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Third-order aberration analysis of tilted-pupil optical systems*

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The aberration field of an optical system with a tilted pupil is explored through expanding the vector expressions of the third-order wavefront aberrations. First, the vector forms of the wavefront aberrations are modified to obtain the aberration expressions with the pupil tilted; full field displays of coma and astigmatism in this situation are given. Then, the third-order aberration formulas with the pupil decentered and tilted simultaneously are derived and discussed. Finally, an example is taken to certify the validity of aberration distribution properties.

Keywords: vector aberration, formula expansion, tilted pupil, imaging system

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1. Introduction

Optical systems have gradually come to have more and more complex structures, so the improvement of the current aberration theory developed for traditional axisymmetrical systems becomes very important. The vector wavefront aberration theory, also known as the nodal aberration theory, is applied broadly in optical engineering. In the 1970s, Shack developed the conventional wavefront formula in vector form, defined the vector aberration theory and gave its third-order form.[1] In the 1980s, Thompson developed the vector wavefront aberration theory to a fifth-order form.[2] In further research, Rogers analyzed the aberrations of unobscured reflective optical systems with this theory.[3] In China, optical researchers also did some systematic research on the vector aberration theory. For instance, Yang *et al.* analyzed the aberration properties of optical systems with a large tilted and decentered refractive plate. With this theory,[4] Sun *et al*. developed third-order aberrations of sub-aperture plane symmetric optical systems.[5] Shi *et al.* also researched the applications of the vector aberration theory.[6,7]

The research in this paper is based on the vector aberration theory as conceived by Shack and developed by Thompson *et al*. Similarly to other Chinese research like Shi's work, which focused on the reflective telescope alignment and Ren's work that focused on the off-axis optical systems, this article is an effort to expand the application of this theory, but focuses on a different aspect.

With the development of optical fabrication and testing, freeform optical systems are more and more widely applied. Wang *et al.*, Wang and Song *et al.* designed space,[8] head-mounted[9] and conformal optical systems[10] containing separate freeform surfaces, respectively. In such optical systems, the structural properties of decentering and tilts are familiar. Therefore, the corresponding aberration theory needs to be further expanded. The work reported in this article focuses on this area.

In actual optical systems, each parameter of every element will affect the aberrations of the whole system. In this article, we discuss the aberration while considering the elements to be integrated as a whole system, so the aberration fields are considered to be essentially independent of the parameters of individual elements. Under this premise, it is convenient to realize the impact of pupil movement, or to control the system aberration in optical design. This article is based on the work in Ref. [11], which discussed the third-order aberration fields of pupil-decentered optical systems, to further study the aberrations introduced by a tilted pupil. Mainly, the spherical aberration, coma and astigmatism variations are discussed. In this article, wavefront aberrations introduced by a tilted pupil are discussed in Section 2 and the case of the pupil being simultaneously decentered and tilted is illuminated in Section 3. To verify the above aberration properties, an example is given in Section 4.

2. Wavefront aberration introduced by a tilted pupil

In optical systems, the vector representation of the wavefront aberration of surface j is[12]

$$W_j = \sum_p^{\infty}\sum_n^{\infty}\sum_m^{\infty}(W_{klm})_j(\boldsymbol{H}\cdot\boldsymbol{H})^p(\boldsymbol{\rho}\cdot\boldsymbol{\rho})^n(\boldsymbol{H}\cdot\boldsymbol{\rho})^m,$$
$$k = 2p+m,\quad l = 2n+m, \tag{1}$$

where $\boldsymbol{H}$ is the normalized field vector and $\boldsymbol{\rho}$ is the normalized aperture vector. The coordinate system is shown in Fig. 1.

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Formula representations in this article are all based on this coordinate system.

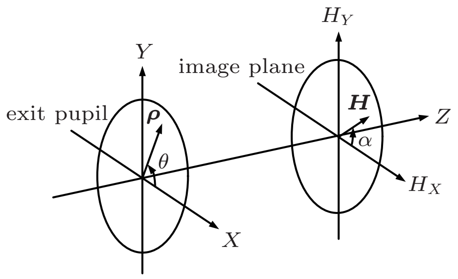

Fig. 1. Coordinate system consists of a pupil and an image plane.

As shown in Ref. [11], if a surface in the system is tilted, the pupil of this surface will differ from the original one and the new pupil is a projection to the old pupil plane. For example, if the original pupil has a circular aperture, then its projection is an ellipse, as shown in Fig. 2.

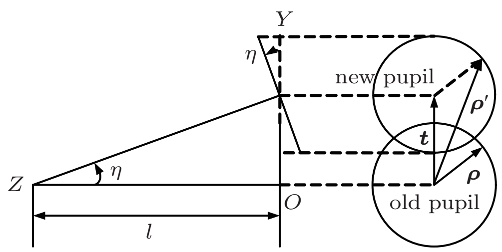

Fig. 2. Pupil before and after tilting.

The new pupil is elliptical and its center is displaced by $\boldsymbol{t}$ from the original pupil center. In order to facilitate the presentation, it is assumed that $\boldsymbol{t}$ is along the Y-axial direction. Consider the elliptic equation in Cartesian coordinates $\frac{x'^2}{\rho^2}+\frac{y'^2}{\rho^2\cos^2\eta}=1$ and as Fig. 2 shows, $x'=x=\rho\cos\theta$ and $y'=\rho\sin\theta\cos\eta$, we obtain

$$\begin{aligned}\rho'^2 &= x'^2+y'^2=\rho^2\cos^2\theta+\rho^2\sin^2\theta\cos^2\eta\\ &= \rho^2(1-\sin^2\theta)+\rho^2\sin^2\theta\cos^2\eta\\ &= \rho^2(1-\sin^2\theta\sin^2\eta).\end{aligned}\tag{2}$$

Then the expression of $\boldsymbol{\rho}'$ is

$$\boldsymbol{\rho}'=\sqrt{1-\sin^2\theta\sin^2\eta}\,\boldsymbol{\rho}+\boldsymbol{t},\quad |\boldsymbol{t}|=l\cdot\tan\eta.\tag{3}$$

By taking $\boldsymbol{\rho}'$ into the aberration expression (1) and setting the coefficient $C=\sqrt{1-\sin^2\theta\sin^2\eta}$, the wavefront aberration of surface j changes to

$$\begin{aligned}W_j &= \sum_p^\infty\sum_n^\infty\sum_m^\infty (W_{klm})_j(\boldsymbol{H}\cdot\boldsymbol{H})^p\\ &\quad\times[(C\boldsymbol{\rho}+\boldsymbol{t})\cdot(C\boldsymbol{\rho}+\boldsymbol{t})]^n[\boldsymbol{H}\cdot(C\boldsymbol{\rho}+\boldsymbol{t})]^m.\end{aligned}\tag{4}$$

Next, the third-order aberration expression of the optical system can be expanded as

$$\begin{aligned}W &= \sum_j W_j=\sum_j W_{020j}(C\boldsymbol{\rho}+\boldsymbol{t})\cdot(C\boldsymbol{\rho}+\boldsymbol{t})\\ &\quad+\sum_j W_{111j}\boldsymbol{H}\cdot(C\boldsymbol{\rho}+\boldsymbol{t})\\ &\quad+\sum_j W_{040j}[(C\boldsymbol{\rho}+\boldsymbol{t})\cdot(C\boldsymbol{\rho}+\boldsymbol{t})]^2\\ &\quad+\sum_j W_{131j}[\boldsymbol{H}\cdot(C\boldsymbol{\rho}+\boldsymbol{t})][(C\boldsymbol{\rho}+\boldsymbol{t})\cdot(C\boldsymbol{\rho}+\boldsymbol{t})]\\ &\quad+\sum_j W_{222j}[\boldsymbol{H}^2\cdot(C\boldsymbol{\rho}+\boldsymbol{t})^2]\\ &\quad+\sum_j W_{220_\mathrm{M}j}(\boldsymbol{H}\cdot\boldsymbol{H})[(C\boldsymbol{\rho}+\boldsymbol{t})\cdot(C\boldsymbol{\rho}+\boldsymbol{t})]\\ &\quad+\sum_j W_{311j}(\boldsymbol{H}\cdot\boldsymbol{H})[\boldsymbol{H}\cdot(C\boldsymbol{\rho}+\boldsymbol{t})],\end{aligned}\tag{5}$$

where $W_{220_\mathrm{M}}=W_{220}+\frac{1}{2}W_{222}$.[13] Expanding Eq. (5) and collecting terms by the relationship with $\boldsymbol{\rho}$, we obtain

$$\begin{aligned}W &= W_{040}C^4\cdot(\boldsymbol{\rho}\cdot\boldsymbol{\rho})^2\\ &\quad+(4W_{040}\boldsymbol{t}+W_{131}\boldsymbol{H})C^3\cdot\boldsymbol{\rho}(\boldsymbol{\rho}\cdot\boldsymbol{\rho})\\ &\quad+\left(\frac{1}{2}W_{222}\boldsymbol{H}^2+W_{131}\boldsymbol{t}\boldsymbol{H}+2W_{040}\boldsymbol{t}^2\right)C^2\cdot\boldsymbol{\rho}^2\\ &\quad+[W_{220_\mathrm{M}}(\boldsymbol{H}\cdot\boldsymbol{H})+2W_{131}(\boldsymbol{t}\cdot\boldsymbol{H})\\ &\quad+(W_{220}+4W_{040}t^2)]C^2\cdot(\boldsymbol{\rho}\cdot\boldsymbol{\rho})\\ &\quad+[2W_{020}\boldsymbol{t}+W_{111}\boldsymbol{H}+4W_{040}t^2\boldsymbol{t}+2W_{131}t^2\boldsymbol{H}+W_{131}t^2\boldsymbol{H}^*\\ &\quad+W_{222}\boldsymbol{H}^2\boldsymbol{t}^*+2W_{220_\mathrm{M}}(\boldsymbol{H}\cdot\boldsymbol{H})\boldsymbol{t}+W_{311}(\boldsymbol{H}\cdot\boldsymbol{H})\boldsymbol{H}]C\cdot\boldsymbol{\rho}\\ &\quad+[W_{020}t^2+W_{111}(\boldsymbol{H}\cdot\boldsymbol{t})+W_{040}t^4+W_{131}t^2(\boldsymbol{t}\cdot\boldsymbol{H})\\ &\quad+\frac{1}{2}W_{222}(\boldsymbol{H}^2\cdot\boldsymbol{t}^2)+W_{220_\mathrm{M}}t^2(\boldsymbol{H}\cdot\boldsymbol{H})\\ &\quad+W_{311}(\boldsymbol{H}\cdot\boldsymbol{H})(\boldsymbol{H}\cdot\boldsymbol{t})].\end{aligned}\tag{6}$$

From formula (6), the third-order system aberration expressions introduced by the tilted pupil are as follows:
spherical aberration:

$$W=W_{040}C^4\cdot(\boldsymbol{\rho}\cdot\boldsymbol{\rho})^2.\tag{7}$$

coma:

$$\begin{aligned}W &= (4W_{040}\boldsymbol{t}+W_{131}\boldsymbol{H})C^3\cdot\boldsymbol{\rho}(\boldsymbol{\rho}\cdot\boldsymbol{\rho})\\ &= W_{131}C^3\left(\boldsymbol{H}+\frac{4W_{040}}{W_{131}}\boldsymbol{t}\right)\cdot\boldsymbol{\rho}(\boldsymbol{\rho}\cdot\boldsymbol{\rho}).\end{aligned}$$

By defining vector $\boldsymbol{c}_\mathrm{c}\equiv-\frac{4W_{040}}{W_{131}}\boldsymbol{t}$, the coma can be described as

$$W=W_{131}C^3(\boldsymbol{H}-\boldsymbol{c}_\mathrm{c})\cdot\boldsymbol{\rho}(\boldsymbol{\rho}\cdot\boldsymbol{\rho}).\tag{8}$$

astigmatism:

$$\begin{aligned}W &= \left(\frac{1}{2}W_{222}\boldsymbol{H}^2+W_{131}\boldsymbol{t}\boldsymbol{H}+2W_{040}\boldsymbol{t}^2\right)C^2\cdot\boldsymbol{\rho}^2\\ &= \frac{1}{2}W_{222}C^2\left(\boldsymbol{H}^2+\frac{2W_{131}\boldsymbol{t}}{W_{222}}\boldsymbol{H}+\frac{4W_{040}\boldsymbol{t}^2}{W_{222}}\right)\cdot\boldsymbol{\rho}^2\\ &= \frac{1}{2}W_{222}C^2\left[\left(\boldsymbol{H}^2+\frac{W_{131}\boldsymbol{t}}{W_{222}}\right)^2\right.\end{aligned}$$

$$-\frac{(W_{131}^2-4W_{040}W_{222})\boldsymbol{t}^2}{W_{222}^2}\Bigg]\cdot\boldsymbol{\rho}^2.$$

By defining vector $\boldsymbol{c}_{\rm a}\equiv-\frac{W_{131}\boldsymbol{t}}{W_{222}}$ and $\boldsymbol{d}_{\rm a}\equiv\sqrt{\frac{W_{131}^2-4W_{040}W_{222}}{W_{222}^2}}\boldsymbol{t}$, astigmatism can be described as

$$W=\frac{1}{2}W_{222}C^2\left[(\boldsymbol{H}-\boldsymbol{c}_{\rm a})^2-\boldsymbol{d}_{\rm a}^2\right]\cdot\boldsymbol{\rho}^2. \tag{9}$$

Here astigmatism has the two-nodal characteristics.[11]

Considering the aberration expressions taken by the decentered pupil in Ref. [11], it can be seen that the third-order aberration expressions with a tilted pupil differ from the decentered pupil by only a single coefficient C^x, where x is the order of ρ in the aberration expression. Since the coefficient C is a function related to θ, the aberration arising from the tilted pupil is no longer axisymmetrical but scales differently in the X and Y directions. If the pupil is a circular aperture, then the wavefront aberration introduced is elliptical; the ellipticity is related to the tilt angle η.

Full field displays of the coma and astigmatism aberrations are shown in Figs. 3 and 4, respectively.

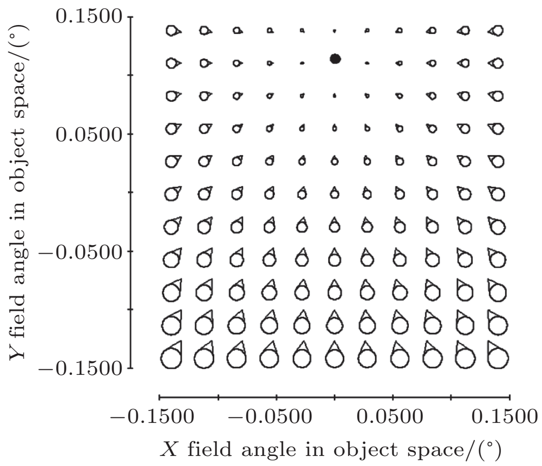

Fig. 3. Full field display of coma.

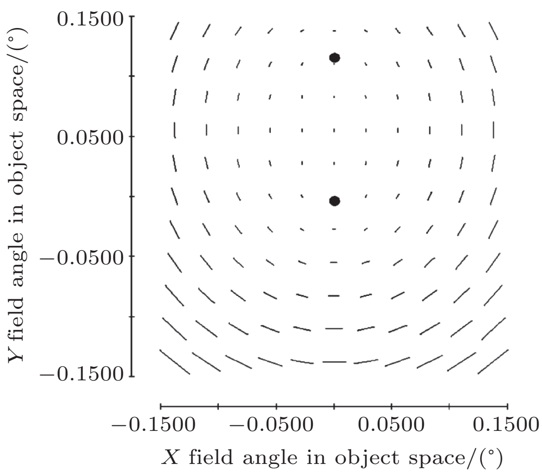

Fig. 4. Full field display of astigmatism.

3. The case of a simultaneously decentered and tilted pupil

In complex optical systems, the pupil is likely to be both decentered and tilted, in which case its impact to the system is a combination of the aberrations of the simple tilted case and the simple decentered case, as $W=W_{\rm d}+W_{\rm t}$. The three main reference aberrations, spherical aberration, coma and astigmatism, are expressed as follows:
spherical aberration:

$$\begin{aligned}W&=W_{\rm ds}+W_{\rm ts}=W_{040}(\boldsymbol{\rho}\cdot\boldsymbol{\rho})^2+W_{040}C^4\cdot(\boldsymbol{\rho}\cdot\boldsymbol{\rho})^2\\&=W_{040}(1+C^4)\cdot(\boldsymbol{\rho}\cdot\boldsymbol{\rho})^2.\end{aligned} \tag{10}$$

coma:

$$\begin{aligned}W&=W_{\rm dc}+W_{\rm tc}=W_{131}(\boldsymbol{H}-\boldsymbol{a}_{\rm c})\cdot\boldsymbol{\rho}(\boldsymbol{\rho}\cdot\boldsymbol{\rho})\\&\quad+W_{131}C^3(\boldsymbol{H}-\boldsymbol{c}_{\rm c})\cdot\boldsymbol{\rho}(\boldsymbol{\rho}\cdot\boldsymbol{\rho})\\&=W_{131}(1+C^3)(\boldsymbol{H}-\boldsymbol{A}_{\rm c})\cdot\boldsymbol{\rho}(\boldsymbol{\rho}\cdot\boldsymbol{\rho}),\end{aligned} \tag{11}$$

where $\boldsymbol{A}_{\rm c}=\frac{\boldsymbol{a}_{\rm c}+C^3\boldsymbol{c}_{\rm c}}{1+C^3}$.
astigmatism:

$$\begin{aligned}W&=W_{\rm da}+W_{\rm ta}\\&=\frac{1}{2}W_{222}\left[(\boldsymbol{H}-\boldsymbol{a}_{\rm a})^2-\boldsymbol{b}_{\rm a}^2\right]\cdot\boldsymbol{\rho}^2\\&\quad+\frac{1}{2}W_{222}C^2\left[(\boldsymbol{H}-\boldsymbol{c}_{\rm a})^2-\boldsymbol{d}_{\rm a}^2\right]\cdot\boldsymbol{\rho}^2\\&=\frac{1}{2}W_{222}(1+C^2)\left[(\boldsymbol{H}-\boldsymbol{A}_{\rm a})^2-\boldsymbol{B}_{\rm a}^2\right]\cdot\boldsymbol{\rho}^2,\end{aligned} \tag{12}$$

where

$$\boldsymbol{A}_{\rm a}=\frac{\boldsymbol{a}_{\rm a}+C^2\boldsymbol{c}_{\rm a}}{1+C^2},$$

$$\boldsymbol{B}_{\rm a}=\sqrt{\frac{(\boldsymbol{a}_{\rm a}+C^2\boldsymbol{c}_{\rm a})^2}{(1+C^2)^2}-\frac{(\boldsymbol{a}_{\rm a}^2-\boldsymbol{b}_{\rm a}^2)+C^2(\boldsymbol{c}_{\rm a}^2-\boldsymbol{d}_{\rm a}^2)}{1+C^2}}.$$

It can be seen from the above expressions, when decentering and tilting coexist, the spherical aberration, coma and astigmatism characteristics are the vector combinations of those in the two individual cases. The performance is similar to the situation with only tilting or only decentering. However, because of the coefficient C, all aberrations reveal the ellipticity. Besides, coma and astigmatism field center vectors must be replaced by the center vectors in this integrated case.

4. Example

To validate the aberration distribution formulas in this paper, the optical design software CODEV is used to modulate a Cassegrain Ritchey–Chretien system. The effective focal length of this system is 1752 mm and the entrance pupil diameter is 150 mm. The shapes of the primary and the secondary mirrors are both conic. The optical layout of the system is shown in Fig. 5.

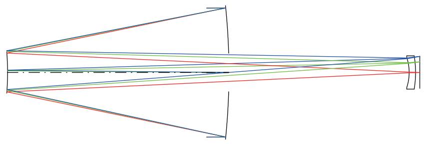

Fig. 5. (color online) Optical layout.

Making the system's pupil decentered and tilted respectively, we then calculate the amounts of aberrations in the $\pm 0.3^{\circ}$ field of view. For the decentered case, it is 1.5 mm, $+X$ direction and for the tilted case, it is 0.3°, $+Y$ direction. It can be computed from optical system parameters

$$W_{040} = -0.005065\lambda, \quad W_{131} = -0.150189\lambda,$$

$$W_{222} = 0.178759\lambda. \tag{13}$$

Then,

$$\boldsymbol{a}_{\mathrm{a}} = 0.8402\boldsymbol{s}, \quad \boldsymbol{b}_{\mathrm{a}} = 0.9051\boldsymbol{s},$$
$$\boldsymbol{c}_{\mathrm{a}} = 0.8402\boldsymbol{t}, \quad \boldsymbol{d}_{\mathrm{a}} = 0.9051\boldsymbol{t}. \tag{14}$$

Taking the astigmatism as an example, we take Eqs. (13) and (14) into Eq. (12) and obtain the astigmatism distribution of $\pm 0.3^{\circ}$ fields, as shown in Table 1, where Mag. represents the magnitude and Ang. represents the angle. Then we compute the same data with the field map tool of CODEV.[14] The results are shown in Table 2.

Table 1. Results calculated from Eq. (12).

X \ Y	−0.3		−0.15		0		0.15		0.3	
	Mag.	Ang.	Mag.	Ang.	Mag.	Ang.	Mag.	Ang.	Mag.	Ang.
0.3	0.053	122.9	0.033	117.9	0.026	113.5	0.018	85.7	0.029	66.9
0.15	0.052	132.3	0.031	–45.5	0.016	–40.3	0.003	40.0	0.020	46.3
0	0.054	–35.5	0.037	–31.3	0.028	–19.2	0.021	6.8	0.029	25.9
–0.15	0.070	–26.1	0.050	–23.3	0.046	–9.0	0.043	4.7	0.042	18.3
–0.3	0.085	–23.2	0.066	–17.1	0.068	–8.5	0.066	4.0	0.063	12.3

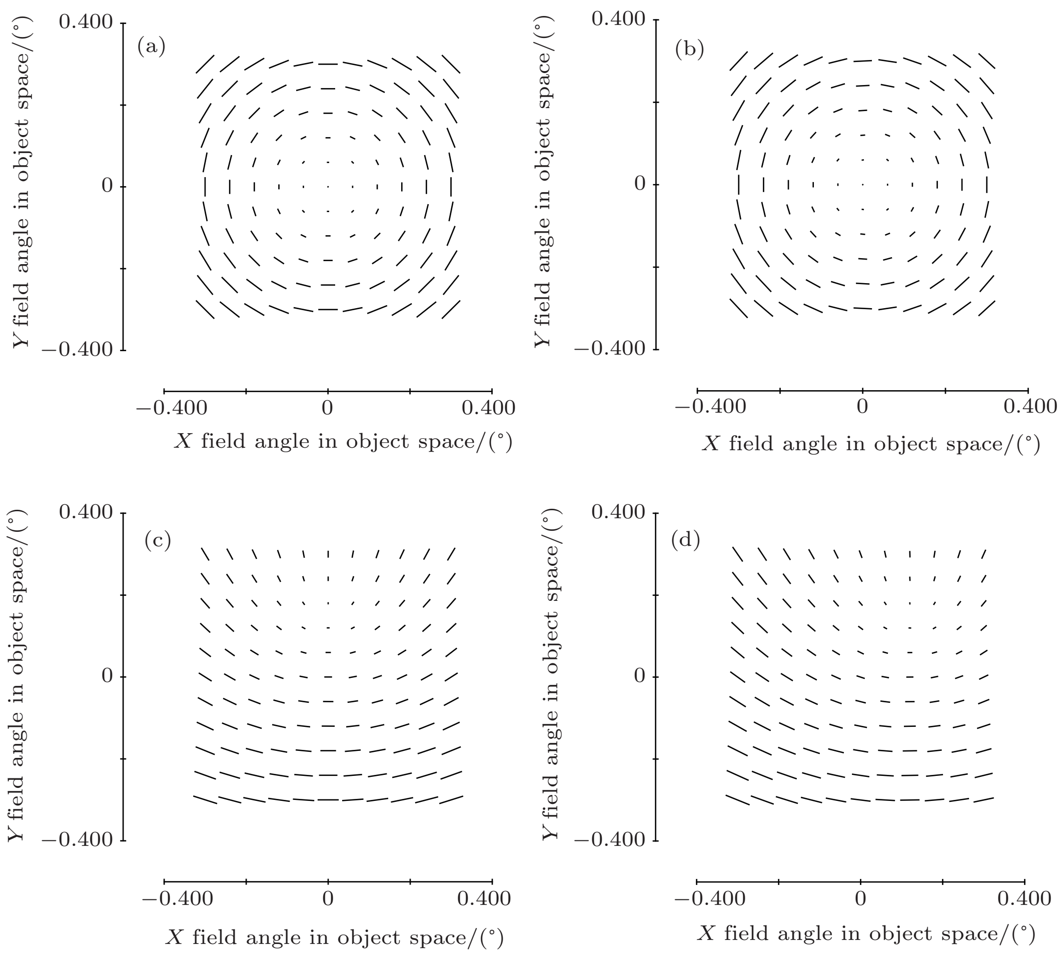

Fig. 6. Full field view of astigmatism in each situation: (a) no decentering, no tilting; (b) with decentering, no tilting; (c) with tilting, no decentering; and (d) with both decentering and tilting.

Table 2. Results calculated from CODEV.

X \ Y	–0.3		–0.15		0		0.15		0.3	
	Mag.	Ang.	Mag.	Ang.	Mag.	Ang.	Mag.	Ang.	Mag.	Ang.
0.3	0.055	124.6	0.037	120.7	0.023	110.7	0.018	85.8	0.029	65.9
0.15	0.052	134.3	0.033	–44.7	0.015	–43.0	0.003	40.5	0.022	46.0
0	0.057	–36.0	0.039	–30.5	0.025	–18.7	0.020	6.2	0.030	26.4
–0.15	0.068	–28.0	0.053	–21.2	0.042	–10.5	0.040	3.8	0.046	16.8
–0.3	0.082	–22.0	0.070	–15.7	0.063	–7.1	0.061	2.8	0.065	11.9

Comparing Tables 1 and 2, it can be seen that the results from formula (12) and from CODEV are almost the same. The full field of view (field aberration map) of each situation is plotted in Fig. 6, where panel (a) is the map with no decentering and no tilting; panel (b) is the map with decentering but no tilting; panel (c) is the map with tilting but no decentering; and panel (d) is the map with decentering and tilting simultaneously.

5. Conclusion

In this paper, through expanding the vector expressions of the third-order wavefront aberrations, the aberration field of a system with a tilted pupil is discussed. It is proved that, in the case of a tilted pupil, the third-order aberrations change with rotationally asymmetrical coefficients, unlike the case of a decentered pupil. Then the situation of a simultaneously decentered and tilted pupil is discussed; the aberration is found to be described by a combination of vectors of the two individual cases. These results will provide guidance for optical system design. Our next step will focus on the high-order aberrations introduced by rotationally asymmetrical surfaces.

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