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Output feedback controller design of discrete-time linear switching systems

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Abstract

This paper presents some stability synthesis results for the discrete-time linear switching systems whose dynamics contain reset functions and are determined by exogenous and uncontrollable events. First, for autonomous linear switching systems, conditions of stability are given in terms of a new definition of δ-controlled invariant set, and existence conditions to obtain such a set are presented. Then, under the assumption that the discrete state is known and the continuous state is unavailable for feedback, this new result is used to find the sufficient conditions for the existence of an observer-based stabilizing controller and dynamic output feedback controller. Such a design can be formulated in terms of linear matrix inequalities, which are numerically feasible with commercially available software. Finally, illustrative examples are given to indicate the effectiveness of the proposed design.

Keywords

Controlled invariance, linear matrix inequality (LMI), output feedback controller design, stability, switching systems

Introduction

Switching systems are a subclass of hybrid systems (Branicky, 1998; Gollu and Varaiya, 1989), which are characterized by a finite state machine (FSM) and a set of dynamical systems, each corresponding to a state of the FSM. The transitions between different modes of the FSM are determined by external uncontrollable events, which act as discrete disturbances and whenever a discrete transition occurs, the continuous state is instantly reset to a new value. Because of the particularity of its discrete transitions and modelling, switching systems have been receiving increasing attention from the control community.

Much of the work in controller synthesis of switching systems is based on a full knowledge of the hybrid state, and several controller design techniques have been developed (Benzaouia, 2007; Fan et al., 2010, 2012; Liu et al., 2012; Santis et al., 2006). However, in many application domains, this unrealistic assumption is not always verified. Hence, to make the hybrid controller synthesis relevant, the design of the output feedback controller is an important step. Depending on the class of hybrid systems under consideration and on the knowledge that is assumed at the hybrid output, different output feedback controller design methods have been given in the literature (Deaecto and Geromel, 2009; Molaei, 2008; Santis et al., 2009; Vidal, 2003, and references therein).

For linear switching systems, in the case of different assumptions, observability and observer design problem have been extensively studied both in the continuous-time and in the discrete-time domains. For the continuous-time linear switching systems, Santis et al. (2003) compared some definitions of observability previously offered and analysed their drawbacks, and presented a novel definition of observability; sufficient and necessary conditions for these properties to hold for switching systems are also given. In Santis and Benedetto (2005), under the assumption that the discrete state of switching systems is known and the conditions ensuring detectability are satisfied, a suitable Luenberger-like observer, solving the problem of estimating the continuous component of hybrid state, is proposed. In the discrete-time case, detection requires a finite, non-zero number of time instants, while in continuous-time it can be achieved in an arbitrarily small amount of time, so in Santis et al. (2007), the notion of Δ-observability is introduced, and completely characterized by computational conditions. In Caravani and Santis (2007, 2008), for linear switching systems without a reset function, in the assumption of full information on the continuous state and no information on the discrete state, the conditions of stabilizability for this class of systems are given in terms of a new definition of control invariance, and tractable parametric procedures for controller synthesis are investigated.

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In this paper, for the discrete-time linear switching systems, under the assumption that the discrete state of switching systems is known and continuous state is not available for feedback, the subsequent logical step-output feedback controller design problem is researched. After considering the relation between the stability and invariant set, new sufficient conditions of stability are proposed. Based on this result, observer-based stabilizing controller design and dynamic output feedback controller design problems are discussed, and all of those approaches can be formulated in terms of linear matrix inequalities (LMIs).

The paper is organized as follows. The next section provides the class of hybrid systems that are considered and the problem statement. Then, main results are presented both for the observer-based stabilizing controller design problem and the dynamic output feedback controller design problem. Two numerical examples are given to illustrate the technique, and then the paper is concluded.

Notation

The letters R, N are the sets of real and non-negative integers. R^n is the set of all n-tuples of real numbers, $R^{m\times n}$ is the set of real matrices with m rows and n columns. Given a countable set τ, the symbol $card(\tau)$ denotes the cardinality of τ and $int(D)$ is the interior of the set D. For a matrix A, the transpose of A is denoted by A^T and $\lambda(A)$ represents the eigenvalue. The notation $A > 0$ (or $A < 0$) means that the matrix A is positive definite (or negative definite). The identity matrix is denoted by I with appropriate dimensions. Here, $diag\{\cdot\}$ stands for a block-diagonal matrix, and $\star$ is used to notify a matrix element that is induced by transposition.

Definitions and problem presentation

Definition 1. The discrete-time linear switching systems are a tuple:

$$H = (\Xi, U, Y, S_d, E, L)$$

where Ξ is the hybrid state space, $\Xi = \bigcup_{i\in J}\{q_i\}\times R^n$, with $J = \{1, \cdots, M_d\}$; Q the discrete state space and $Q = \{q_i, i \in J\}$; R^n the continuous state space associated with discrete state q_i; U the continuous control input space, and $U = \bigcup_{i\in J} R^m$; Y the continuous output space, $Y = \bigcup_{i\in J} R^p$; and S_d the finite family of modes, $S_i \in S_d$ is a mapping that associates a discrete-time linear system to every discrete state $q_i \in Q$. Its state-space representation is described below:

$$\begin{cases} x(t+1) = A_i x(t) + B_i u_i(t) \\ y(t) = C_i x(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the continuous state vector, $u_i(t) \in R^m$ is the continuous control input vector, $y(t) \in R^p$ is the continuous output vector, the matrices A_i, B_i and C_i are constant ones that have the corresponding dimensions; E the collection of discrete transitions, and $E \subset Q\times Q$; and L the reset function $L : E\times\Xi \rightarrow \Xi$. When a discrete transition $e = (q_i, q_s) \in E$ occurs for $\xi = (q_i, x) \in \Xi$, the reset function has the following formulation:

$$L(e, \xi) = (q_s, L_{is}x), L_{is} \in R^{n\times n} \tag{2}$$

Next, some definitions that are often used to analyse and design switching systems are given. As introduced in Benzaouia (2007) and Fan et al. (2010), the hybrid time basis τ and the controlled execution of linear switching systems H are given below.

Definition 2. A hybrid time basis τ is an infinite or finite sequence of sets $I_j = \{t : t_j \leqslant t \leqslant \overline{t_j}\}$, with $\overline{t_j} = t_{j+1}$ for $j \in L = \{0, \ldots, l\}$, and if $card(\tau) = l + 1 < \infty$ then $\overline{t_l}$ can be finite or infinite. The times $\overline{t_j}$ will be called switching times, and the set of all hybrid time bases is denoted by T.

Definition 3. The controlled execution of linear switching systems H is a collection $\chi = (\xi_0, \tau, u_i, \xi)$, where $\xi_0 = (q_0, x_0)$ is the initial hybrid state; $\tau \in T$ is the hybrid time basis; $u_i : N \rightarrow R^m$ is the continuous control input function; and $\xi : N\times N \rightarrow \Xi$ is defined as,

$$\xi(t_0, 0) = \xi_0 = (q_0, x_0)$$

$$\xi(t, j) = (q(j), x(t, j)),\ q(j) = q_i$$

$$\xi(t_{j+1}, j+1) = L(e_j, \xi(t_j, j))$$

$$e_j = (q(j), q(j+1))$$

where $q : N \rightarrow Q$ is a map associating to each time interval a discrete state, $x(t, j)$ is the unique solution at time t of linear dynamical system S_i with initial time t_j, initial state $x(t_j, j)$ and continuous control inputs $u_i(\nu), \nu \in [t_j, t)$.

Before describing the problems in this paper, the following assumptions are made:

1) Switching among modes is uncontrolled but can be observed, which implies the discrete state of switching systems is known.
2) At each discrete time-point only one member of S_d is active.
3) The minimum dwell time $\delta \in N$ of each subsystem S_i is described below:

$$t_{j+1} - t_j \geqslant \delta, \forall j \in L, \delta \geqslant 1$$

The existence of a minimum dwell time is a widely used assumption in the analysis of switching systems, and models the inertia of the system for reacting to an external discrete input (Lygeros et al., 1999; Morse, 1996).

For the discrete-time linear switching systems H defined above, the considered problems in this paper can be formulated as follows.

Problem 1. For the discrete-time linear switching systems H, design a continuous state observer with the ith subsystem described by the equation:

$$\hat{x}(t+1) = A_i\hat{x}(t) + B_i u_i(t) + F_i(C_i\hat{x}(t) - y(t)) \tag{3}$$

and find M_d system-specific observer-based feedback control laws of the form:

$$u_i(t) = K_i\hat{x}(t,j), t_j \leqslant t < \overline{t}_j, q(j) = q_i \quad (4)$$

$$j = 0, \cdots, l, \ i = 1, \cdots, M_d$$

such that the stability of closed-loop linear switching systems is guaranteed, where the matrices F_i and K_i have to be designed.

Problem 2. For the discrete-time linear switching systems H, design a dynamic output feedback controller with the ith subsystem described by the equation:

$$\begin{cases} \hat{x}(t+1) = A_{ic}\hat{x}(t) + B_{ic}y(t) \\ u_i(t) = C_{ic}\hat{x}(t) \end{cases} \quad (5)$$

such that the stability of closed-loop linear switching systems is guaranteed, where the matrices A_{ic}, B_{ic} and C_{ic} have to be designed.

Main results

This section will present the main results of this paper. First, we characterize the new sufficient conditions to guarantee stability of linear switching systems. Then, this result is used to design the observer-based stabilizing controller and dynamic output feedback controller.

For the discrete-time linear switching systems, the stability condition of existence of a bounded controlled invariant set is conservative, because it requires the controlled execution of linear switching systems is guaranteed in an invariant set at the time interval $[t_j, t_{j+1}]$. Considering the assumption of minimum dwell time $\delta \in N$ in each subsystem, in order to guarantee the stability of systems, it only needs to constraint the controlled execution of linear switching systems in an invariant set at the time interval $[t_j + \delta, t_{j+1}]$, in which switching among different modes may occur (Caravani and Santis, 2008, 2007). Based on this idea, for linear switching systems considered in this paper, first, we give the following definitions.

Definition 4. A set $\psi = \bigcup_{i\in J} q_i \times \psi_i \subset \Xi$ is called a δ-controlled invariant with respect to $\Sigma = \bigcup_{i\in J} q_i \times \Sigma_i$ if there exists a control strategy ϕ such that starting from any hybrid state in $(q(j), \Sigma_i)$, there is

$$\xi(t,j) \in \psi_i, \forall t \in [t_j + \delta, t_{j+1}], \forall j \in L$$

for any controlled execution of H. If ψ is a δ-controlled invariant with respect to some non-empty set Σ, we call the set ψ δ-controlled invariant.

Definition 5. A linear switching system H is δ-stabilizable if there exists a control strategy such that

$$\forall \varepsilon > 0, \exists \rho > 0, \xi(t,j) \in \varepsilon\tilde{B}, \forall t \in I_j, j \in L$$

for all controlled executions with initial hybrid state $\xi_0 \in \rho\tilde{B}$, where $\tilde{B} = \bigcup_{i\in J} q_i \times B$ is the hybrid unit ball and $B = \{x \in R^n : \| x \| \leqslant 1\}$. Moreover, for a scalar α, it has $\alpha\tilde{B} = \bigcup_{i\in J} q_i \times \alpha B$.

The existence of a bounded δ-controlled invariant (contractive) set is related to the δ-stability (asymptotic δ-stability) of linear switching systems, and we have the following result.

Theorem 1. A linear switching system H is δ-stabilizable if and only if there exists a set $\psi = \bigcup_{i\in J} q_i \times \psi_i$, ψ_i bounded, which is δ-controlled invariant with respect to a set Σ, with the origin in $int(\Sigma_i)$.

Proof. Necessity: assume H is δ-stabilizable. Starting from any initial state $\xi_0 \in \rho\tilde{B}$ at time t_j, because the discrete state is known at all time, therefore, in the time interval $[t_j + 1, t_j + \delta]$, the controlled execution of H should stay in set εB_i, because a switching may never occur from mode q_i to another mode q_s. Then in the time interval $[t_j + \delta, t_{j+1}]$, it has to evolve in a subset ψ_i of εB_i, which is controlled invariant for the ith subsystem. When a switching from q_i to q_s occurs at time t_{j+1}, in $[t_{j+1} + 1, t_{j+1} + \delta]$, the control strategy has to guarantee the continuous state stay in εB_s, and controlled invariant with respect to a subset ψ_s of εB_s in $[t_{j+1} + \delta, t_{j+2}]$, because a switching from mode q_s to mode q_j maybe occur. Then, the set $\psi = \bigcup_{i\in J} q_i \times \psi_i$ is δ-controlled invariant with respect to $\rho\tilde{B}$, and $0 \in int(\rho\tilde{B})$, ψ_i is bounded. Hence the result follows (sufficiency is obvious).

From Theorem 1, in order to guarantee the δ-stability of linear switching systems, it needs to find a bounded δ-controlled invariant set. In the following, we present a class of δ-controlled invariant set, which is easy to obtain its existence conditions.

Theorem 2. The hybrid region $\psi = \bigcup_{i\in J} q_i \times \psi_i \subset \Xi$ is a δ-controlled invariant one, if there exist the bounded sets $\Omega_1 (0 \in int(\Omega_1))$, Ω_2 and ψ_i, such that the following conditions hold:

1) For any continuous state $x(t_j) \in \Omega_1$, it should have $x(t_j + \Delta) \in \Omega_2$ for any $\Delta = 1, 2, \cdots \delta$;
2) At the time instant $t_j + \delta$, it has $x(t_j + \delta) \in \psi_i$;
3) In the time interval $[t_j + \delta, t_{j+1}]$, ψ_i is a bounded controlled invariant set for the ith subsystem;
4) At the switching time t_{j+1}, the continuous state after reset belongs to the set Ω_1, which means $L_{is}x \in \Omega_1$.

Proof. From Definition 4, it is easy to see that the hybrid region $\psi = \bigcup_{i\in J} q_i \times \psi_i$ is δ-controlled invariant with respect to initial states in set $\Omega = \bigcup_{i\in J} q_i \times \Omega_1$.

Remark 1. Without loss of generality, the symbols Ω_1, Ω_2 should be Ω_{i1}, Ω_{i2}, which have relations with the discrete state q_i; here, for the sake of computational simplicity, we consider the common ones. The set such as the form in Theorem 2 is one kind of δ-controlled invariant set, certainly paid by certain conservatism; however, the design freedom of sets Ω_1, Ω_2 and ψ_i reduces the conservatism of the conditions. The existing methods, such as common invariant set or hybrid Lyapunov method, are particular cases of δ-controlled invariant set, when it has $\Omega_1 = \Omega_2 = \psi_i$. Therefore, the method in this paper is less conservative, especially in the presence of various state constraints on the different time interval of

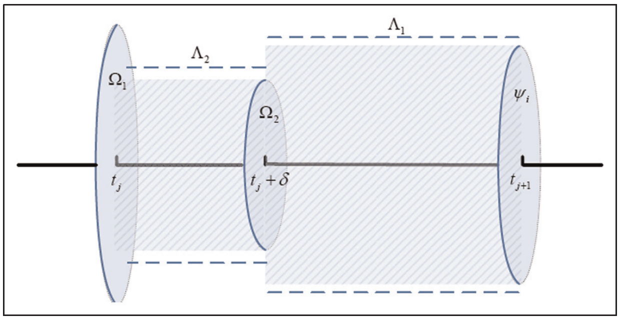

Figure 1. The δ-controlled invariant set with various state constraints.

$I_j, \forall j \in L$. (Please see Figure 1 with different constraint sets Λ_1 and Λ_2 as an example).

Define autonomous linear switching systems $H_0 = (\Xi, \phi, S_d, E, L)$ with the ith subsystem below:

$$x(t+1) = A_i x(t), i = 1, \cdots, M_d$$

where the symbols Ξ, S_d, E, L hold as in Definition 1, and ϕ represents the null set.

In the following, the sets we considered are all invariant ellipsoids. Though the solutions when sets are selected in the family of ellipsoids are inevitably expressed by sufficient condition, it is easy to obtain the tractable parametric procedures for controller synthesis. In detail, the considered sets have the following representations:

$$\Omega_1 = \{x(t) : x(t)^T R_1 x(t) \leqslant \varepsilon_1\}$$
$$\Omega_2 = \{x(t) : x(t)^T R_2 x(t) \leqslant \varepsilon_2\}$$
$$\psi_i = \{x(t) : x(t)^T Z_i x(t) \leqslant 1\}$$

where R_1, R_2 and Z_i are positive definite matrices, $\varepsilon_1, \varepsilon_2$ are positive constants. In the case of invariant ellipsoids, a pictorial representation of δ-controlled invariant set in Theorem 2 is shown in Figure 1.

Based on Theorems 1 and 2, the following theorem gives the sufficient conditions to judge the δ-stability of autonomous linear switching systems H_0.

Theorem 3. The autonomous linear switching systems H_0 are δ-stabilizable, if there exist the positive definite matrices P_{i1}, P_{i2}, Z_i and positive constant $\gamma \geqslant 1$, such that the following conditions hold:

$$A_i^T P_{i2} A_i - \gamma P_{i2} \leqslant 0 \tag{6}$$

$$P_{i1} - \gamma^\delta P_{i2} \geqslant 0 \tag{7}$$

$$\lambda_{max}(\tilde{P}_{i1})/\lambda_{min}(\tilde{P}_{i2}) \leqslant \varepsilon_2/\varepsilon_1 \tag{8}$$

$$R_2/\varepsilon_2 - Z_i > 0 \tag{9}$$

$$Z_i - A_i^T Z_i A_i > 0 \tag{10}$$

$$Z_i - L_{is}^T R_1 L_{is}/\varepsilon_1 > 0 \tag{11}$$

where $\tilde{P}_{i1} = R_1^{-1/2} P_{i1} R_1^{-1/2}$, $\tilde{P}_{i2} = R_2^{-1/2} P_{i2} R_2^{-1/2}$.

Proof. Let us assume $x(t_j)^T R_1 x(t_j) \leqslant \varepsilon_1$, we want to prove that if conditions (6)–(8) hold, then $x(t)^T R_2 x(t) \leqslant \varepsilon_2$ for all $t = t_j + 1, \cdots, t_j + \delta, j \in L$.

From condition (6), we can obtain

$$x(t+1)^T P_{i2} x(t+1) = x(t)^T A_i^T P_{i2} A_i x(t) \leqslant \gamma x(t)^T P_{i2} x(t)$$

which are simple calculations, and using the fact that $\gamma \geqslant 1$, we deduce that

$$x(t)^T P_{i2} x(t) \leqslant \gamma^{t-t_j} x(t_j)^T P_{i2} x(t_j) \leqslant \gamma^\delta x(t_j)^T P_{i2} x(t_j) \tag{12}$$

Because $\tilde{P}_{i1} = R_1^{-1/2} P_{i1} R_1^{-1/2}$ and $\tilde{P}_{i2} = R_2^{-1/2} P_{i2} R_2^{-1/2}$, we have

$$x(t)^T P_{i2} x(t) \geqslant \lambda_{min}(\tilde{P}_{i2}) x(t)^T R_2 x(t) \tag{13}$$

$$\varepsilon_1 \lambda_{max}(\tilde{P}_{i1}) \geqslant \lambda_{max}(\tilde{P}_{i1}) x(t_j)^T R_1 x(t_j) \geqslant x(t_j)^T P_{i1} x(t_j) \tag{14}$$

From condition (7), thus

$$x(t_j)^T P_{i1} x(t_j) - \gamma^\delta x(t_j)^T P_{i2} x(t_j) \geqslant 0 \tag{15}$$

Then, from (12)–(15), we have

$$\begin{aligned} &\varepsilon_1 \lambda_{max}(\tilde{P}_{i1}) \geqslant x(t_j)^T P_{i1} x(t_j) \geqslant \gamma^\delta x(t_j)^T \\ &P_{i2} x(t_j) \geqslant \lambda_{min}(\tilde{P}_{i2}) x(t)^T R_2 x(t) \end{aligned} \tag{16}$$

Together with condition (8), it follows that

$$x(t)^T R_2 x(t) \leqslant \varepsilon_2, \forall t = t_j + 1, \cdots, t_j + \delta \tag{17}$$

Then, condition 1 in Theorem 2 is obtained. Conditions (9)–(11) imply that an ellipsoid is contained in another one or an ellipsoid is an invariant one, which could guarantee the conditions 2–4 in Theorem 2. Therefore, the hybrid region $\psi = \bigcup_{i \in J} q_i \times \psi_i$ is a δ-controlled invariant set in Theorem 2. From Theorem 1, the autonomous linear switching systems H_0 are δ- stabilizable.

Remark 2. As a special case, when the sets Ω_1 and Ω_2 share the same positive definite matrix R, also with $\varepsilon_1 < \varepsilon_2$, conditions (6)–(8) can be degraded to the sufficient conditions that guarantee the finite-time stability of the ith subsystem in the time interval $[t_j, t_j + \delta]$ (Amato and Ariola, 2005; Amato et al., 2001).

Remark 3. In the case of various state constraints on $I_j, \forall j \in L$ (Figure 1), in order to achieve the constrained stability problem, a δ-controlled invariant set that has less conservatism can be obtained from the following steps:

1) Choose the sets $\Omega_2 = \psi_i = \Lambda_2$, and use Theorem 3 to judge whether it has a feasible solution.
2) If a feasible solution is found, go to step 3, else go to step 4.
3) Enlarge the set $\psi_i \subset \Lambda_1$, and use Theorem 3 to judge whether a δ-controlled invariant set can be found. Repeat this process until the set $\psi_i \subset \Lambda_1$ is as large as possible and Theorem 3 has a feasible solution. Then a δ-controlled invariant set can be obtained.

4) Lessen the initial sets $\Omega_2 = \psi_i \subset \Lambda_2$ until Theorem 3 has a feasible solution. Then enlarge the set $\psi_i \subset \Lambda_1$ according to step 3.

Observer-based stabilizing controller design

In this section, we will use the results above to design the continuous state observer and stabilizing controller.

Next, let $H^o = (\Xi, Q, U, S_d^o, E, L)$ be a class of linear switching systems with the ith subsystem described by the equation:

$$\hat{x}(t+1) = A_i\hat{x}(t) + B_iu_i(t) + F_i(C_i\hat{x}(t) - C_ix(t))$$

and the hybrid state of the system H^o is $\xi = (q(j), \hat{x}(t,j))$. Define error vector $\bar{e}(t) = \hat{x}(t) - x(t)$, together with H, let $H^c = (\Xi, Q, U, S_d^c, E, L^c)$ be the closed-loop linear switching systems, with the ith subsystem in S_d^c described by

$$z(t+1) = A_i^c z(t), i = 1, \cdots, M_d$$

where $A_i^c, z(t)$ and the reset function L^c are given below:

$$A_i^c = \begin{bmatrix} A_i + B_iK_i & B_iK_i \\ 0 & A_i + F_iC_i \end{bmatrix}, z(t) = \begin{bmatrix} x(t) \\ \bar{e}(t) \end{bmatrix}$$

$$L^c(e, (q(j), z(t,j))) = (q_s, \begin{bmatrix} L_{is} & 0 \\ 0 & L_{is} \end{bmatrix} z), L_{is} \in R^{n\times n}$$

For the closed-loop linear switching systems H^c, taking into account its structure, in order to design a δ-controlled invariant set guaranteeing its stability, we consider the sets Ω_1, Ω_2 as follows:

$$\Omega_1 = \{z(t) : \begin{bmatrix} x(t) \\ \bar{e}(t) \end{bmatrix}^T \begin{bmatrix} R_1 & 0 \\ 0 & R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ \bar{e}(t) \end{bmatrix} \leqslant c_1\} \quad (18)$$

$$\Omega_2 = \{z(t) : \begin{bmatrix} x(t) \\ \bar{e}(t) \end{bmatrix}^T \begin{bmatrix} R_2 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} x(t) \\ \bar{e}(t) \end{bmatrix} \leqslant c_2\} \quad (19)$$

Remark 4. Note that the bound in the initial condition for $x(t)$ of H^c is

$$x(t)^T R_1 x(t) + \bar{e}(t)^T R_1 \bar{e}(t) \leqslant c_1$$

which implies $x(t)^T R_1 x(t) \leqslant c_1/2$.

Using the results in Theorem 3, for any continuous state in set Ω_1, in the time interval $[t_j+1, t_j+\delta]$, the controlled execution of H^c is contained in set Ω_2, if there exist positive definite matrices P_{i1}, P_{i2} and positive constant $\gamma \geqslant 1$, such that the following conditions hold:

$$\begin{bmatrix} A_i + B_iK_i & B_iK_i \\ 0 & A_i + F_iC_i \end{bmatrix}^T P_{i2} \begin{bmatrix} A_i + B_iK_i & B_iK_i \\ 0 & A_i + F_iC_i \end{bmatrix} - \gamma P_{i2} \leqslant 0 \quad (20)$$

$$P_{i1} - \gamma^\delta P_{i2} \geqslant 0 \quad (21)$$

$$\lambda_{max}(\tilde{P}_{i1})/\lambda_{min}(\tilde{P}_{i2}) \leqslant c_2/c_1 \quad (22)$$

where $\tilde{P}_{i1} = \begin{bmatrix} R_1 & 0 \\ 0 & R_1 \end{bmatrix}^{-1/2} P_{i1} \begin{bmatrix} R_1 & 0 \\ 0 & R_1 \end{bmatrix}^{-1/2}$, $\tilde{P}_{i2} = \begin{bmatrix} R_2 & 0 \\ 0 & R_2 \end{bmatrix}^{-1/2} P_{i2} \begin{bmatrix} R_2 & 0 \\ 0 & R_2 \end{bmatrix}^{-1/2}$.

Using the method in reference Salim and Sette (2008), in the following, we present the sufficient conditions to guarantee condition (20), which is shown in the following theorem.

Theorem 4. If there exist the positive definite matrices Q_{i1}, Q_{i2}, real matrices Y_{i1}, Y_{i2} and positive constants $\alpha, \beta, \gamma \geqslant 1$ such that the following conditions hold:

$$\begin{bmatrix} -Q_{i1} & A_iQ_{i1} + B_iY_{i1} & B_iY_{i1} \\ \star & -\gamma Q_{i1} & 0 \\ \star & \star & -\alpha I \end{bmatrix} \leqslant 0 \quad (23)$$

$$\begin{bmatrix} Q_{i1} & I \\ \star & (2\beta - \alpha)I \end{bmatrix} > 0 \quad (24)$$

$$\begin{bmatrix} -\gamma Q_{i2} & A_i^TQ_{i2} + C_i^TY_{i2} & \beta I \\ \star & -Q_{i2} & 0 \\ \star & \star & -Q_{i1} \end{bmatrix} \leqslant 0 \quad (25)$$

then condition (20) can be guaranteed.

Proof. The proof is similar to the one in Salim and Sette (2008), so the detailed proof is omitted here.

Theorem 5. Inequality (22) can be guaranteed, if there exists a positive constant λ, such that the following conditions hold:

$$P_{i1} \leqslant \begin{bmatrix} R_1 & 0 \\ 0 & R_1 \end{bmatrix} \quad (26)$$

$$P_{i2} \geqslant \lambda \begin{bmatrix} R_2 & 0 \\ 0 & R_2 \end{bmatrix} \quad (27)$$

$$c_1 \leqslant c_2\lambda \quad (28)$$

Proof. From conditions (26) and (27), we have

$$\lambda_{max}(\tilde{P}_{i1}) \leqslant 1, \lambda_{min}(\tilde{P}_{i2}) \geqslant \lambda$$

Then together with (28), we deduce that

$$\frac{c_1}{\lambda_{min}(\tilde{P}_{i2})} \leqslant \frac{c_1}{\lambda} \leqslant c_2 \leqslant \frac{c_2}{\lambda_{max}(\tilde{P}_{i1})}$$

Combing the results of Theorems 4 and 5, we have the following theorem to guarantee conditions (20)–(22).

Theorem 6. If there exist the positive definite matrices $Q_{i1}, Q_{i2}, T_{i1}, T_{i2}$, real matrices Y_{i1}, Y_{i2} and positive constants $\alpha, \beta, \mu, \gamma \geqslant 1$ such that the following conditions hold:

$$\begin{bmatrix} -Q_{i1} & A_iQ_{i1} + B_iY_{i1} & B_iY_{i1} \\ \star & -\gamma Q_{i1} & 0 \\ \star & \star & -\alpha I \end{bmatrix} \leqslant 0 \quad (29)$$

$$\begin{bmatrix} Q_{i1} & I \\ \star & (2\beta - \alpha)I \end{bmatrix} > 0 \quad (30)$$

$$\begin{bmatrix} -\gamma Q_{i2} & A_i^T Q_{i2} + C_i^T Y_{i2} & \beta I \\ \star & -Q_{i2} & 0 \\ \star & \star & -Q_{i1} \end{bmatrix} \leqslant 0 \quad (31)$$

$$\begin{bmatrix} Q_{i1}/\gamma^{\delta} - T_{i1} & 0 \\ 0 & T_{i2} - \gamma^{\delta} Q_{i2} \end{bmatrix} \geqslant 0 \quad (32)$$

$$\begin{bmatrix} T_{i1} - R_1^{-1} & 0 \\ 0 & R_1 - T_{i2} \end{bmatrix} \geqslant 0 \quad (33)$$

$$\begin{bmatrix} Q_{i1} - \mu R_2^{-1} & 0 & 0 \\ \star & -Q_{i2} & I \\ \star & \star & -\mu R_2^{-1} \end{bmatrix} \leqslant 0 \quad (34)$$

$$\mu c_1 - c_2 \leqslant 0 \quad (35)$$

Then for any continuous state in set Ω_1, the controlled execution of H^c can be guaranteed in set Ω_2 for $t \in [t_j + 1, t_j + \delta]$, and the gains $K_i = Y_{i1} Q_{i1}^{-1}, F_i = Q_{i2}^{-1} Y_{i2}^T$.

Proof. Defining the positive definite matrices $P_{i1} = diag(T_{i1}^{-1}, T_{i2})$ and $P_{i2} = diag(Q_{i1}^{-1}, Q_{i2})$, from Theorems 4 and 5, and also with Schur complement formula, sufficient conditions (29)–(35) are easy to obtain.

Remark 5. For a fixed $\gamma \geqslant 1$, the above inequalities (29)–(35) are reduced to feasibility problems involving LMIs, which are numerically feasible with commercially available software.

Compared with the sets Ω_1, Ω_2, the set $\psi = \bigcup_{i \in J} q_i \times \psi_i$ has a certain design freedom, and therefore we propose the following theorem.

Theorem 7. For the gains $F_i = Q_{i2}^{-1} Y_{i2}^T$ and $K_i = Y_{i1} Q_{i1}^{-1}$ obtained from Theorem 6, if there exists the positive definite matrix Z_i such that the following conditions hold:

$$\frac{1}{c_2} \begin{bmatrix} R_2 & 0 \\ 0 & R_2 \end{bmatrix} - Z_i > 0 \quad (36)$$

$$Z_i - \begin{bmatrix} A_i + B_i K_i & B_i K_i \\ 0 & A_i + F_i C_i \end{bmatrix}^T Z_i \begin{bmatrix} A_i + B_i K_i & B_i K_i \\ 0 & A_i + F_i C_i \end{bmatrix} > 0 \quad (37)$$

$$Z_i - \frac{1}{c_1} \begin{bmatrix} L_{is} & 0 \\ 0 & L_{is} \end{bmatrix}^T \begin{bmatrix} R_1 & 0 \\ 0 & R_1 \end{bmatrix} \begin{bmatrix} L_{is} & 0 \\ 0 & L_{is} \end{bmatrix} > 0 \quad (38)$$

Then the closed-loop linear switching systems H^c with gains $F_i = Q_{i2}^{-1} Y_{i2}^T$ and $K_i = Y_{i1} Q_{i1}^{-1}$ are observer-based δ-stabilizable.

Proof. From Theorems 3 and 6, the δ-stability of closed-loop linear switching systems H^c can be obtained.

Based on the above analysis, a procedure for the design of observer-based stabilizing controller can be summarized as below.

Algorithm 1.

1) For any given positive definite matrices R_1, R_2 and positive constants $c_1, c_2, \gamma \geqslant 1$, according to Theorem 6, obtain the gains F_i and K_i.
2) For the given gains F_i and K_i, if the conditions (36)–(38) in Theorem 7 are feasible, go to step 3. Else go to step 4.
3) The closed-loop linear switching systems H^c with gains F_i and K_i are observer-based δ-stabilizable.
4) Adjust the suitable parameters $c_1, c_2, \gamma \geqslant 1$ and matrices R_1, R_2, then go to step 1.

Remark 6. Theorems 6 and 7 are based on the existence of some matrices and positive scalars, and then Algorithm 1 is just based on the trial-and-error method.

Dynamic output feedback controller design

In the following, we will use the results in Theorem 3 to design the dynamic output feedback controller.

Let $H^c = (\Xi, Q, U, S_d^c, E, L)$ be a class of linear switching systems with the ith subsystem described by the equation:

$$\begin{cases} \hat{x}(t+1) = A_{ic}\hat{x}(t) + B_{ic} y(t) \\ u_i(t) = C_{ic}\hat{x}(t) \end{cases}$$

and the hybrid state of the system H^c is $\xi = (q(j), \hat{x}(t,j))$. Together with H, let $H^{cl} = (\Xi, Q, U, S_d^{cl}, E, L^{cl})$ be the closed-loop linear switching systems with the ith subsystem in S_d^{cl} described by

$$z(t+1) = A_i^{cl} z(t), i = 1, \cdots, M_d$$

where $A_i^{cl}, z(t)$ and the reset function L^{cl} are given below:

$$A_i^{cl} = \begin{bmatrix} A_i & B_i C_{ic} \\ B_{ic} C_i & A_{ic} \end{bmatrix}, z(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}.$$

$$L^{cl}(e, (q(j), z(t,j))) = (q_s, \begin{bmatrix} L_{is} & 0 \\ 0 & L_{is} \end{bmatrix} z), L_{is} \in R^{n \times n}$$

Similarly, in order to design a δ-controlled invariant set guaranteeing the stability of closed-loop linear switching systems H^{cl}, we also consider the sets Ω_1, Ω_2 as follows:

$$\Omega_1 = \{z(t) : \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}^T \begin{bmatrix} R_1 & 0 \\ 0 & R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \leqslant c_1\} \quad (39)$$

$$\Omega_2 = \{z(t) : \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}^T \begin{bmatrix} R_2 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \leqslant c_2\} \quad (40)$$

Remark 7. Note that the bound in the initial condition for $x(t)$ of H^{cl} is

$$x(t)^T R_1 x(t) + \hat{x}(t)^T R_1 \hat{x}(t) \leqslant c_1$$

which implies $x(t)^T R_1 x(t) \leqslant c_1$.

From Theorem 3, we can obtain that, the closed-loop linear switching systems H^{cl} are δ-stabilizable, if there exist the positive definite matrices P_{i1}, P_{i2}, Z_i and positive constant $\gamma \geqslant 1$, such that the following conditions hold:

$$A_i^{cl\,T} P_{i2} A_i^{cl} - \gamma P_{i2} \leqslant 0$$

$$P_{i1} - \gamma^{\delta} P_{i2} \geqslant 0$$

$$\lambda_{max}(\tilde{P}_{i1}) / \lambda_{min}(\tilde{P}_{i2}) \leqslant c_2 / c_1$$

$$\frac{1}{c_2}\begin{bmatrix} R_2 & 0 \\ 0 & R_2 \end{bmatrix} - Z_i > 0$$

$$Z_i - A_i^{cl^T} Z_i A_i^{cl} > 0$$

$$Z_i - \frac{1}{c_1}\begin{bmatrix} L_{is} & 0 \\ 0 & L_{is} \end{bmatrix}^T \begin{bmatrix} R_1 & 0 \\ 0 & R_1 \end{bmatrix}\begin{bmatrix} L_{is} & 0 \\ 0 & L_{is} \end{bmatrix} > 0$$

The first three conditions above can be expressed by sufficient ones, which are shown in the following theorem.

Theorem 8. If there exist the positive definite matrices $X_{i2}, Y_{i2}, T_{i1}, H_{i1}$, matrices $G_{i1}, \hat{A}_{ic}, \hat{B}_{ic}, \hat{C}_{ic}$ and positive constants $\mu, \gamma \geqslant 1$, such that the following conditions hold:

$$\begin{bmatrix} -\gamma Y_{i2} & -\gamma I & A_i Y_{i2} + B_i \hat{C}_{ic} & A_i \\ \star & -\gamma X_{i2} & \hat{A}_{ic} & X_{i2} A_i + \hat{B}_{ic} C_i \\ \star & \star & -Y_{i2} & -I \\ \star & \star & \star & -X_{i2} \end{bmatrix} \leqslant 0 \quad (41)$$

$$\begin{bmatrix} \gamma^\delta X_{i2} - H_{i1} & -H_{i1} - G_{i1}^T & 0 \\ \star & -H_{i1} - T_{i1} - G_{i1} - G_{i1}^T & I \\ \star & \star & -Y_{i2}/\gamma^\delta \end{bmatrix} \leqslant 0 \quad (42)$$

$$\begin{bmatrix} T_{i1} - R_1 & G_{i1} \\ \star & H_{i1} - R_1 \end{bmatrix} \leqslant 0 \quad (43)$$

$$\begin{bmatrix} -2Y_{i2} & -I & Y_{i2} & Y_{i2} & Y_{i2} \\ \star & -X_{i2} & I & 0 & I \\ \star & \star & -Y_{i2} & 0 & 0 \\ \star & \star & \star & -\mu R_2^{-1} & 0 \\ \star & \star & \star & \star & -\mu R_2^{-1} \end{bmatrix} \leqslant 0 \quad (44)$$

$$\mu c_1 - c_2 \leqslant 0 \quad (45)$$

then for any continuous state in set Ω_1, the controlled execution of H^{cl} can be guaranteed in set Ω_2 for any $t \in [t_j + 1, t_j + \delta]$, and the matrices A_{ic}, B_{ic}, C_{ic} can be obtained uniquely:

$$\begin{bmatrix} A_{ic} & B_{ic} \\ C_{ic} & 0 \end{bmatrix} = \begin{bmatrix} Y_{i2}^{-1} - X_{i2} & X_{i2} B_i \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} \hat{A}_{ic} - X_{i2} A_i Y_{i2} & \hat{B}_{ic} \\ \hat{C}_{ic} & 0 \end{bmatrix}\begin{bmatrix} Y_{i2} & 0 \\ C_i Y_{i2} & I \end{bmatrix}^{-1}$$

Proof. Let

$$P_{i1} = \begin{bmatrix} T_{i1} & G_{i1} \\ G_{i1}^T & H_{i1} \end{bmatrix}, P_{i2} = \begin{bmatrix} X_{i2} & M_{i2} \\ M_{i2}^T & V_{i2} \end{bmatrix}, P_{i2}^{-1} = \begin{bmatrix} Y_{i2} & N_{i2} \\ N_{i2}^T & W_{i2}^{-1} \end{bmatrix}$$

and it is easy to check that:

$$N_{i2} M_{i2}^T = I - Y_{i2} X_{i2}$$
$$P_{i2}\begin{bmatrix} Y_{i2} \\ N_{i2}^T \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Therefore, take the transform as follows:

$$\hat{A}_{ic} = M_{i2} A_{ic} N_{i2}^T + M_{i2} B_{ic} C_i Y_{i2} + X_{i2} B_i C_{ic} N_{i2} + X_{i2} A_i Y_{i2}$$
$$\hat{B}_{ic} = M_{i2} B_{ic}$$
$$\hat{C}_{ic} = C_{ic} N_{i2}^T$$

If the matrices M_{i2}, N_{i2} are full row rank, the matrices $\hat{A}_{ic}, \hat{B}_{ic}, \hat{C}_{ic}$ and X_{i2}, Y_{i2} are known, then the matrices A_{ic}, B_{ic}, C_{ic} can be calculated. Moreover, if we design the full order output feedback controller, we can assume $N_{i2} = N_{i2}^T = Y_{i2}$, because of $P_{i2} P_{i2}^{-1} = I$, it has:

$$Q_{i2} = Y_{i2}^{-1}, M_{i2} = Q_{i2} - X_{i2}, V_{i2} = -M_{i2}, W_{i2} = Q_{i2} - Q_{i2} X_{i2}^{-1} Q_{i2}$$

and the matrices A_{ic}, B_{ic}, C_{ic} can be obtained from $\hat{A}_{ic}, \hat{B}_{ic}, \hat{C}_{ic}$ uniquely.

From the condition $A_i^{cl^T} P_{i2} A_i^{cl} - \gamma P_{i2} \leqslant 0$ and using the Schur complement formula, it follows that

$$\begin{bmatrix} -\gamma P_{i2} & P_{i2} A_i^{cl} \\ \star & -P_{i2} \end{bmatrix} \leqslant 0$$

Define $\Pi_{i2} = \begin{bmatrix} Y_{i2} & I \\ Y_{i2} & 0 \end{bmatrix}$, and multiplying $diag\{\Pi_{i2}^T, \Pi_{i2}^T\}$, $diag\{\Pi_{i2}, \Pi_{i2}\}$ on both sides of above inequality, then it is easy to obtain

$$\begin{bmatrix} -\gamma \Pi_{i2}^T P_{i2} \Pi_{i2} & \Pi_{i2}^T P_{i2} A_i^{cl} \Pi_{i2} \\ \star & -\Pi_{i2}^T P_{i2} \Pi_{i2} \end{bmatrix} = \begin{bmatrix} -\gamma Y_{i2} & -\gamma I & A_i Y_{i2} + B_i \hat{C}_{ic} & A_i \\ \star & -\gamma X_{i2} & \hat{A}_{ic} & X_{i2} A_i + \hat{B}_{ic} C_i \\ \star & \star & -Y_{i2} & -I \\ \star & \star & \star & -X_{i2} \end{bmatrix} \leqslant 0 \quad (46)$$

If from (46), the matrices X_{i2}, Y_{i2} and $\hat{A}_{ic}, \hat{B}_{ic}, \hat{C}_{ic}$ are calculated, then we obtain that

$$\begin{bmatrix} A_{ic} & B_{ic} \\ C_{ic} & 0 \end{bmatrix} = \begin{bmatrix} Q_{i2} - X_{i2} & X_{i2} B_i \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} \hat{A}_{ic} - X_{i2} A_i Y_{i2} & \hat{B}_{ic} \\ \hat{C}_{ic} & 0 \end{bmatrix}\begin{bmatrix} Y_{i2} & 0 \\ C_i Y_{i2} & I \end{bmatrix}^{-1}$$

From the condition $P_{i1} - \gamma^\delta P_{i2} \geqslant 0$, we have

$$\begin{bmatrix} \gamma^\delta X_{i2} - T_{i1} & \gamma^\delta Y_{i2}^{-1} - \gamma^\delta X_{i2} - G_{i1} \\ \star & \gamma^\delta X_{i2} - \gamma^\delta Y_{i2}^{-1} - H_{i1} \end{bmatrix} \leqslant 0 \quad (47)$$

Multiplying $\begin{bmatrix} I & 0 \\ I & I \end{bmatrix}$ and $\begin{bmatrix} I & I \\ 0 & I \end{bmatrix}$ on both side of (47), then

$$\begin{bmatrix} \gamma^\delta X_{i2} - T_{i1} & T_{i1} + G_{i1} - \gamma^\delta Y_{i2}^{-1} \\ \star & \gamma^\delta Y_{i2}^{-1} - H_{i1} - T_{i1} - G_{i1} - G_{i1}^T \end{bmatrix} \leqslant 0 \quad (48)$$

By the Schur complement formula, (48) can be converted to

$$\begin{bmatrix} \gamma^\delta X_{i2} - H_{i1} - \gamma^\delta Y_{i2}^{-1} & -H_{i1} - G_{i1}^T & 0 \\ \star & -H_{i1} - T_{i1} - G_{i1} - G_{i1}^T & I \\ \star & \star & -Y_{i2}/\gamma^\delta \end{bmatrix} \leqslant 0 \quad (49)$$

Because $-(\gamma^{\delta}X_{i2}-H_{i1}-\gamma^{\delta}Y_{i2}^{-1})>-(\gamma^{\delta}X_{i2}-H_{i1})$, then one sufficient condition of (49) is

$$\begin{bmatrix} \gamma^{\delta}X_{i2}-H_{i1} & -H_{i1}-G_{i1}^T & 0 \\ \star & -H_{i1}-T_{i1}-G_{i1}-G_{i1}^T & I \\ \star & \star & -Y_{i2}/\gamma^{\delta} \end{bmatrix} \leqslant 0$$

From the conditions in Theorem 5, it follows

$$\begin{bmatrix} T_{i1}-R_1 & G_{i1} \\ G_{i1}^T & H_{i1}-R_1 \end{bmatrix} \leqslant 0 \tag{50}$$

$$\begin{bmatrix} X_{i2} & M_{i2} \\ M_{i2}^T & V_{i2} \end{bmatrix} \geqslant \lambda \begin{bmatrix} R_2 & 0 \\ 0 & R_2 \end{bmatrix} \tag{51}$$

By the Schur complement formula, (51) is equivalent to

$$\begin{bmatrix} -X_{i2} & X_{i2}-Y_{i2}^{-1} & 0 & I & 0 \\ \star & -X_{i2} & I & 0 & I \\ \star & \star & -Y_{i2} & 0 & 0 \\ \star & \star & \star & -R_2^{-1}/\lambda & 0 \\ \star & \star & \star & \star & -R_2^{-1}/\lambda \end{bmatrix} \leqslant 0 \tag{52}$$

Multiplying $diag\{\begin{bmatrix} I & I \\ 0 & I \end{bmatrix}, I, I, I\}$ and $diag\{\begin{bmatrix} I & I \\ 0 & I \end{bmatrix}, I, I, I\}^T$ on both side of (52), we have

$$\begin{bmatrix} -2Y_{i2}^{-1} & -Y_{i2}^{-1} & I & I & I \\ \star & -X_{i2} & I & 0 & I \\ \star & \star & -Y_{i2} & 0 & 0 \\ \star & \star & \star & -R_2^{-1}/\lambda & 0 \\ \star & \star & \star & \star & -R_2^{-1}/\lambda \end{bmatrix} \leqslant 0 \tag{53}$$

Also, multiplying $diag\{Y_{i2}, I, I, I, I\}$ on both sides of (53), we deduce that

$$\begin{bmatrix} -2Y_{i2} & -I & Y_{i2} & Y_{i2} & Y_{i2} \\ \star & -X_{i2} & I & 0 & I \\ \star & \star & -Y_{i2} & 0 & 0 \\ \star & \star & \star & -R_2^{-1}/\lambda & 0 \\ \star & \star & \star & \star & -R_2^{-1}/\lambda \end{bmatrix} \leqslant 0 \tag{54}$$

After considering $\mu = \frac{1}{\lambda}$, then we can obtain conditions (41)–(45).

Also, compared with the sets Ω_1, Ω_2, the set $\psi = \bigcup_{i \in J} q_i \times \psi_i$ has a certain design freedom; then we provide the following theorem.

Theorem 9. For the matrices A_{ic}, B_{ic}, C_{ic} obtained from Theorem 8, if there exists the positive definite matrix Z_i such that the following conditions hold:

$$\frac{1}{c_2}\begin{bmatrix} R_2 & 0 \\ 0 & R_2 \end{bmatrix} - Z_i > 0 \tag{55}$$

$$Z_i - \begin{bmatrix} A_i & B_iC_{ic} \\ B_{ic}C_i & A_{ic} \end{bmatrix}^T Z_i \begin{bmatrix} A_i & B_iC_{ic} \\ B_{ic}C_i & A_{ic} \end{bmatrix} > 0 \tag{56}$$

$$Z_i - \frac{1}{c_1}\begin{bmatrix} L_{is} & 0 \\ 0 & L_{is} \end{bmatrix}^T \begin{bmatrix} R_1 & 0 \\ 0 & R_1 \end{bmatrix} \begin{bmatrix} L_{is} & 0 \\ 0 & L_{is} \end{bmatrix} > 0 \tag{57}$$

then the closed-loop linear switching systems H^{cl} are δ-stabilizable via the dynamic output feedback controller with matrices A_{ic}, B_{ic}, C_{ic}.

From Theorems 8 and 9, a procedure for the design of dynamic output feedback controller can be summarized as below.

Algorithm 2.

1) For any given positive definite matrices R_1, R_2, and positive constants $c_1, c_2, \gamma \geqslant 1$, according to Theorem 8, obtain the matrices A_{ic}, B_{ic}, C_{ic}.
2) For the obtained matrices A_{ic}, B_{ic}, C_{ic}, if the conditions in Theorem 9 are feasible, go to step 3. Else go to step 4.
3) The closed-loop linear switching systems H^{cl} are δ-stabilizable via the dynamic output feedback controller with matrices A_{ic}, B_{ic}, C_{ic}.
4) Adjust the suitable parameters $c_1, c_2, \gamma \geqslant 1$ and matrices R_1, R_2, then go to step 1.

Similarly, Theorems 8 and 9 are based on the existence of some matrices and positive scalars, and Algorithm 2 is also based on the trial-and-error method.

Numerical example

The objective of this section is to illustrate the observer-based stabilizing controller and dynamic output feedback controller design approaches; two numerical examples are given.

Example for observer-based stabilizing controller design

Consider the following discrete-time linear switching system with two subsystems. The matrices of the first and second subsystems are described by:

$$A_1 = \begin{bmatrix} 0.2 & 0.3 \\ 0 & 1.1 \end{bmatrix}, B_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix},$$
$$C_1 = [-1 \quad -1], L_{12} = \begin{bmatrix} -0.2 & 0.7 \\ 0.1 & 0.6 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 1.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix},$$
$$C_2 = [0 \quad 0.5], L_{21} = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.65 \end{bmatrix}$$

and the minimum dwell time is $\delta = 2$.

In order to design the observer-based stabilizing controller, the considered sets Ω_1, Ω_2 and scalar γ in step 1 of Algorithm 1 are as follows:

$$R_1 = \begin{bmatrix} 1 & -0.6 \\ -0.6 & 1 \end{bmatrix}, R_2 = \begin{bmatrix} 0.16 & -0.05 \\ -0.05 & 0.3 \end{bmatrix},$$
$$c_1 = 6, c_2 = 4.6, \gamma = 1.1$$

Then, use of the Matlab LMI Toolbox to check the conditions in Theorem 6 leads to the following results:

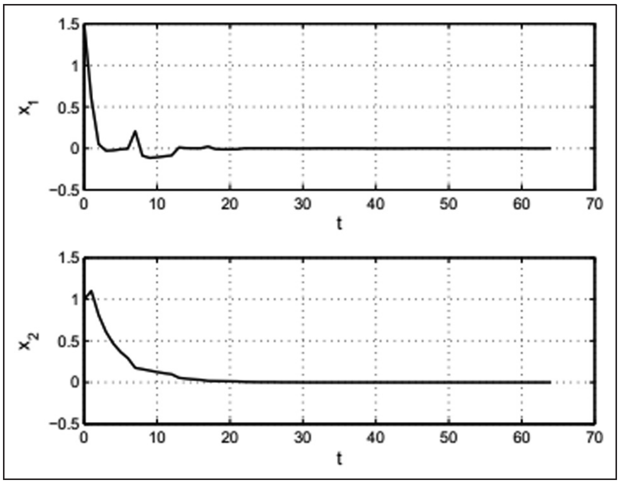

Figure 2. The controlled execution from the initial state $[1.5, 1]^T$.

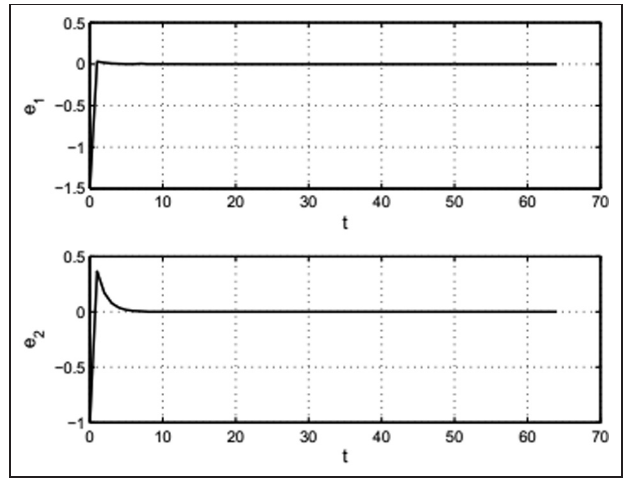

Figure 3. The evolution of estimation errors.

$$F_1 = \begin{bmatrix} 0.2523 \\ 0.5859 \end{bmatrix}, F_2 = \begin{bmatrix} -0.3481 \\ -2.4013 \end{bmatrix},$$
$$K_1 = [-0.0420, 0.2915], K_2 = [0.0783, -1.5081]$$

According to step 2 of Algorithm 1, for the obtained matrices F_1, K_1, F_2, K_2 and from Theorem 7, we have:

$$Z_1 = \begin{bmatrix} 0.0274 & -0.0097 & 0.0002 & 0 \\ \star & 0.0609 & 0.0006 & 0.0001 \\ \star & \star & 0.0297 & -0.0110 \\ \star & \star & \star & 0.0605 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} 0.0157 & 0.0007 & -0.0001 & 0.0007 \\ \star & 0.0546 & 0.0002 & -0.0009 \\ \star & \star & 0.0260 & -0.0072 \\ \star & \star & \star & 0.0591 \end{bmatrix}$$

Therefore, from Algorithm 1, we have designed a δ-controlled invariant set, which can guarantee the δ-stability of closed-loop systems.

Here, we take the initial state $[1.5, 1]^T$ in ellipsoid $\{x : x(t)^T R_1 x(t) \leq c_1/2\}$ for simulation, and consider the following hybrid time basis and switching strategy:

$$[t_0 = 0, \overline{t_0} = 6], [t_1 = 6, \overline{t_1} = 11], [t_2 = 11, \overline{t_2} = 14],$$
$$[t_3 = 14, \overline{t_3} = 18], [t_4 = 18, \overline{t_4} = 23],$$
$$[t_5 = 23, \overline{t_5} = 26], [t_6 = 26, \overline{t_6} = 31], [t_7 = 31, \overline{t_7} = 37],$$
$$[t_8 = 37, \overline{t_8} = 40], [t_9 = 40, \overline{t_9} = 45], [t_{10} = 45, \overline{t_{10}} = 49],$$
$$[t_{11} = 49, \overline{t_{11}} = 52]$$

The initial active system is the 1th subsystem. The simulation results are shown in Figures 2 and 3. It can be seen that the continuous state of linear switching system is reconstructed, and the controlled execution from the initial state $[1.5, 1]^T$ is asymptotically stable, independently of the external uncontrollable events, as expected, and the observer-based output feedback stabilization is successful.

Example for dynamic output feedback controller design

Consider the following discrete-time linear switching system with two subsystems. The matrices of the first and second subsystems are described by:

$$A_1 = \begin{bmatrix} -0.2 & 0 \\ 0.5 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} -0.6 \\ -0.4 \end{bmatrix},$$
$$C_1 = [-0.8 \quad -1], L_{12} = \begin{bmatrix} 0.2 & 0.1 \\ -0.1 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.1 & -0.2 \\ 0 & 0.6 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix},$$
$$C_2 = [1 \quad -0.5], L_{21} = \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & 0.15 \end{bmatrix}$$

and the minimum dwell time is $\delta = 2$.

In order to design the dynamic output feedback controller, the considered sets Ω_1, Ω_2 and scalar γ in step 1 of Algorithm 2 are as follows:

$$R_1 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix}, R_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.06 \end{bmatrix}, c_1 = 2.2, c_2 = 4, \gamma = 1.1$$

Also, use of the Matlab LMI Toolbox to check the conditions in Theorem 8 leads to the following results:

$$A_{1c} = \begin{bmatrix} -0.1612 & 0.2958 \\ 0.5983 & -0.1083 \end{bmatrix},$$
$$B_{1c} = \begin{bmatrix} 0.0236 \\ 0.8553 \end{bmatrix}, C_{1c} = [0.1043, -0.4205]$$
$$A_{2c} = \begin{bmatrix} -0.0835 & -0.0243 \\ 0.1588 & 0.2322 \end{bmatrix}, B_{2c} = \begin{bmatrix} 0.2030 \\ -0.4154 \end{bmatrix},$$
$$C_{2c} = [-0.0796, 0.0632]$$

According to step 2 of Algorithm 2, for the obtained matrices A_{1c}, B_{1c}, C_{1c} and A_{2c}, B_{2c}, C_{2c} from Theorem 9, we have:

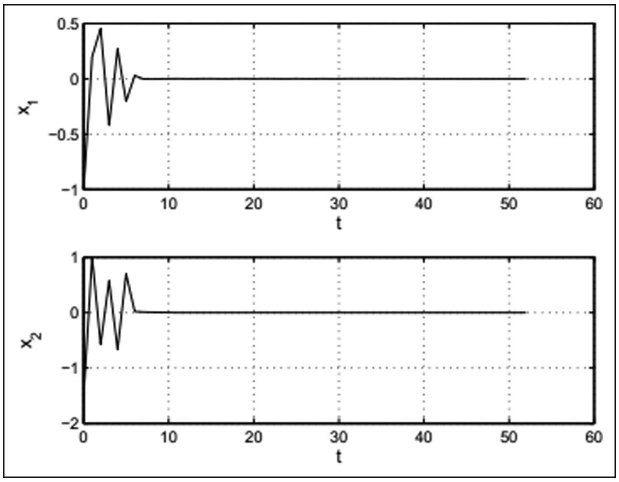

Figure 4. The controlled execution from the initial state $[-1, -1.5]^T$.

$$Z_1 = \begin{bmatrix} 0.0229 & 0.0017 & -0.0013 & 0.0026 \\ \star & 0.0113 & 0.0024 & -0.0042 \\ \star & \star & 0.0206 & 0.0048 \\ \star & \star & \star & 0.0070 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} 0.0243 & -0.0002 & 0 & 0 \\ \star & 0.0127 & 0 & -0.0001 \\ \star & \star & 0.0243 & -0.0002 \\ \star & \star & \star & 0.0125 \end{bmatrix}$$

Therefore, from Algorithm 2, we have designed a δ-controlled invariant set, which can guarantee the δ-stability of closed-loop systems.

In order to show the technique, consider the initial state $[-1, -1.5]^T$ in ellipsoid $\{x : x(t)^T R_1 x(t) \leq c_1\}$. For simulation, we choose the following hybrid time basis and switching strategy:

$$\begin{aligned} &[t_0 = 0, \overline{t_0} = 5], [t_1 = 5, \overline{t_1} = 9], [t_2 = 9, \overline{t_2} = 13], \\ &[t_3 = 13, \overline{t_3} = 16], [t_4 = 16, \overline{t_4} = 20], [t_5 = 20, \overline{t_5} = 22], \\ &[t_6 = 22, \overline{t_6} = 25], [t_7 = 25, \overline{t_7} = 29], [t_8 = 29, \overline{t_8} = 32], \\ &[t_9 = 32, \overline{t_9} = 35], [t_{10} = 35, \overline{t_{10}} = 37], [t_{11} = 37, \overline{t_{11}} = 40] \end{aligned}$$

and the initial active system is the 1th subsystem. The simulation results are given in Figures 4 and 5. They show that the controlled execution from the initial state $[-1, -1.5]^T$ is asymptotically stable, independently of the external uncontrollable events.

Conclusions

In this paper, output stabilization problem for the discrete-time linear switching systems is considered in the framework of invariant set theory. A new stability condition, which is related with the existence of a δ-controlled invariant set, is proposed, and sufficient conditions to obtain such a set are presented. Compared with the existing methods, the stability condition in our paper is more general. Then this new result is used to design the observer-based stabilizing controller and dynamic output feedback controller. The proposed method

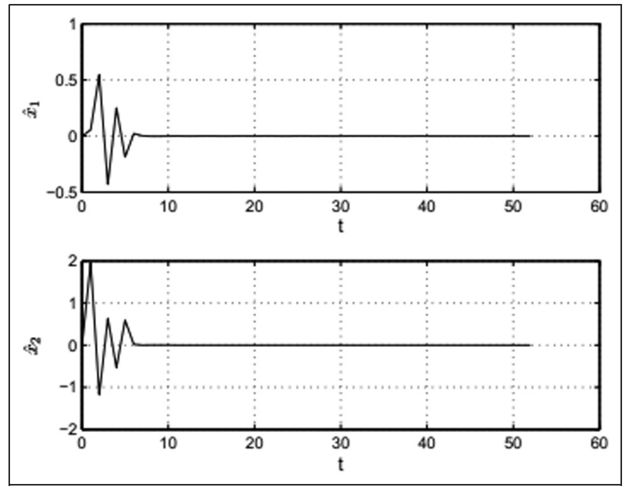

Figure 5. The state evolution of dynamic output feedback controller.

can be converted to the feasible problem of LMI, which are numerically feasible. From the numerical examples, it is obvious that the controller obtained can stabilize the system. Our future work is to find how to design an LMI-based output feedback controller for the discrete-time linear switching systems in the case of no information on the discrete and continuous states.

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