Optics Communications 321 (2014) 211–218

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

# Study photonic crystals defect model property with quantum theory

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ARTICLE INFO

*Article history:*
Received 26 September 2013
Received in revised form
16 January 2014
Accepted 3 February 2014
Available online 14 February 2014

*Keywords:*
Photonic crystals
Quantum transmissivity
Quantum dispersion relation

ABSTRACT

In this paper, we have presented a quantum theory approach to study one-dimensional photonic crystals with and without defect layer. We give quantum dispersion relation, quantum transmissivity, reflectivity and absorptivity, and compare them with the classical dispersion relation, transmissivity, reflectivity and absorptivity. By the calculation, we find that the classical and quantum dispersion relation, transmissivity reflectivity and absorptivity are identical. With the quantum theory new approach, we can study two-dimensional and three-dimensional photonic crystals in the future.

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## 1. Introduction

Photonic crystals have generated a surge of interest in the last decades because they offer the possibility to control the propagation of light to an unprecedented level [1–4]. In its simplest form, a photonic crystal is an engineered inhomogeneous periodic structure made up of two or more materials with very different dielectric constants. When an electromagnetic wave (EM) propagates in such a structure whose period is comparable to the wavelength of the wave, unexpected behaviors occur. Among the most interesting ones are the possibility of forming a complete photonic band gap (CPBG) [5,6], which forbids the radiation propagation in a specific range of frequencies. The existence of PBGs will lead to many interesting phenomena. In the past ten years has been developed an intensive effort to study and micro-fabricate PBG materials in one, two or three dimensions, e.g., modification of spontaneous emission [7,8] and photon localization [9–12].

The existence of PBGs will lead to many interesting phenomena, e.g., modification of spontaneous emission [13,14] and photon localization [15–17]. Thus numerous applications of photonic crystals have been proposed in improving the performance of optoelectronic and microwave devices such as high-efficiency semiconductor lasers, right emitting diodes, wave guides, optical filters, high-Q resonators, antennas, frequency-selective surface, optical limiters and amplifiers [18,19]. Other applications of PCs have been proposed and designed in the SLED to realize high power [20–22]. These applications would be significantly enhanced if the band structure of the photonic crystal could be tuned.

The theory calculations of PCs have many numerical methods, such as the plane-wave expansion method (PWE) [23], the finite-difference time-domain method (FDTD) [24], the transfer matrix method (TMM) [25], the finite element method (FE) [26], the scattering matrix method [27], the Green's function method [28], etc. These methods are classical electromagnetism theory. Obviously, the full quantum theory of PCs is necessary. In Refs. [29,30], the authors give the quantum wave equation of single photon. In Ref. [31], we give the quantum wave equations of free and non-free photon. In this paper, we have studied the 1D PCs by the quantum wave equations of photon [31], and given quantum dispersion relation, quantum transmissivity, reflectivity and absorptivity, and compare them with the classical dispersion relation, transmissivity, reflectivity and absorptivity. By the calculation, we find that the classical and quantum dispersion relation, transmissivity reflectivity and absorptivity are identical. With the new approach, we can study two-dimensional and three-dimensional photonic crystals.

## 2. The quantum wave equation and probability current density of photon

The quantum wave equations of free and non-free photon have been obtained in Ref. [31], they are

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t) = c\hbar\nabla\times\vec{\psi}(\vec{r},t), \tag{1}$$

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http://dx.doi.org/10.1016/j.optcom.2014.02.012
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and

$$i\hbar\frac{\partial}{\partial t}\overrightarrow{\psi}(\overrightarrow{r},t)=c\hbar\nabla\times\overrightarrow{\psi}(\overrightarrow{r},t)+V\overrightarrow{\psi}(\overrightarrow{r},t), \tag{2}$$

where $\overrightarrow{\psi}(\overrightarrow{r},t)$ is the vector wave function of photon, and $V$ is the potential energy of photon in medium. In the medium of refractive index $n$, the photon's potential energy $V$ is [31]

$$V=\hbar\omega(1-n). \tag{3}$$

The conjugate of Eq. (2) is

$$-i\hbar\frac{\partial}{\partial t}\overrightarrow{\psi}^*(\overrightarrow{r},t)=c\hbar\nabla\times\overrightarrow{\psi}^*(\overrightarrow{r},t)+V\overrightarrow{\psi}^*(\overrightarrow{r},t). \tag{4}$$

Multiplying Eq. (2) by $\overrightarrow{\psi}^*$, Eq. (4) by $\overrightarrow{\psi}$, and taking the difference, we get

$$i\hbar\frac{\partial}{\partial t}(\overrightarrow{\psi}^*\cdot\overrightarrow{\psi})=c\hbar(\overrightarrow{\psi}^*\cdot\nabla\times\overrightarrow{\psi}-\overrightarrow{\psi}\cdot\nabla\times\overrightarrow{\psi}^*)=c\hbar\nabla\cdot(\overrightarrow{\psi}\times\overrightarrow{\psi}^*), \tag{5}$$

i.e.

$$\frac{\partial\rho}{\partial t}+\nabla\cdot J=0, \tag{6}$$

where

$$\rho=\overrightarrow{\psi}^*\cdot\overrightarrow{\psi}, \tag{7}$$

and

$$J=ic\overrightarrow{\psi}\times\overrightarrow{\psi}^*, \tag{8}$$

are the probability density and probability current density, respectively.

By the method of separation variable

$$\overrightarrow{\psi}(\overrightarrow{r},t)=\overrightarrow{\psi}(\overrightarrow{r})f(t), \tag{9}$$

the time-dependent equation (2) becomes the time-independent equation

$$c\hbar\nabla\times\overrightarrow{\psi}(\overrightarrow{r})+V\overrightarrow{\psi}(\overrightarrow{r})=E\overrightarrow{\psi}(\overrightarrow{r}), \tag{10}$$

where $E$ is the energy of photon in medium.

By taking curl in (10), when $\partial V/\partial x_i=0, (i=1,2,3)$, Eq. (10) becomes

$$(\hbar c)^2(\nabla(\nabla\cdot\overrightarrow{\psi}(\overrightarrow{r}))-\nabla^2\overrightarrow{\psi}(\overrightarrow{r}))=(E-V)^2\overrightarrow{\psi}(\overrightarrow{r}). \tag{11}$$

Choosing transverse gange

$$\nabla\cdot\overrightarrow{\psi}(\overrightarrow{r})=0, \tag{12}$$

Eq. (11) becomes

$$\nabla^2\overrightarrow{\psi}(\overrightarrow{r})+\left(\frac{E-V}{\hbar c}\right)^2\overrightarrow{\psi}(\overrightarrow{r})=0. \tag{13}$$

With Eqs. (12) and (13), we should study one-dimensional PCs by the quantum theory approach.

## 3. The quantum theory of one-dimensional photonic crystals

For one-dimensional photonic crystals, we should define and calculate its quantum dispersion relation and quantum transmissivity. The one-dimensional PCs structure is shown in Fig. 1.

In Fig. 1, $\overrightarrow{\psi}_I$, $\overrightarrow{\psi}_R$, $\overrightarrow{\psi}_T$ are the wave functions of incident, reflection and transmission photon, respectively, and they can be written as

$$\overrightarrow{\psi}(\overrightarrow{r},t)=\overrightarrow{\psi}_0e^{i(\overrightarrow{k}\cdot\overrightarrow{r}-\omega t)}=\psi_x\overrightarrow{i}+\psi_y\overrightarrow{j}+\psi_z\overrightarrow{k}, \tag{14}$$

By transverse gange $\nabla\cdot\overrightarrow{\psi}(\overrightarrow{r})=0$, we get

$$k_x\psi_x+k_y\psi_y+k_z\psi_z=0. \tag{15}$$

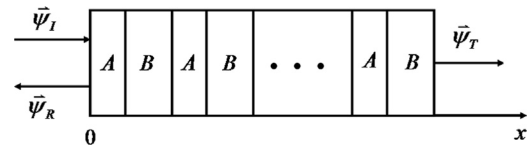


**Fig. 1.** The structure of one-dimensional photonic crystals.

In Fig. 1, the photon travels along with the $x$-axis, the wave vector $k_y=k_z=0$ and $k_x\neq 0$. By Eq. (15), we have

$$\psi_x=0, \tag{16}$$

so the total wave function of photon is

$$\overrightarrow{\psi}=\overrightarrow{\psi}_y\overrightarrow{j}+\overrightarrow{\psi}_z\overrightarrow{k}, \tag{17}$$

Eq. (13) becomes two component equations

$$\nabla^2\psi_y+\left(\frac{E-V}{\hbar c}\right)^2\psi_y=0, \tag{18}$$

and

$$\nabla^2\psi_z+\left(\frac{E-V}{\hbar c}\right)^2\psi_z=0. \tag{19}$$

In Fig. 1, the wave functions of incident, reflection and transmission photon can be written as

$$\overrightarrow{\psi_I}=F_ye^{i(\overrightarrow{k}\cdot\overrightarrow{r}-\omega t)}\overrightarrow{j}+F_ze^{i(\overrightarrow{k}\cdot\overrightarrow{r}-\omega t)}\overrightarrow{k}, \tag{20}$$

$$\overrightarrow{\psi_R}=F'_ye^{i(\overrightarrow{k}\cdot\overrightarrow{r}-\omega t)}\overrightarrow{j}+F'_ze^{i(\overrightarrow{k}\cdot\overrightarrow{r}-\omega t)}\overrightarrow{k}, \tag{21}$$

$$\overrightarrow{\psi_T}=D_ye^{i(\overrightarrow{k}\cdot\overrightarrow{r}-\omega t)}\overrightarrow{j}+D_ze^{i(\overrightarrow{k}\cdot\overrightarrow{r}-\omega t)}\overrightarrow{k}, \tag{22}$$

where $F_y$, $F_z$, $F'_y$, $F'_z$, $D_y$, and $D_z$ are their amplitudes.

The component form of Eq. (1) is

$$\begin{cases} i\hbar\frac{\partial}{\partial t}\psi_x=\hbar c\left(\frac{\partial\psi_z}{\partial y}-\frac{\partial\psi_y}{\partial z}\right) \\ i\hbar\frac{\partial}{\partial t}\psi_y=\hbar c\left(\frac{\partial\psi_x}{\partial z}-\frac{\partial\psi_z}{\partial x}\right), \\ i\hbar\frac{\partial}{\partial t}\psi_z=\hbar c\left(\frac{\partial\psi_y}{\partial x}-\frac{\partial\psi_x}{\partial y}\right) \end{cases} \tag{23}$$

substituting Eqs. (14) and (16) into (23), we have

$$\psi_z=i\psi_y, \tag{24}$$

the probability current density becomes

$$J=ic\overrightarrow{\psi}\times\overrightarrow{\psi}^*=2c|\psi_z|^2\overrightarrow{i}=2c|\psi_{0z}|^2\overrightarrow{i}, \tag{25}$$

where

$$\psi_z=\psi_{0z}e^{i(\overrightarrow{k}\cdot\overrightarrow{r}-\omega t)}, \tag{26}$$

the $\psi_{0z}$ is $\psi_z$ amplitude.

For the incident, reflection and transmission photon, their probability current density $J_I$, $J_R$, $J_T$ are

$$J_I=2c|F_z|^2, \tag{27}$$

$$J_R=2c|F'_z|^2, \tag{28}$$

$$J_T=2c|D_z|^2, \tag{29}$$

We can define quantum transmissivity $T$ and quantum reflectivity $R$ as

$$T=\frac{J_T}{J_I}=\left|\frac{D_z}{F_z}\right|^2, \tag{30}$$

$$R=\frac{J_R}{J_I}=\left|\frac{F'_z}{F_z}\right|^2. \tag{31}$$

## 4. The quantum transmissivity and quantum dispersion relation

Since the probability current densities are relevant to the $z$ component amplitudes of wave function, we should only solve the $z$ component equation (19) for the one-dimensional PCs, which is shown in Fig. 2.

With Eq. (19), the photon's quantum wave equation in mediums $A$ and $B$ are

$$\frac{\partial^2\psi_A}{\partial x^2}+k_A^2\psi_A=0 \quad (0<x<a), \tag{32}$$

$$\frac{\partial^2\psi_B}{\partial x^2}+k_B^2\psi_B=0 \quad (a<x<a+b), \tag{33}$$

where

$$k_A=\frac{E-V_a}{\hbar c}=\frac{E-\hbar\omega(1-n_a)}{\hbar c}=\frac{\omega}{c}n_a=\frac{2\pi}{\lambda}n_a, \tag{34}$$

$$k_B=\frac{E-V_b}{\hbar c}=\frac{E-\hbar\omega(1-n_b)}{\hbar c}=\frac{\omega}{c}n_b=\frac{2\pi}{\lambda}n_b, \tag{35}$$

where $\lambda=2\pi c/\omega$ is the photon wave length in vacuum, $V_a=\hbar\omega(1-n_a)(V_b=\hbar\omega(1-n_b))$ is the potential energy of photon in medium $A(B)$, and $n_a(n_b)$ is the refractive index of medium $A(B)$. In order to simplify, the index $z$ is omitted, i.e., $\psi_{zA}(\psi_{zB})$ is written as $\psi_A(\psi_B)$. The solutions of Eqs. (32) and (33) are

$$\psi_A=A_1e^{ik_Ax}+A_2e^{-ik_Ax} \quad (0<x<a), \tag{36}$$

$$\psi_B=B_1e^{ik_Bx}+B_2e^{-ik_Bx} \quad (a<x<a+b). \tag{37}$$

By Bloch law, there is

$$\begin{aligned}\psi(a+b<x<2a+b)&=\psi(0<x<a)e^{ik(a+b)}\\&=(A_1e^{ik_A(x-(a+b))}\\&\quad+A_2e^{-ik_A(x-(a+b))})e^{ik(a+b)},\end{aligned} \tag{38}$$

where $k$ is Bloch wave vector.

At $x=a$, by the continuation of wave function and its derivative, we have

$$A_1e^{ik_Aa}+A_2e^{-ik_Aa}=B_1e^{ik_Ba}+B_2e^{-ik_Ba}, \tag{39}$$

$$ik_AA_1e^{ik_Aa}-ik_AA_2e^{-ik_Aa}=ik_BB_1e^{ik_Ba}-ik_BB_2e^{-ik_Ba}, \tag{40}$$

At $x=a+b$, by the continuation of wave function and its derivative, we have

$$A_1e^{ik(a+b)}+A_2e^{ik(a+b)}=B_1e^{ik_B(a+b)}+B_2e^{-ik_B(a+b)}, \tag{41}$$

$$ik_AA_1e^{ik(a+b)}-ik_AA_2e^{ik(a+b)}=ik_BB_1e^{ik_B(a+b)}-ik_BB_2e^{-ik_B(a+b)}, \tag{42}$$

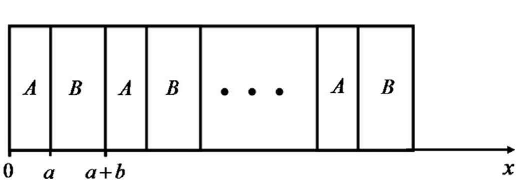


**Fig. 2.** The structure of one-dimensional photonic crystals.

and we obtain the following equations set:

$$\begin{cases}A_1e^{ik_Aa}+A_2e^{-ik_Aa}=B_1e^{ik_Ba}+B_2e^{-ik_Ba}\\ik_AA_1e^{ik_Aa}-ik_AA_2e^{-ik_Aa}=ik_BB_1e^{ik_Ba}-ik_BB_2e^{-ik_Ba}\\A_1e^{ik(a+b)}+A_2e^{ik(a+b)}=B_1e^{ik_B(a+b)}+B_2e^{-ik_B(a+b)}\\ik_AA_1e^{ik(a+b)}-ik_AA_2e^{ik(a+b)}=ik_BB_1e^{ik_B(a+b)}-ik_BB_2e^{-ik_B(a+b)},\end{cases} \tag{43}$$

the necessary and sufficient condition of Eq. (43) nonzero solution is its coefficient determinant equal to zero

$$\begin{vmatrix}e^{ik_Aa} & e^{-ik_Aa} & -e^{ik_Ba} & -e^{-ik_Ba}\\k_Ae^{ik_Aa} & -k_Ae^{-ik_Aa} & -k_Be^{ik_Ba} & k_Be^{-ik_Ba}\\e^{ik(a+b)} & e^{ik(a+b)} & -e^{ik_B(a+b)} & -e^{-ik_B(a+b)}\\k_Ae^{ik(a+b)} & -k_Ae^{ik(a+b)} & -k_Be^{ik_B(a+b)} & k_Be^{-ik_B(a+b)}\end{vmatrix}=0, \tag{44}$$

simplifying Eq. (44), we obtain the quantum dispersion relation

$$\cos(k(a+b))=\cos(k_Aa)\cos(k_Bb)-\frac{1}{2}\left(\frac{1}{k_A}+\frac{1}{k_B}\right)\sin(k_Aa)\sin(k_Bb). \tag{45}$$

In the following, we should give the wave function of photon in every medium, and the transmission wave function. In Fig. 3, we give the simplification form of wave function in every medium, such as symbols $A_{k_A}^1$ and $A_{-k_A}^1$ express simplifying wave function of medium $A$ in the first period, they express wave function

$$\psi_{A^1}(x)=A_{k_A}^1e^{ik_Ax}+A_{-k_A}^1e^{-ik_Ax}, \tag{46}$$

in medium $B$ of first period, the symbols $B_{k_A}^1$ and $B_{-k_A}^1$ express wave function

$$\psi_{B^1}(x)=B_{k_B}^1e^{ik_Bx}+B_{-k_B}^1e^{-ik_Bx}, \tag{47}$$

in medium $A$ of second period, the symbols $A_{k_A}^2$ and $A_{-k_A}^2$ express wave function

$$\psi_{A^2}(x)=A_{k_A}^2e^{ik_Ax}+A_{-k_A}^2e^{-ik_Ax}, \tag{48}$$

similarly, in medium $B$ of second period, the symbols $B_{k_A}^2$ and $B_{-k_A}^2$ express wave function

$$\psi_{B^2}(x)=B_{k_B}^2e^{ik_Bx}+B_{-k_B}^2e^{-ik_Bx}, \tag{49}$$

and so on.

In the incident area, the total wave function $\psi_{tot}(x)$ is the superposition of incident and reflection wave function, it is

$$\psi_{tot}(x)=\psi_I(x)+\psi_R(x)=Fe^{iKx}+F'e^{-iKx}, \tag{50}$$

where $K$ is the wave vector of incident, reflection, and transmission photon. In the following, we should use the condition of wave function and its derivative continuation at interface of two mediums.

(1) At $x=0$, by the continuation of wave function and its derivative, we have

$$F+F'=A_{k_A}^1+A_{-k_A}^1, \tag{51}$$

$$iKF-iKF'=ik_AA_{k_A}^1-ik_AA_{-k_A}^1, \tag{52}$$

we obtain

$$A_{k_A}^1=\frac{1}{2}\left[\left(1+\frac{K}{k_A}\right)F+\left(1-\frac{K}{k_A}\right)F'\right], \tag{53}$$

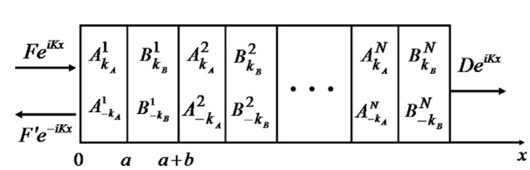


**Fig. 3.** The quantum structure of one-dimensional photonic crystals.

$$A^1_{-k_A}=\frac{1}{2}\left[\left(1-\frac{K}{k_A}\right)F+\left(1+\frac{K}{k_A}\right)F'\right], \tag{54}$$

Eqs. (53) and (54) can be written as matrix form

$$\begin{pmatrix} A^1_{k_A} \\ A^1_{-k_A} \end{pmatrix}=\frac{1}{2}\begin{pmatrix} 1+K/k_A & 1-K/k_A \\ 1-K/k_A & 1+K/k_A \end{pmatrix}\begin{pmatrix} F \\ F' \end{pmatrix}=M^1_A\begin{pmatrix} F \\ F' \end{pmatrix}, \tag{55}$$

where $M^1_A$ is the quantum transform matrix of the first period medium $A$, it is

$$M^1_A=\frac{1}{2}\begin{pmatrix} 1+K/k_A & 1-K/k_A \\ 1-K/k_A & 1+K/k_A \end{pmatrix}, \tag{56}$$

(2) At $x=a$, by the continuation of wave function and its derivative, we have

$$A^1_{k_A}e^{ik_Aa}+A^1_{-k_A}e^{-ik_Aa}=B^1_{k_B}e^{ik_Ba}+B^1_{-k_B}e^{-ik_Ba}, \tag{57}$$

$$\frac{k_A}{k_B}(A^1_{k_A}e^{ik_Aa}-A^1_{-k_A}e^{-ik_Aa})=B^1_{k_B}e^{ik_Ba}-B^1_{-k_B}e^{-ik_Ba}, \tag{58}$$

we get

$$B^1_{k_B}=\frac{1}{2}e^{i(k_A-k_B)a}\left(1+\frac{k_A}{k_B}\right)A^1_{k_A}+\frac{1}{2}e^{-i(k_A+k_B)a}\left(1-\frac{k_A}{k_B}\right)A^1_{-k_A}, \tag{59}$$

$$B^1_{-k_B}=\frac{1}{2}e^{i(k_A+k_B)a}\left(1-\frac{k_A}{k_B}\right)A^1_{k_A}+\frac{1}{2}e^{i(k_B-k_A)a}\left(1+\frac{k_A}{k_B}\right)A^1_{-k_A}, \tag{60}$$

Eqs. (59) and (60) can be written as matrix form

$$\begin{pmatrix} B^1_{k_B} \\ B^1_{-k_B} \end{pmatrix}=\frac{1}{2}\begin{pmatrix} e^{i(k_A-k_B)a}(1+k_A/k_B) & e^{-i(k_A+k_B)a}(1-k_A/k_B) \\ e^{i(k_A+k_B)a}(1-k_A/k_B) & e^{i(k_B-k_A)a}(1+k_A/k_B) \end{pmatrix}\begin{pmatrix} A^1_{kA} \\ A^1_{-kA} \end{pmatrix}$$
$$=M^1_B\begin{pmatrix} A^1_{kA} \\ A^1_{-kA} \end{pmatrix}, \tag{61}$$

where $M^1_B$ is the quantum transform matrix of the first period medium $B$, it is

$$M^1_B=\frac{1}{2}\begin{pmatrix} e^{i(k_A-k_B)a}(1+k_A/k_B) & e^{-i(k_A+k_B)a}(1-k_A/k_B) \\ e^{i(k_A+k_B)a}(1-k_A/k_B) & e^{i(k_B-k_A)a}(1+k_A/k_B) \end{pmatrix}, \tag{62}$$

(3) At $x=a+b$, by the continuation of wave function and its derivative, we have

$$B^1_{k_B}e^{ik_B(a+b)}+B^1_{-k_B}e^{-ik_B(a+b)}=A^2_{k_A}e^{ik_A(a+b)}+A^2_{-k_A}e^{-ik_A(a+b)}, \tag{63}$$

$$\frac{k_B}{k_A}(B^1_{k_B}e^{ik_B(a+b)}-B^1_{-k_B}e^{-ik_B(a+b)})=A^2_{k_A}e^{ik_A(a+b)}-A^2_{-k_A}e^{-ik_A(a+b)}, \tag{64}$$

we get

$$A^2_{k_A}=\frac{1}{2}e^{i(k_B-k_A)(a+b)}\left(1+\frac{k_B}{k_A}\right)B^1_{k_B}+\frac{1}{2}e^{-i(k_A+k_B)(a+b)}\left(1-\frac{k_B}{k_A}\right)B^1_{-k_B}, \tag{65}$$

$$A^2_{-k_A}=\frac{1}{2}e^{i(k_A+k_B)(a+b)}\left(1-\frac{k_B}{k_A}\right)B^1_{k_B}+\frac{1}{2}e^{i(k_A-k_B)(a+b)}\left(1+\frac{k_B}{k_A}\right)B^1_{-k_B}, \tag{66}$$

Eqs. (65) and (66) can be written as matrix form

$$\begin{pmatrix} A^2_{k_A} \\ A^2_{-k_A} \end{pmatrix}=\frac{1}{2}\begin{pmatrix} e^{i(k_B-k_A)(a+b)}(1+k_B/k_A) & e^{-i(k_A+k_B)(a+b)}(1-k_B/k_A) \\ e^{i(k_A+k_B)(a+b)}(1-k_B/k_A) & e^{i(k_A-k_B)(a+b)}(1+k_B/k_A) \end{pmatrix}\begin{pmatrix} B^1_{kB} \\ B^1_{-kB} \end{pmatrix}$$
$$=M^2_A\begin{pmatrix} B^1_{kB} \\ B^1_{-kB} \end{pmatrix}, \tag{67}$$

where $M^2_A$ is the quantum transform matrix of the second period medium $A$, it is

$$M^2_A=\frac{1}{2}\begin{pmatrix} e^{i(k_B-k_A)(a+b)}(1+k_B/k_A) & e^{-i(k_A+k_B)(a+b)}(1-k_B/k_A) \\ e^{i(k_A+k_B)(a+b)}(1-k_B/k_A) & e^{i(k_A-k_B)(a+b)}(1+k_B/k_A) \end{pmatrix}, \tag{68}$$

(4) At $x=2a+b$, by the continuation of wave function and its derivative, we get

$$\begin{pmatrix} B^2_{k_B} \\ B^2_{-k_B} \end{pmatrix}=\frac{1}{2}\begin{pmatrix} e^{i(k_A-k_B)(2a+b)}(1+k_A/k_B) & e^{-i(k_A+k_B)(2a+b)}(1-k_A/k_B) \\ e^{i(k_A+k_B)(2a+b)}(1-k_A/k_B) & e^{i(k_B-k_A)(2a+b)}(1+k_A/k_B) \end{pmatrix}\begin{pmatrix} A^2_{kA} \\ A^2_{-kA} \end{pmatrix}$$
$$=M^2_B\begin{pmatrix} A^2_{kA} \\ A^2_{-kA} \end{pmatrix}, \tag{69}$$

where $M^2_B$ is the quantum transform matrix of the second period medium $B$, it is

$$M^2_B=\frac{1}{2}\begin{pmatrix} e^{i(k_A-k_B)(2a+b)}(1+k_A/k_B) & e^{-i(k_A+k_B)(2a+b)}(1-k_A/k_B) \\ e^{i(k_A+k_B)(2a+b)}(1-k_A/k_B) & e^{i(k_B-k_A)(2a+b)}(1+k_A/k_B) \end{pmatrix}, \tag{70}$$

(5) At $x=2(a+b)$, by the continuation of wave function and its derivative, we get

$$\begin{pmatrix} A^3_{k_A} \\ A^3_{-k_A} \end{pmatrix}=\frac{1}{2}\begin{pmatrix} e^{i(k_B-k_A)2(a+b)}(1+k_B/k_A) & e^{-i(k_A+k_B)2(a+b)}(1-k_B/k_A) \\ e^{i(k_A+k_B)2(a+b)}(1-k_B/k_A) & e^{i(k_A-k_B)2(a+b)}(1+k_B/k_A) \end{pmatrix}\begin{pmatrix} B^2_{kB} \\ B^2_{-kB} \end{pmatrix}$$
$$=M^3_A\begin{pmatrix} B^2_{kB} \\ B^2_{-kB} \end{pmatrix}, \tag{71}$$

where $M^3_A$ is the quantum transform matrix of the third period medium $A$, it is

$$M^3_A=\frac{1}{2}\begin{pmatrix} e^{i(k_B-k_A)2(a+b)}(1+k_B/k_A) & e^{-i(k_A+k_B)2(a+b)}(1-k_B/k_A) \\ e^{i(k_A+k_B)2(a+b)}(1-k_B/k_A) & e^{i(k_A-k_B)2(a+b)}(1+k_B/k_A) \end{pmatrix} \tag{72}$$

(6) Similarly, at $x=3a+2b$, by the continuation of wave function and its derivative, we get

$$\begin{pmatrix} B^3_{k_B} \\ B^3_{-k_B} \end{pmatrix}=\frac{1}{2}\begin{pmatrix} e^{i(k_A-k_B)(3a+2b)}(1+k_A/k_B) & e^{-i(k_A+k_B)(3a+2b)}(1-k_A/k_B) \\ e^{i(k_A+k_B)(3a+2b)}(1-k_A/k_B) & e^{i(k_B-k_A)(3a+2b)}(1+k_A/k_B) \end{pmatrix}$$
$$\begin{pmatrix} A^3_{kA} \\ A^3_{-kA} \end{pmatrix}=M^3_B\begin{pmatrix} A^3_{kA} \\ A^3_{-kA} \end{pmatrix}, \tag{73}$$

where $M^3_B$ is the quantum transform matrix of the third period medium $B$, it is

$$M^3_B=\frac{1}{2}\begin{pmatrix} e^{i(k_A-k_B)(3a+2b)}(1+k_A/k_B) & e^{-i(k_A+k_B)(3a+2b)}(1-k_A/k_B) \\ e^{i(k_A+k_B)(3a+2b)}(1-k_A/k_B) & e^{i(k_B-k_A)(3a+2b)}(1+k_A/k_B) \end{pmatrix}. \tag{74}$$

By the above calculation, we can obtain the results of transform matrices:

(1) For the transform matrix $M^1_A$ of the first period medium $A$ is independent form.
(2) For the transform matrices $M^N_A$ of the $N$-th period $(N\geq 2)$, they can be written as

$$M^N_A=\frac{1}{2}\begin{pmatrix} e^{i(k_B-k_A)(N-1)(a+b)}(1+k_B/k_A) & e^{-i(k_A+k_B)(N-1)(a+b)}(1-k_B/k_A) \\ e^{i(k_A+k_B)(N-1)(a+b)}(1-k_B/k_A) & e^{i(k_A-k_B)(N-1)(a+b)}(1+k_B/k_A) \end{pmatrix}, \tag{75}$$

(3) For the transform matrices $M_B^N$ of the $N$-th period ($N \geq 1$), they can be written as

$$M_B^N = \frac{1}{2}\begin{pmatrix} e^{i(k_A-k_B)(N(a+b)-b)}(1+k_A/k_B) & e^{-i(k_A+k_B)(N(a+b)-b)}(1-k_A/k_B) \\ e^{i(k_A+k_B)(N(a+b)-b)}(1-k_A/k_B) & e^{i(k_B-k_A)(N(a+b)-b)}(1+k_A/k_B) \end{pmatrix}. \quad (76)$$

By the quantum transform matrices, we can give their relations:

(1) The representation of the first period quantum transform matrices is

$$\begin{pmatrix} A_{k_A}^1 \\ A_{-k_A}^1 \end{pmatrix} = M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix}, \quad (77)$$

$$\begin{pmatrix} B_{k_B}^1 \\ B_{-k_B}^1 \end{pmatrix} = M_B^1 \begin{pmatrix} A_{kA}^1 \\ A_{-kA}^1 \end{pmatrix} = M_B^1 M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix} = M^1 \begin{pmatrix} F \\ F' \end{pmatrix}. \quad (78)$$

(2) The representation of the second period quantum transform matrices is

$$\begin{pmatrix} A_{k_A}^2 \\ A_{-k_A}^2 \end{pmatrix} = M_A^2 \begin{pmatrix} B_{kB}^1 \\ B_{-kB}^1 \end{pmatrix} = M_A^2 M_B^1 M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix} = M_A^2\, M^1 \begin{pmatrix} F \\ F' \end{pmatrix}, \quad (79)$$

$$\begin{pmatrix} B_{k_B}^2 \\ B_{-k_B}^2 \end{pmatrix} = M_B^2 \begin{pmatrix} A_{kA}^2 \\ A_{-kA}^2 \end{pmatrix} = M_B^2 M_A^2 M_B^1 M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix} = M^2 M^1 \begin{pmatrix} F \\ F' \end{pmatrix}. \quad (80)$$

(3) Similarly, the representation of the $N$-th period quantum transform matrices is

$$\begin{pmatrix} A_{k_A}^N \\ A_{-k_A}^N \end{pmatrix} = M_A^N M_B^{N-1} M_A^{N-1} \cdots M_A^2 M_B^1 M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix} = M_A^N M^{N-1} \cdots M^2\, M^1 \begin{pmatrix} F \\ F' \end{pmatrix}, \quad (81)$$

$$\begin{pmatrix} B_{k_B}^N \\ B_{-k_B}^N \end{pmatrix} = M_B^N M_A^N M_B^{N-1} M_A^{N-1} \cdots M_A^2 M_B^1 M_A^1 \begin{pmatrix} F \\ F' \end{pmatrix} = M^N M^{N-1} \cdots M^2 M^1 \begin{pmatrix} F \\ F' \end{pmatrix} = M \begin{pmatrix} F \\ F' \end{pmatrix}, \quad (82)$$

where

$$M = M^N M^{N-1} \cdots M^2 M^1 = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}, \quad (83)$$

is the total quantum transform matrix of N period, and $M^1 = M_B^1 M_A^1$ is the first period quantum transform matrix, $M^2 = M_B^2 M_A^2$ is the second period quantum transform matrix, and $M^N = M_B^N M_A^N$ is the $N$-th period quantum transform matrix.

By Eqs. (82) and (83), we can give the wave function of $N$-th period in medium $B$, it is

$$\psi_B^N(x) = B_{kB}^N e^{ik_B x} + B_{-kB}^N e^{-ik_B x} = (m_1 F + m_2 F')e^{ik_B x} + (m_3 F + m_4 F')e^{-ik_B x}. \quad (84)$$

In Fig. 3, the transmission wave function is

$$\psi_D(x) = De^{iKx}. \quad (85)$$

At $x = N(a+b)$, by the continuation of wave function and its derivative, we have

$$(m_1 F + m_2 F')e^{ik_B N(a+b)} + (m_3 F + m_4 F')e^{-ik_B N(a+b)} = De^{iKN(a+b)}, \quad (86)$$

and

$$\frac{k_B}{K}(m_1 F + m_2 F')e^{ik_B N(a+b)} - \frac{k_B}{K}(m_3 F + m_4 F')e^{-ik_B N(a+b)} = De^{iKN(a+b)}, \quad (87)$$

we can obtain

$$r = \frac{F'}{F} = \frac{m_1(K-k_B)e^{ik_B N(a+b)} + m_3(K+k_B)e^{-ik_B N(a+b)}}{m_2(k_B-K)e^{ik_B N(a+b)} - m_4(K+k_B)e^{-ik_B N(a+b)}}, \quad (88)$$

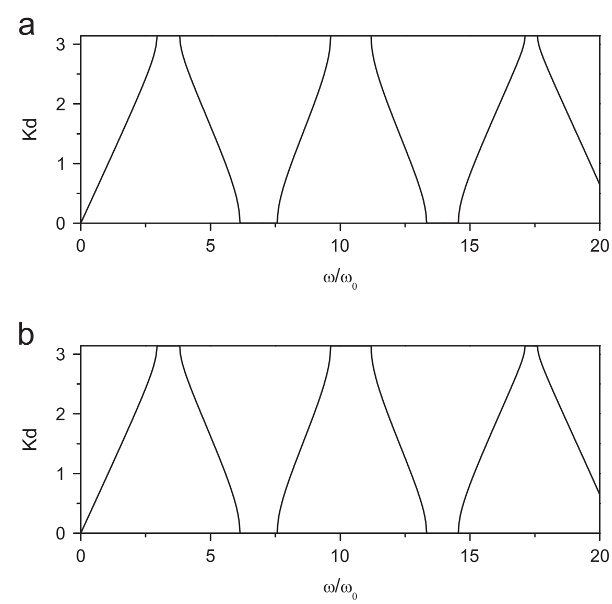


**Fig. 4.** Comparing quantum dispersion relation (a) with classical dispersion relation (b).

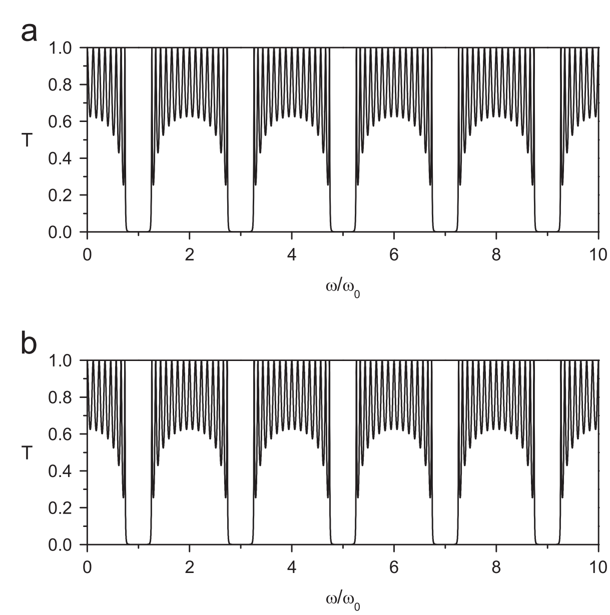


**Fig. 5.** Comparing quantum transmissivity (a) with classical transmissivity (b) without defect layer.

By Eqs. (86)–(88), we have

$$t=\frac{D}{F}=\left(m_1+m_2\frac{F'}{F}\right)e^{i(k_B-K)N(a+b)}+\left(m_3+m_4\frac{F'}{F}\right)e^{-i(k_B+K)N(a+b)}, \tag{89}$$

and the quantum transmissivity $T$, quantum reflectivity $R$ and quantum absorptivity are

$$T=|t|^2, \tag{90}$$

$$R=|r|^2, \tag{91}$$

$$A=1-T-R. \tag{92}$$

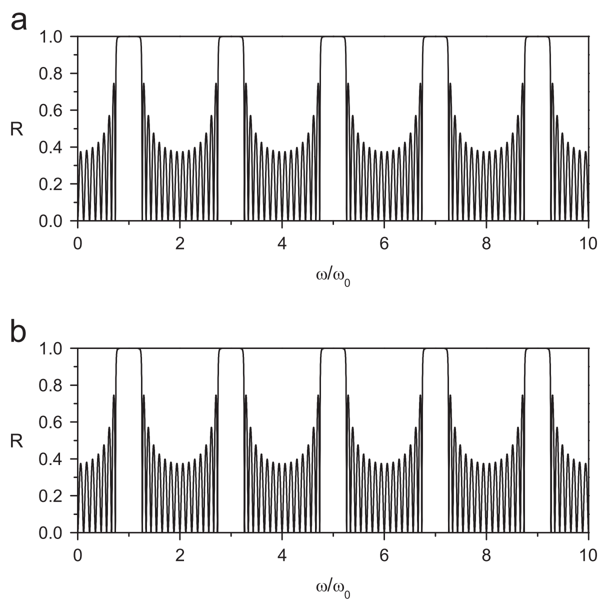


**Fig. 6.** Comparing quantum reflectivity (a) with classical reflectivity (b) without defect layer.

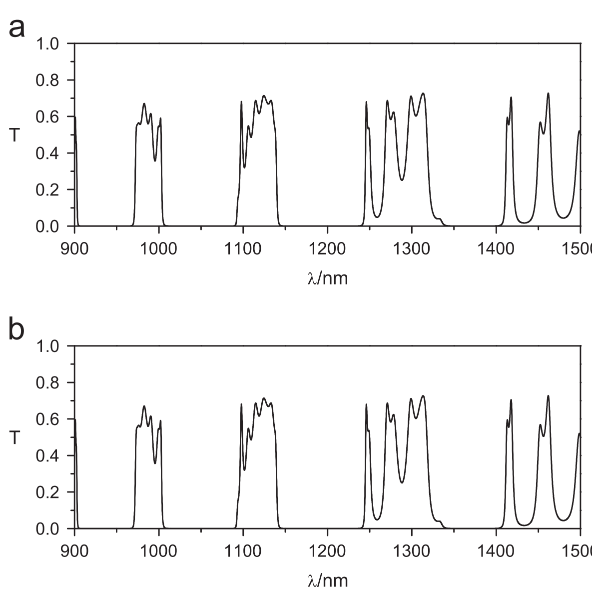


**Fig. 7.** Comparing quantum transmissivity (a) with classical transmissivity (b) with defect layer.

## 5. Numerical result

In this section, we report our numerical results of quantum transmissivity and quantum dispersion relation with defect layer and without defect layer. Firstly, we consider without defect layer, the main parameters are: medium $B$ refractive index is $n_b$=2.97, and its thickness is $b=130$ nm. The medium $A$ refractive index is $n_a$=1.40, and its thickness is $a=277$ nm. The central frequency is $\omega_0=271$ THz, and the periodicity $N$=8. In numerical calculation, we compare quantum dispersion relation, quantum transmissivity and quantum reflectivity with classical dispersion relation, transmissivity and reflectivity. With Eq. (45), we can study the quantum dispersion relation, and compare it with classical dispersion relation, which is shown in Fig. 4. In Fig. 4(a) and (b), quantum dispersion relation and classical dispersion relation, respectively, are shown. We can find that the dispersion relation of classical and quantum is identical. With Eqs. (88)–(91), we can calculate the quantum transmissivity and reflectivity, and compare it with

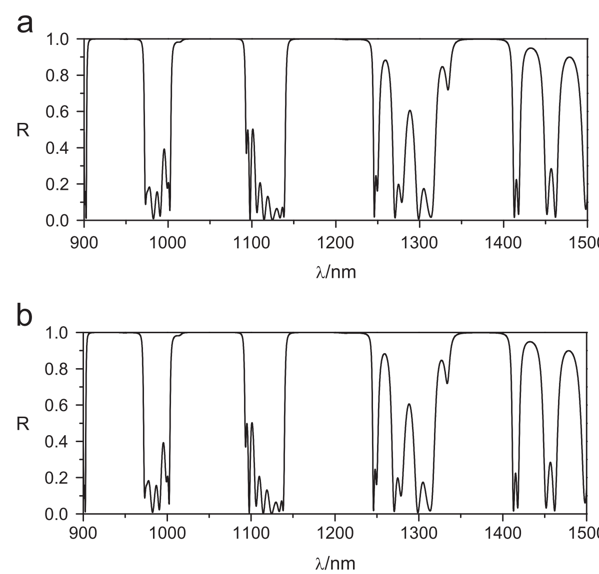


**Fig. 8.** Comparing quantum reflectivity (a) with classical reflectivity (b) with defect layer.

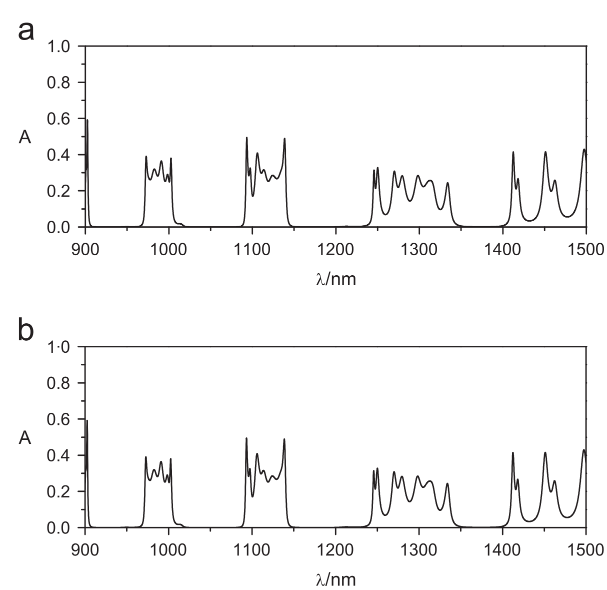


**Fig. 9.** Comparing quantum absorptivity (a) with classical absorptivity (b) with defect layer.

classical transmissivity and reflectivity, which are shown in Figs. 5 and 6. Fig. 5(a) (6(a)) and Fig. 5(b) (6(b)) are quantum transmissivity (reflectivity) and classical transmissivity (reflectivity), respectively. We can find that the transmissivity and reflectivity of classical and quantum are identical. Then, we consider with defect layer, the main parameters are: medium $B$ refractive index is $n_b=1.34$, and its thickness is $b=576$ nm. The medium $A$ refractive index is $n_a=4.86$, and its thickness is $a=810$ nm. The defect layer $D$ refractive index is $n_d=1.66+0.03i$, i.e., plural refractive indices of positive imaginary part, and its thickness is $d=1000$ nm. The central frequency is $\omega_0=271$ THz, and the periodicity $N=8$. In numerical calculation, we compare quantum transmissivity, reflectivity and absorptivity with classical transmissivity, reflectivity and absorptivity, which are shown in Figs. 7–9, respectively. We can find that the transmissivity, reflectivity and absorptivity of classical and quantum are identical. In Fig. 10, the defect layer $D$ refractive index is $n_d=1.66$, i.e., real refractive indices. We can find that the transmissivity of classical and quantum is identical, and the magnitude and position of defect model are identical. In Figs. 11–12, the defect layer $D$ refractive index is $n_d=2.06-0.01i$, i.e., plural refractive indices of negative imaginary part. We can find that the transmissivity is larger than 1, and the absorptivity is smaller than 0, and the transmissivity and absorptivity of classical and quantum are identical.

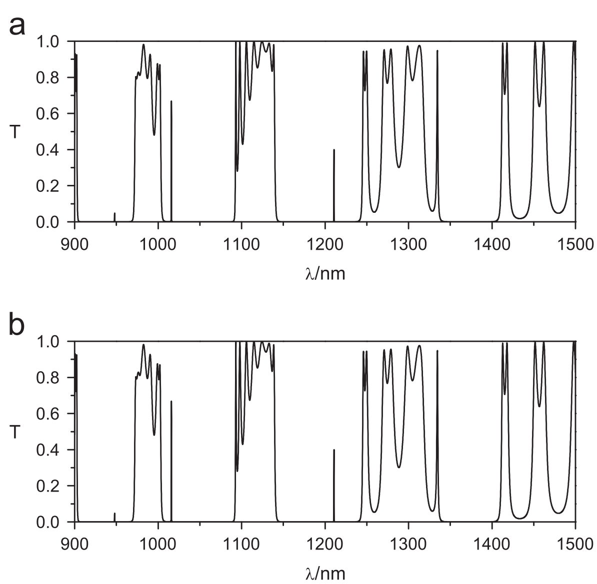


**Fig. 10.** Comparing quantum transmissivity (a) with classical transmissivity (b) with defect layer.

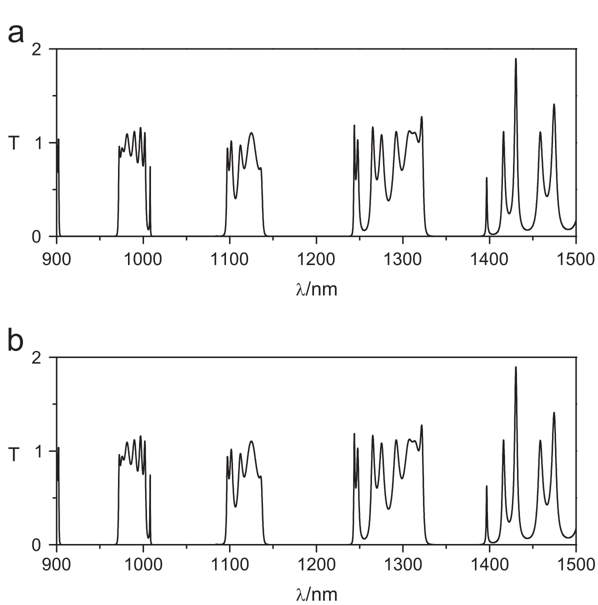


**Fig. 11.** Comparing quantum transmissivity (a) with classical transmissivity (b) with defect layer.

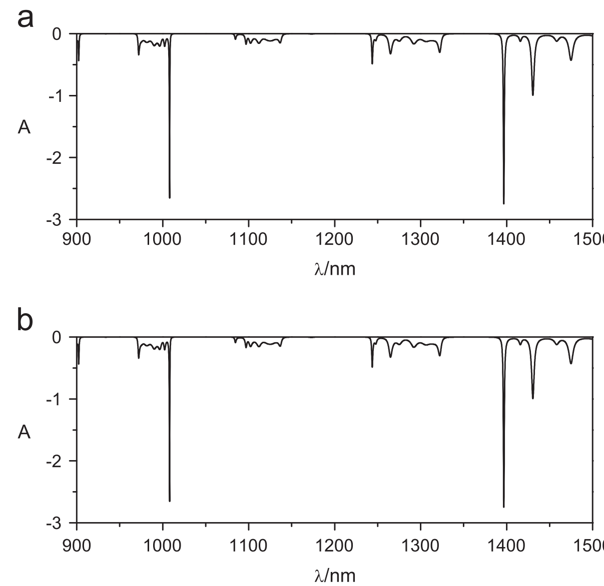


**Fig. 12.** Comparing quantum absorptivity (a) with classical absorptivity (b) with defect layer.

## 6. Conclusion

In summary, we have presented a quantum theory approach to study one-dimensional photonic crystals with and without defect layer. We give quantum dispersion relation, quantum transmissivity, reflectivity and absorptivity, and compare them with the classical dispersion relation, transmissivity, reflectivity and absorptivity. By the calculation, we find the classical and quantum dispersion relation, transmissivity reflectivity, absorptivity and the magnitude and position of defect model are identical. With the quantum theory new approach, we can study two-dimensional and three-dimensional photonic crystals in the future.

## Acknowledgement

This work is supported by Scientific and Technological Development Foundation of Jilin Province, Grant Number 20130101031JC.

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