t) \right]^{\dagger}.$$
 (4b)

Then we rewrite equation (2) in the form

$$\frac{d}{dt}\boldsymbol{\Psi} = \mathbf{L}\boldsymbol{\Psi} + \mathbf{I},\tag{5}$$

where $\Psi = (\rho_{01}, \rho_{02}, \rho_{03}, \rho_{10}, \rho_{11}, \rho_{12}, \rho_{13}, \rho_{20}, \rho_{21}, \rho_{22}, \rho_{23}, \rho_{30}, \rho_{31}, \rho_{32}, \rho_{33})^T$, and **L** is a 15 × 15 matrix. The elements of **L** and **I** can be found explicitly from equation (2).

By means of the quantum regression theorem [45, 46], the steady spectrum of the fluorescence spectrum from the state $|1\rangle$ to the ground state $|0\rangle$ can be obtained

$$S(\delta_k) = \operatorname{Re}\left\{M_{11}\bar{\rho}_{11} + M_{12}\bar{\rho}_{12} + M_{13}\bar{\rho}_{13} + \sum_l N_{ll}I_l\bar{\rho}_{10}\right\}, (6)$$

where

$$M_{ij} = \left[\left. (z - L)^{-1} \right|_{z = i\delta_k} \right]_{i,j}, N_{ij} = \left[L^{-1} \left. (z - L)^{-1} \right|_{z = i\delta_k} \right]_{i,j}$$
(7)

 $\bar{\rho}_{ij}$ is the steady state population (i = j) and atomic coherence $(i \neq j)$, which can be obtained by setting $\dot{\rho}_{ij} = 0$ and solving equation (2) numerically. δ_k is the detuning between the fluorescence and the transition $|1\rangle \rightarrow |0\rangle$. In the next part, we will calculate the fluorescence spectrum by tuning the tunneling coupling, which depends on the barrier characteristics and the external electric field.

3. Results and discussions

In TQDs, the tunneling couplings T_1 and T_2 depend on the barrier characteristics and the external electric field. Detunings of the tunneling are relative to the energy splitting ω_{12} and ω_{23} , which can be done by manipulation of the external electric field that changes the effective confinement potential. (In the present paper, we mainly consider the case of resonant tunneling coupling.) In addition, we only consider the population decay rates. The realistic values of the parameters are according to [47] and references therein [48–50]. And for simplicity, all the parameters are scaled by the decay rate Γ_{10} . Although some of the values of parameters are for DQDs, it can be inferred that the tunneling, energy splitting and decay rates of TQDs have the same values as that of DQDs.

First, in the absence of two tunneling couplings, the electron cannot tunnel from QD 1 to other QDs. Thus the system is like a single QD with one exited state $|1\rangle$ and one ground state $|0\rangle$, and a Mollow-type resonant fluorescence spectrum is obtained, as shown in figure 2(a). Such results have been achieved in an earlier experiment in reference [19]. Then we fix $T_2 = 0$ and tune T_1 from 0 to 0.5, and therefore the electron can tunnel from QD 1 to QD 2, which is similar to a DQD system. In this case the quenching of the fluorescence emission is obtained (figure 2(b)). When the detuning of the



Figure 2. Fluorescence spectrum $S(\omega)$ for $\Omega_c = 2$, $T_2 = 0$, $\delta_2 = \delta_3 = 0$, $\Gamma_{10} = 1$, $\Gamma_{20} = \Gamma_{30} = 10^{-4}\Gamma_{10}$, (a) $T_1 = 0$, $\delta_1 = 0$, (b) $T_1 = 0.5$, $\delta_1 = 0$, (c) $T_1 = 0.5$, $\delta_1 = 0.5$, (d) $T_1 = 0.2$, $\delta_1 = 0.2$.

coupling laser is nonresonant, the fluorescence emission can take place again and develop line width narrowing of the central peak, as shown in figures 2(c) and (d). As can be seen from the figure, with the smaller value for the tunneling, we can obtain the narrower line width of the central peak.

In order to interpret the above results, we investigate the properties of the dressed levels under the coupling of the laser field Ω_c and the tunneling T_1 . The energy eigenvalues of these dressed states [51] can be written as follows:

$$\Psi_{+}\rangle = \sin\theta\sin\phi \mid 0\rangle + \cos\phi \mid 1\rangle + \cos\theta\sin\phi \mid 2\rangle, (8a)$$

$$|\Psi_0\rangle = \cos\theta |0\rangle - \sin\theta |2\rangle,$$
 (8b)

$$|\Psi_{-}\rangle = \sin\theta\cos\phi |0\rangle - \sin\phi |1\rangle + \cos\theta\cos\phi |2\rangle, (8c)$$

where

$$\tan 2\phi = \frac{\sqrt{4\left(\Omega_c^2 + T_1^2\right)}}{\delta_1},\tag{9a}$$

$$\tan \theta = \frac{\Omega_c}{T_1},\tag{9b}$$

and the eigenvalues of $|\Psi_i\rangle$ (i = +, 0, -) is

$$\lambda_0 = 0, \tag{10a}$$

$$\lambda_{\pm} = \frac{\left(\delta_{1} \pm \sqrt{\delta_{1}^{2} + 4\left(\Omega_{c}^{2} + T_{1}^{2}\right)}\right)}{2}.$$
 (10b)

So we can see that both the state $|1\rangle$ and state $|0\rangle$ (see figure 1(*c*)) are split into three dressed levels (see figure 3(*a*)), which are $|\Psi_+\rangle$, $|\Psi_0\rangle$ and $|\Psi_-\rangle$ for the bare-state level $|1\rangle$, and $|\Psi'_+\rangle$, $|\Psi'_0\rangle$ and $|\Psi'_-\rangle$ for the bare-state level $|0\rangle$. Therefore the fluorescence emission from the state $|1\rangle$ to the ground state $|0\rangle$ has nine allowed dipole transitions in the dressed state representation. Note that though $|\Psi'_i\rangle$ and $|\Psi'_i\rangle$ are different in constant energy by the energy difference between level $|1\rangle$ and $|0\rangle$, the dressed levels $|\Psi'_i\rangle$ have the same expressions and eigenvalues as the dressed levels $|\Psi'_i\rangle$ with i = +, 0, -.

Thus we can explain the quenching and narrowing of the fluorescence spectrum obtained in figure 2. First we plot in figure 3(*b*) the steady state population of the dressed date $|\Psi_i\rangle$ as a function of the coupling laser detuning δ_1 . As can be seen, when $\delta_1 = 0$, all the population is in the dressed state $|\Psi_0\rangle$. So the possible transition is $|\Psi_0\rangle \rightarrow |\Psi'_+\rangle$, $|\Psi_0\rangle \rightarrow |\Psi'_0\rangle$ and $|\Psi_0\rangle \rightarrow |\Psi'_-\rangle$. However, the transitions can occur only if dipole moment $|\langle \Psi'_j| \mathbf{P} | \Psi'_i \rangle| \neq 0$, where P = $\mu_{01} | 0 \rangle \langle 1 | + \mu_{02} | 0 \rangle \langle 2 |$ is the transition dipole moment operator of the bare state basis. And in the case of $\Gamma_{10} \gg \Gamma_{20}$, $P = \mu_{01} | 0 \rangle \langle 1 |$. As can be seen from equation (8), the bare state | 1 \rangle has no contribution to the



Figure 3. (*a*) Dressed state under the coupling of the laser field Ω_c and the tunneling T_i . Properties of dressed levels: (*b*) Steady state populations of the dressed state $\rho_{\Psi_i}(i = +, 0, -)$ as a function of the tunneling δ_i . (*c*) Decay rates of the dressed state $R_{ii}(i = +, 0, -)$ as a function of the tunneling T_i . (*d*) Fluorescence spectrum $S(\omega)$ for varying δ_i . The other parameters are the same as those in figure 2.

dressed state $|\Psi_0\rangle$, therefore $|\langle \Psi'_j| \mathbf{P} |\Psi_i\rangle| = 0$ for the above three possible transitions and none of the transition can occur. In such a case the population is trapped in state $|\Psi_0\rangle$, and as a result the quenching of the fluorescence emission is obtained.

Once the population is in state $|\Psi_0\rangle$, they will be trapped there, which is known as coherent population trapping (CPT). The condition of $\delta_1 = \delta_2 = 0$ is the CPT condition. As the state $|\Psi_0\rangle$ is completely decoupled from the light field, we define the state $|\Psi_0\rangle$ as the dark state. So the quenching of the fluorescence emission is the result of the dark state, which is induced by the tunneling coupling between the dots. And unlike the dark state obtained in atomic systems, in QD systems we can use tunneling to achieve the necessary coherence, requiring no additional laser field. And when $\delta_1 \neq 0$, the CPT condition is not satisfied and the dark state is destroyed. Thus all the dressed state $|\Psi_i\rangle$ is populated [figure 3(b)]. As a result the fluorescence emission takes place (figures 2(c) and (d)). The fluorescence emission for varying δ_1 is shown in figure 3(d), given that it goes from nothing to a multiplet.

It is well known that the widths of the fluorescence peaks are determined by the decay rate of the transition between the dressed levels $|\Psi_i\rangle$ to $|\Psi_i'\rangle$, which is proportional to the

squared dipole moments $|\langle \Psi'_i| \mathbf{P} | \Psi'_i \rangle|^2$. The central peak in figures 2(c) and (d) arises from the transition between identical dressed levels of adjacent manifolds, which are $|\Psi_i\rangle \rightarrow |\Psi_i'\rangle$ (i = +, 0, -). (The corresponding transitions are shown in figure 3(a) by dashed line.) We plot the decay rate R_{ii} as a function of T_1 in figure 3(c) to explain the line widths of the central peak under the nonresonant coupling laser. When T_1 is small ($T_1 = 0.2$), the value of R_{00} is close to zero, which gives rise to a very sharp feature at the line center. And when T_1 is increased to $T_1 = 0.5$, though the value of R_{00} is increased a little bit, it is still small enough to give rise to the narrowing of the central peak. In a wide range of T_1 , the value of R_{++} and R_{--} are much larger than those of R_{00} , thus the spectral feature at the line center consists of a sharp peak superimposed on a broad profile contributed by the transitions $|\Psi_{+}\rangle \rightarrow |\Psi_{+}\rangle$ and $|\Psi_{-}\rangle \rightarrow |\Psi_{-}\rangle$. So we conclude that the narrowing of the spectrum is due to the slow decay of the dressed state $|\Psi_0\rangle$, which originates from the coupling of the tunneling T_1 .

Next, we investigate the fluorescence spectrum when both tunnelings are applied. In this case the electron can tunnel from QD 1 to QD 2 and from QD 2 to QD 3, which forms the TQD system. Under the resonant coupling of the laser field, when $T_2 = 0.2$, by comparing with figure 2(b) the



Figure 4. Fluorescence spectrum $S(\omega)$ for $\Omega_c = 2$, $T_1 = 0.5$, $\delta_2 = \delta_3 = 0$, $\Gamma_{10} = 1$, $\Gamma_{20} = \Gamma_{30} = 10^{-4}\Gamma_{10}$, (a) $T_2 = 0.2$, $\delta_1 = 0$, (b) $T_2 = 0.5$, $\delta_1 = 0$, (c) $T_2 = 1$, $\delta_1 = 0$, (d) $T_2 = 0.2$, $\delta_1 = 0.2$.

fluorescence spectrum appears again, with one narrow central peak and two pairs of sidebands (figure 4(a)). In the case of $T_2 = 0.5$, as can be seen from figure 4(b), the intensity of the fluorescence is increased and the line width of the central peak is a little broadened. At the same time, one pair of narrow inner sidebands arises. When the tunneling is increased to $T_2 = 1$, the intensity of the fluorescence emission continues to increase, as well as the line width of the central peak and the inner sideband. Also the location of the pair of inner narrow sidebands gets farther away from the central peak (figure 4(c)). When the detuning of the coupling field is tuned to $\delta_1 = T_2$, the fluorescence emission disappears again, as shown in figure 4(d).

The above results can also be interpreted in the dressed state picture. Under the resonant coupling of the laser field and the tunneling, the energy eigenvalues of the dressed states can be written as follows

$$\lambda_4 = -\lambda_1 = \kappa_+, \tag{11a}$$

$$\lambda_3 = -\lambda_2 = \kappa_{-},\tag{11b}$$

where

$$\sqrt{\frac{\left(\Omega_c^2 + T_1^2 + T_2^2\right) \pm \sqrt{\left(\Omega_c^2 + T_1^2 + T_2^2\right)^2 - 4\Omega_c^2 T_2^2}}{2}}.$$
 (12)

Then the dressed levels can be expressed as

$$\Psi_{1} \rangle = C_{10} | 0 \rangle + C_{11} | 1 \rangle + C_{12} | 2 \rangle + C_{13} | 3 \rangle, \quad (13a)$$

$$\Psi_{2} \rangle = -C_{20} | 0 \rangle + C_{21} | 1 \rangle - C_{22} | 2 \rangle + C_{23} | 3 \rangle, \quad (13b)$$

$$|\Psi_{3}\rangle = -C_{30} |0\rangle + C_{31} |1\rangle - C_{32} |2\rangle + C_{33} |3\rangle, (13c)$$

$$|\Psi_4\rangle = C_{40} |0\rangle + C_{41} |1\rangle + C_{42} |2\rangle + C_{43} |3\rangle, \quad (13d)$$

where

$$C_{i0} = \frac{1}{D_i} \frac{\Omega_c \left(\lambda_i^2 - T_2^2\right)}{T_i T_2 \lambda_i},$$
(14a)

$$C_{i1} = \frac{1}{D_i} \frac{\lambda_i^2 - T_2^2}{T_1 T_2},$$
(14b)

$$C_{i2} = \frac{1}{D_i} \frac{\lambda_i}{T_2},\tag{14c}$$

$$C_{i3} = \frac{1}{D_i},\tag{14d}$$

$$D_i = \sqrt{1 + \left(\frac{\lambda_i}{T_2}\right)^2 + \left(\frac{\lambda_i^2 - T_2^2}{nT_2}\right)^2 + \left(\frac{\alpha_c(\lambda_i^2 - T_2^2)}{nT_2\lambda_i}\right)^2}.$$
 (14e)

For simplicity, we rewrite equation (13) in the form

$$|\Psi_i\rangle = \sum C_{ik} |k\rangle.$$
 (*i* = 1,2,3,4; *k* = 0,1,2,3) (15)

Similar to the DQD system, both state $|1\rangle$ and state $|0\rangle$ are split into four dressed levels (see figure 5(*a*)), which are $|\Psi_1\rangle$, $|\Psi_2\rangle$, $|\Psi_3\rangle$ and $|\Psi_4\rangle$ for the bare-state level $|1\rangle$, while $|\Psi_1'\rangle$, $|\Psi_2'\rangle$, $|\Psi_3'\rangle$ and $|\Psi_4'\rangle$ for the bare-state level $|0\rangle$. Therefore the fluorescence from the state $|1\rangle$ to the state $|0\rangle$ has sixteen dipole transitions in the dressed state representation. Also the dressed levels $|\Psi_i\rangle$ have the same expressions and the eigenvalues as the dressed levels $|\Psi_i'\rangle$ with i = 1, 2, 3, 4.

First we explain the appearance of the fluorescence emission when tunneling T_2 is applied. In figure 5(*b*), we plot the steady state population of the dressed level $|\Psi_i\rangle$ as a function of T_2 for the resonant coupling of the laser field. From the figure one can see that, when $T_2 \neq 0$, all the dressed states are populated. However, all the dipole moments $|\langle \Psi'_j | \mathbf{P} | \Psi_i \rangle| \neq 0$. So all the transitions can occur and the electrons cannot get trapped in any of the dressed states. As a result the dark state is destroyed and fluorescence emission takes place.

Next we interpret the emerging of the narrow inner sideband (figures 4(b) and (c)). The pair of inner peaks comes from the transition of $|\Psi_2\rangle \rightarrow |\Psi_3\rangle$ and $|\Psi_3\rangle \rightarrow |\Psi_2\rangle$ and the position of the peaks depend on the difference between the energies λ_2 and λ_3 . (The corresponding transitions are shown in figure 5(a) by a dotted line.) We show in figure 5(c) the eigenenergies λ_i as a function of the tunneling T_2 . As can be seen from the figure, the energies of the dressed levels $|\Psi_1\rangle$ and $|\Psi_4\rangle$ depend weakly on T_2 , while the difference between the energies of the dressed levels $|\Psi_2\rangle$ and $|\Psi_3\rangle$ is increased with an increasing value of T_2 . Therefore the position of the narrow inner peaks become farther away from the central peak as T_2 is increased. So when T_2 is small, the inner sideband is so close to the central peak that it cannot be separated from the central peak (figure 4(a)). While for the larger value of T_2 , the narrow sideband gets farther away from the center and can be seen clearly (figures 4(b) and (c)).

Last we explain the narrowing of the central peak and the inner sideband obtained in figure 4. As we discussed above, the widths of the fluorescence peaks is proportional to the squared dipole moments $|\langle \Psi'_j \rangle| \mathbf{P} | \Psi_i \rangle|$. In the TQD system, the transition dipole moment operator is $P = \boldsymbol{\mu}_{01} | 0 \rangle \langle 1 | + \boldsymbol{\mu}_{02} | 0 \rangle \langle 2 | + \boldsymbol{\mu}_{03} | 0 \rangle \langle 3 |$. In the case of $\Gamma_{10} \gg \Gamma_{20}(\Gamma_{30}), P = \boldsymbol{\mu}_{01} | 0 \rangle \langle 1 |$. Thus the decay rate R_{ij} can be calculated with the expression

$$R_{ij} = \left| C_{j0} \right|^2 \boldsymbol{\mu}_{01}^2 \left| C_{i1} \right|^2 \quad (i, j = 1, 2, 3, 4).$$
(16)

The central peak comes from the transition $|\Psi_i'\rangle \rightarrow |\Psi_i'\rangle$ (i = 1-4), which are shown in figure 5(*a*) by the dashed line. We plot in figure 5(*d*) the decay rate R_{ii} as a function of T_2 in order to explain the line widths of the central peak. When T_2 is small, the value of $R_{22}(R_{33})$ is very small, which corresponds to the narrowing of the central peak. When

 T_2 is increased, the value of $R_{22}(R_{33})$ does not change much and still gives rise to a narrowed central peak. With all values of T_2 , the value of $R_{11}(R_{44})$ is much larger than those of $R_{22}(R_{33})$. Similar to the case of that for DQDs, the spectral feature at the line center consists of a sharp peak superimposed on a broad profile. The narrowing of the inner sideband, which comes from the transition $|\Psi_2\rangle \rightarrow |\Psi_3'\rangle$ and $|\Psi_3\rangle \rightarrow |\Psi_2'\rangle$, is also due to the slow decay rate of the corresponding transition.

Alternatively, the TQD system can also be analyzed by diagonalizing the interaction with the tunneling T_2 , and the corresponding dressed state is shown in figure 5(e). Under the coupling of T_2 , the state $|2\rangle$ splits into two dressed levels $|2_+\rangle$ with splitting of $2T_2$, therefore two Λ type subsystems show up clearly. In both Λ subsystems, the tunneling T_1 is nonresonant with the transition $|1\rangle \rightarrow |2_+\rangle$ or $|1\rangle \rightarrow |2_-\rangle$, with detuning $\delta_2 = \pm T_2$. So with the resonant laser coupling, the CPT condition is not satisfied in both Λ systems, and as a result the dark state vanishes and fluorescence emission takes place (figures 4(a)-(c)). As the strength of the tunneling T_2 is increased, the splitting between the level $|2_{\perp}\rangle$ and $|2_{\perp}\rangle$ becomes larger, thus the two Λ systems are farther away from the CPT condition, therefore the intensity of the fluorescence is increased. When the detuning of the laser field is tuned to $\delta_1 = \pm T_2$, the dark state will be set up in one of the Λ type subsystems and the fluorescence emission disappears (figure 4(d)).

To finish our discussion, we investigate the influence of the decay rate $\Gamma_{20}(\Gamma_{30})$ on fluorescence emission. The fluorescence comes from the spontaneous emission from the dressed state $|\Psi_i\rangle$ to the dressed state $|\Psi_i\rangle$, which can occur only if dipole moment $|\langle \Psi'_i| \mathbf{P} | \Psi_i \rangle| \neq 0$. For $\Gamma_{10} \gg \Gamma_{20}(\Gamma_{30})$, once the CPT condition is satisfied, all the dipole moments are zero and the quenching of the fluorescence emission can be obtained. Otherwise when $\Gamma_{20}(\Gamma_{30})$ can be compared with Γ_{10} , the transition dipole moment operator is P = $\boldsymbol{\mu}_{01} \mid 0 \rangle \langle 1 \mid + \boldsymbol{\mu}_{02} \mid 0 \rangle \langle 2 \mid + \boldsymbol{\mu}_{03} \mid 0 \rangle \langle 3 \mid, \text{ and } |\langle \boldsymbol{\Psi}_{1}' \mid \mathbf{P} \mid \boldsymbol{\Psi}_{1}' \rangle |$ are not zero even under the CPT condition. Therefore the dark states are destroyed and the complete quenching of fluorescence emission cannot be obtained. In figure 6 we display the fluorescence spectrum with various values of $\Gamma_{20}(\Gamma_{30})$. As can be seen from the figure, the bigger the value of $\Gamma_{20}(\Gamma_{30})$, the greater the population of the bare state $|1\rangle$, and therefore the stronger the intensity of the fluorescence emission. For the real experimental value of QDMs, $\Gamma_{20}(\Gamma_{30}) = 10^{-4}\Gamma_{10}$ [47–50], this is small enough to acquire the quenching of fluorescence emission.

4. Conclusions

In this paper, we investigate the resonance fluorescence spectrum from TQDs controlled by tunneling and demonstrate that it is possible to use tunneling to induce dark states. With the tunneling coupling, interesting features such as quenching and narrowing of the fluorescence can be obtained. We also interpret the results in the dressed state basis of the two tunneling couplings and the laser field. The quenching of





Figure 5. (*a*) Dressed state under the coupling of the laser field Ω_c and two tunneling values T_1 and T_2 . Properties of dressed levels as a function of the tunneling coupling T_2 : (*b*) steady state populations of the dressed state $|\rho_{\Psi_i}\rangle$ (i = 1-4), (*c*) the eigenenergies λ_{Ψ_i} , (*d*) decay rates of the dressed state R_{ii} (i = 1-4). (*e*) Dressed state under the coupling tunneling T_2 . The parameters are the same as those in figure 4.

the fluorescence is due to the tunneling induced dark states, while the narrowing of the central peak is owing to the slow decay rate of the dressed levels. By comparing with the atomic system, QDMs offers many degrees of freedom for real applications, such as being easily controllable in size and in the energy levels spacing. And most importantly, QDMs adopted here possess some unique advantages because that quantum coherence can be induced by using tunneling instead



Figure 6. Fluorescence spectrum $S(\omega)$ for $\Omega_c = 2$, $T_1 = 0.5$, $T_2 = 0.5$, $\delta_1 = 0.5$, $\delta_2 = \delta_3 = 0$, $\Gamma_{10} = 1$, (a) $\Gamma_{20} = \Gamma_{30} = 10^{-4}\Gamma_{10}$, (b) $\Gamma_{20} = \Gamma_{30} = 10^{-1}\Gamma_{10}$. (c) $\Gamma_{20} = \Gamma_{30} = 10^{-1}\Gamma_{10}$. (d) Steady state populations of bare state | 1 \rangle as a function of Γ_{20} .

of coupling lasers. Such tunneling controllable QDMs will open the way to applications of quantum optics and photonics.

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