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Method for measuring the intensity distribution of a small beam spot with phase retrieval

Peng Sun [a,b,*], Jin Guo [a], Tingfeng Wang [a], Lisheng Liu [a], Jianxin Feng [a]

[a] *Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun, Jilin 130033, China*
[b] *University of Chinese Academy of Sciences, Beijing 100039, China*

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ABSTRACT

A method for measuring the intensity distribution of a small beam spot is presented. The proposed method involves several processes. First, an iterative phase retrieval algorithm is employed to calculate the phase distribution from two known intensity distributions at the out-of-focus distance. Then, based on the obtained phase distribution, the intensity distribution near the small beam spot is computed via the Fresnel-like transform. Finally, by comparing the obtained intensity with the known intensity near the small beam spot, we can determine the accuracy of our results. A simple model is established and the feasibility of the proposed approach is demonstrated.

1. Introduction

In general, a laser beam can be focused to a very small spot in a highly concentrated area. This property makes the laser beam very useful for many applications in physics, medicine, chemistry, and industry. Because lasers can be used in various applications, the measurement of the intensity distribution of a beam spot is very important. Several methods have been proposed to measure the intensity distribution of the beam spot. One of the traditional methods uses a mechanical scanning device, which consists of a rotating drum containing a knife-edge, pinhole, or slit that moves in front of a detector. Although this traditional method can provide excellent resolution (sometimes better than 1 μm), it works only with a continuous wave (CW) laser and not with a pulsed laser. Additionally, it is typically limited to two axes for measurement and integrates the beam along those axes [1]. CCD cameras can be used to provide whole two-dimensional laser beam measurements and work with both pulsed and CW lasers. However, CCD cameras require a small focused spot to be re-imaged using a microscope objective to provide a magnified image for viewing on the camera because the resolution is limited by the pixel size [1,2]. This process causes a change in the actual intensity distribution in space and introduces image aberrations that affect the measurement [2]. In addition, these traditional methods cannot be used in applications that require strict limitations for direct measurements of the beam spot. Because there are many drawbacks in most of the current methods, we propose an appropriate approach in this paper to obtain the intensity distribution of a small beam spot. The proposed method works with both pulsed and CW lasers and is suitable for applications where direct measurements can be challenging.

The outline of this paper is as follows. The fundamental principle of our approach is described in Section 2. In Section 3, we establish a simple model and apply the model to a lens of focal length *f*. The corresponding results are obtained and discussed in Section 4. We summarize our results in Section 5.

2. Fundamental principle and method

2.1. Fundamental principle

Let us assume that the intensity distribution can be readily measured at certain locations along the direction of the beam propagation using the method mentioned above. The phase distribution can be obtained readily at these locations by wavefront sensing [3]. Thus, the field distribution can be achieved at these corresponding locations. We perform the Fresnel-like transform [4] for the field distribution to obtain the intensity distribution of the small beam spot, which is difficult to measure using the methods mentioned above. In this paper, we use the iterative phase retrieval method [5–13] to obtain the phase distribution at the corresponding locations.

* Corresponding author at: Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Dong Nanhu Road 3888, Changchun, Jilin 130033, China. Tel.: +86 15948760180.
E-mail address: sunpeng.sadan@163.com (P. Sun).

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The concrete method is described as follows. First, an iterative phase retrieval algorithm is used to calculate the phase distribution from two known intensity distributions at the out-of-focus distance. Then, the Fresnel-like transform is employed to compute the intensity distribution near the small beam spot using the phase distribution obtained previously. Finally, by comparing the obtained intensity with the known intensity near the small beam spot, we determine whether the result is acceptable or not. The whole process is shown in Fig. 1. The notations 1–5 denote five different locations along the direction of beam propagation. The small beam spot, which is difficult to measure with detectors, is located at point 3. The intensity distributions at four other points can be obtained using detectors. The five locations are separated by free space or the complex optical system characterized by an ABCD ray-transfer matrix. E_1, E_2, E_3, and E_4 are the optical elements that can be represented by the ABCD ray-transfer matrices and Z_1, Z_2, …, Z_8 are the optical distances between the optical elements. An iterative phase retrieval algorithm is utilized to calculate the phase distribution at point 1 from the two known intensity distributions at points 1 and 5. After computing the field distribution at point 1, we perform the Fresnel-like transform for the field distribution at point 1 to compute the intensity distributions at points 2 and 4. Finally, the obtained and known intensity distributions are compared at points 2 and 4. From this comparison, we can determine the accuracy of our results.

2.2. Collins' formula

According to Collins [4], the Fresnel-like transform relating the fields across the input and output planes can be expressed as follows [14]:

$$U(x,y) = -\frac{ik}{2\pi\sqrt{B_x B_y}}\exp(-ikL)\iint dx_0 dy_0 U_i(x_0,y_0) \times\exp[-\frac{ik}{2B_x}(D_x x^2 - 2xx_0 + A_x x_0^2) - \frac{ik}{2B_y}(D_y y^2 - 2yy_0 + A_y y_0^2)] \quad (1)$$

where $U_i(x_0, y_0)$ is the field at the point (x_0, y_0) of the input plane; $U(x, y)$ is the field at the point (x, y) of the output plane; $k=2\pi/\lambda$ is the wavenumber; λ is the wavelength; L is the optical distance along the axis; $A_{x,y}$, $B_{x,y}$, and $D_{x,y}$ are the elements of the x- and y-axis ray-transfer matrix.

2.3. Gerchberg–Saxton algorithm

The Gerchberg–Saxton algorithm [7] was originally developed to solve the problem of reconstructing the phase from two intensity measurements. The algorithm consists of the following four steps: 1) Fourier transform an estimate $g(x)$ of the object $f(x)$; 2) replace the modulus $|G(u)|$ of the resulting computed Fourier transform $G(u)$ with the known Fourier modulus $|F(u)|$ to form an estimate $G'(u)$ of the Fourier transform $G(u)$; 3) inverse Fourier transform the estimate $G'(u)$ of the Fourier transform $G(u)$; and 4) replace the modulus $|g'(x)|$ of the resulting computed image $g'(x)$ with the known object modulus $|f(x)|$ to form a new estimate of the object $f(x)$ [10]. This process is depicted in Fig. 2.

The equations [10] for the kth iteration can be written as

$$G_k(u) = |G_k(u)|\exp[i\phi_k(u)] = F[g_k(x)] = \int_{-\infty}^{\infty} g_k(x)\exp(-i2\pi ux)dx \quad (2)$$

$$G'_k(u) = |F(u)|\exp[i\phi_k(u)] \quad (3)$$

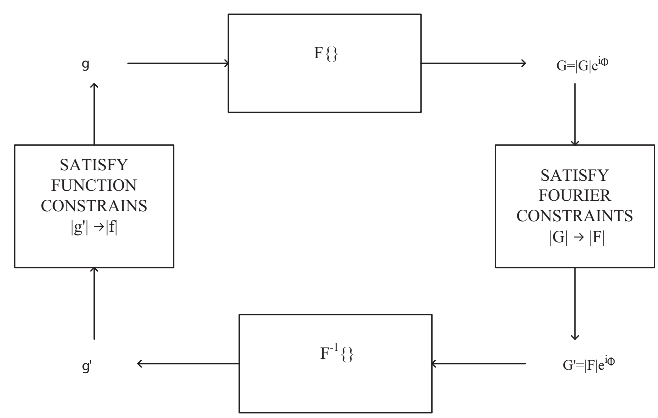

Fig. 2. Schematic diagram of Gerchberg–Saxton algorithm.

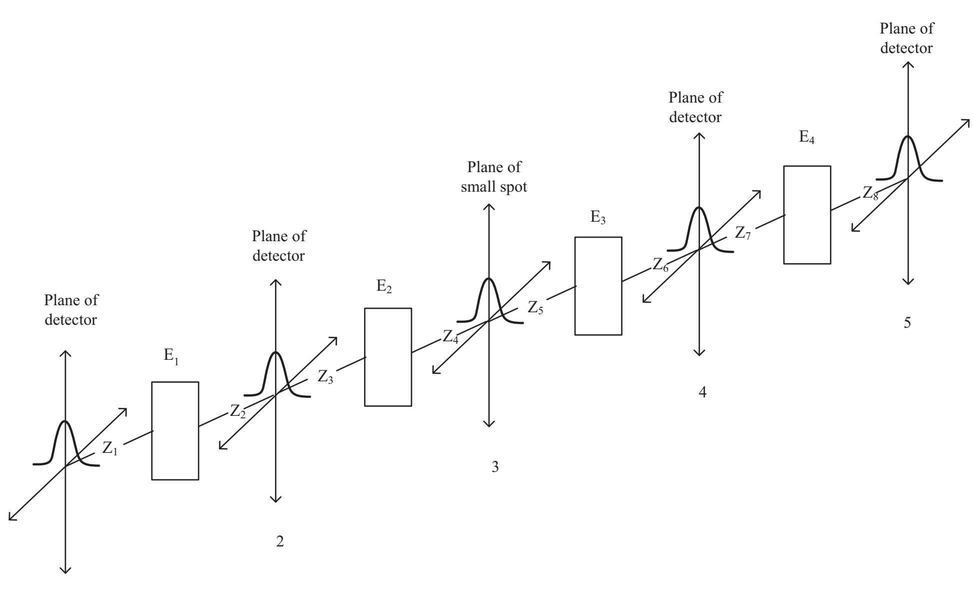

Fig. 1. Schematic diagram of computing process.

$$g'_k(x) = |g'_k(x)|\exp[i\theta'_k(x)] = F^{-1}[G'_k(u)] = \int_{-\infty}^{\infty} G'_k(u)\exp(i2\pi ux)du \tag{4}$$

$$g_{k+1}(x) = |f(x)|\exp[i\theta_{k+1}(x)] = |f(x)|\exp[i\theta'_k(x)] \tag{5}$$

where x is the two-dimensional spatial coordinate and u is the two-dimensional spatial frequency coordinate.

3. Model

A simple model (Fig. 3) is established to demonstrate the feasibility of our method. In our study, the beam is focused using a lens of focal length f and the locations 1–5 in Fig. 1 are set as follows. The point 1 is located at the plane of lens, location 2 lies in front of the focal plane; location 3 is at the focal plane, location 4 lies at the back of the focal plane, and location 5 lies at a point twice the focal length away from the lens. We can also use a different optical system in place of the lens. Although complicated, the same method is applicable, and the operation is similar.

An iterative phase retrieval algorithm is first used to calculate the phase distribution at point 1 from the two known intensity distributions at points 1 and 5 as described previously. In this model, the known intensity distributions at point 1 (near-field) and point 5 (far-field) are flat-top and super-Gaussian functions, respectively. The objective of the phase retrieval method is to obtain the near-field phase distribution at point 1 using the known intensity distributions. The common phase retrieval methods are based on the Gerchberg–Saxton theory [7]. In its basic form, the Gerchberg–Saxton method consists of starting with an initial distribution, propagating back and forth between the near-field and far-field, and normalizing the intensity distribution to the desired form at each plane before returning to the other field, while leaving the phase unchanged.

The iterative process is depicted in Fig. 3. The known intensity distributions in the near-field and far-field are represented by the solid lines at notations 1 and 3, respectively. To start, we assume the random phase (dotted line) at notation 1 and calculate the intensity in the far-field at notation 2. The calculated intensity and phase distribution are represented by the solid and dotted lines, respectively. We then normalize the far-field intensity to the desired shape (known intensity distribution in this model) and leave the phase unchanged. This intensity distribution is indicated in notations 2 and 3. The desired shape in both fields is represented by the dash-dotted line, which is depicted in notations 2 and 4. The beam is then back-propagated to the near-field at notation 4. The intensity does not perfectly match the known distribution. We normalize the near-field to the known envelope, leaving the phase unchanged as indicated in notations 4 and 1, and repeat the process until convergence. At convergence, the phase at notation 1 is required to compute the intensity distribution of the small beam spot. Using the obtained phase distribution, the field distribution can be achieved at location 1 in Fig. 1. Finally, we perform the Fresnel-like transform for the field distribution to obtain the intensity distribution of the small beam spot. To demonstrate the feasibility of our approach, the intensity distribution near the small beam spot is computed by performing the Fresnel-like transform for the obtained field distribution. The calculated and known intensity distributions are compared at points 2 and 4. From the comparison, we can determine the accuracy of our results.

4. Results and discussion

We use the following parameters: focal length $f = 180$ cm, wavelength $\lambda = 532$ nm, numerical aperture NA = 0.02, distance from location 2 to the focal plane $\delta_1 = 30$ cm, distance from location 4 to the focal plane $\delta_2 = 30$ cm, distance from location 5 to the focal plane $f = 180$ cm, and the ray-transfer matrix of this model

$$M = \begin{pmatrix} -1 & 360 \\ -1/180 & 1 \end{pmatrix} \tag{6}$$

The known intensity distributions at locations 1 and 5 are shown in Figs. 4 and 5, respectively. To obtain accurate results via numerical computation, it is important to choose appropriate mesh spacings and mesh dimensions for a given problem. Based on a previous study [15], we choose the appropriate mesh spacings and mesh dimensions at locations 1 and 5 to use in this model.

Based on our method, the intensity distributions at points 2, 3, and 4 can be computed with the known intensity distributions at points 1 and 5. If we denote the known intensity by $f(x)$, where x is

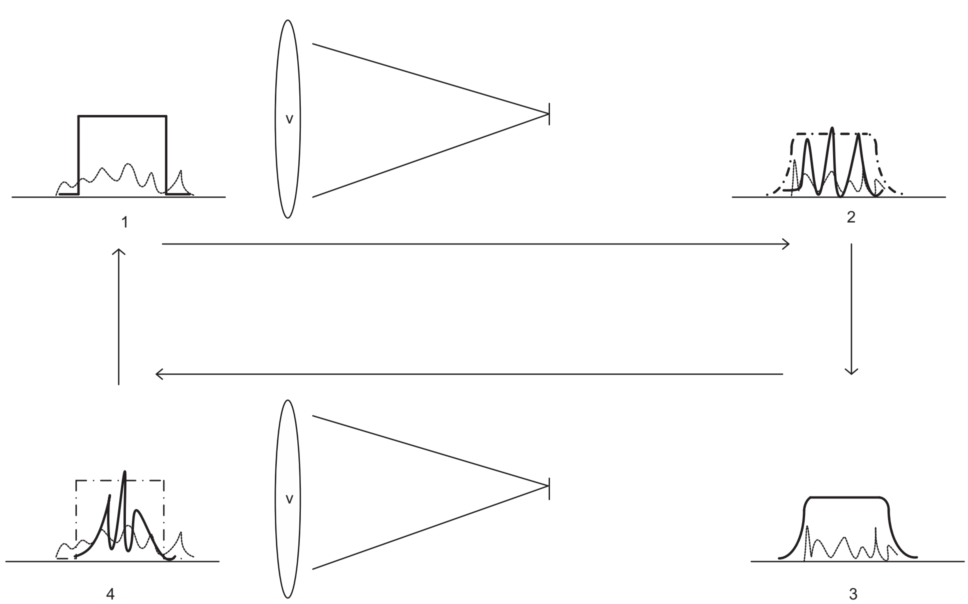

Fig. 3. Diagram of iterative phase retrieval process.

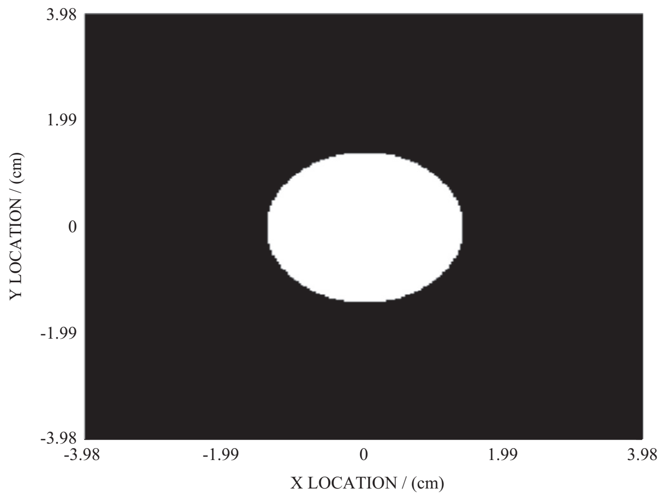

Fig. 4. The known near-field intensity distribution at location 1.

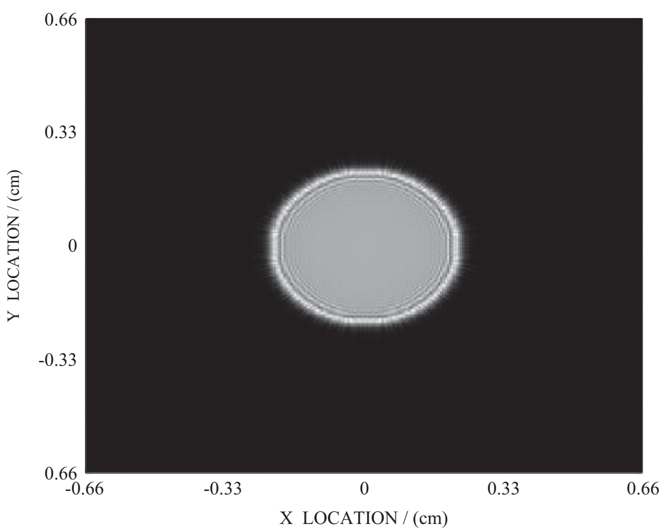

Fig. 5. The known far-field intensity distribution at location 5.

a two-dimensional vector, and the calculated intensity by $g(x)$, then the normalized RMS error E [10] is

$$E=\left(\int_{-\infty}^{\infty}|g(x)-f(x)|^2dx\Big/\int_{-\infty}^{\infty}|f(x)|^2dx\right)^{1/2} \tag{7}$$

We perform only 128 iterations for simplicity. The recovered wave-front at location 1 is shown in Fig. 6, and the RMS error E at locations 2 and 4 are depicted in Figs. 7 and 8, respectively. We observe that the error decreases rapidly during the first few iterations and reaches 0.0033 for location 2 and 0.0035 for location 4 when it approaches the 128th and final iteration.

The beam width is very significant in the beam profile measurement. Therefore, we calculate the beam width along with the calculated intensity at points 2 and 4. The beam width is defined using the encircled-energy criterion [16]. The beam width is 0.2035 cm at location 2 and 0.2239 cm at location 4 at the final iteration (128). The practical beam widths calculated using the known intensity at points 2 and 4 are 0.2034 cm and 0.2238 cm, respectively. Notably, our approach is applicable and effective for comparison with the data mentioned above.

In addition, the intensity distribution at the focal plane also demonstrates the feasibility of our method. The corresponding beam width at the 128th iteration is 0.0120 cm and the practical

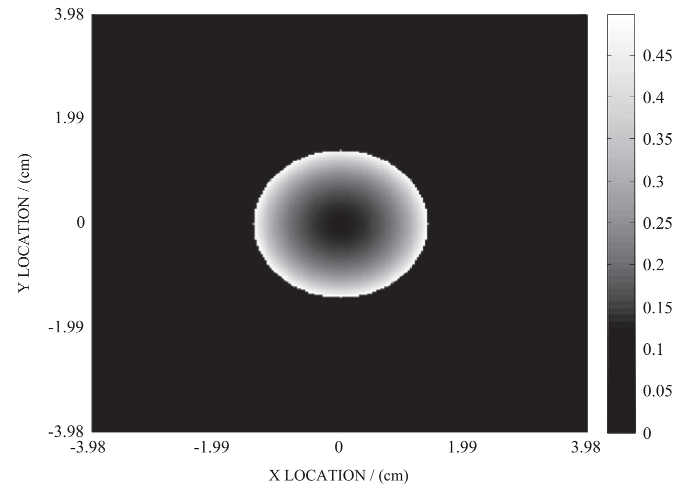

Fig. 6. Diagram of the recovered wave-front at location 1.

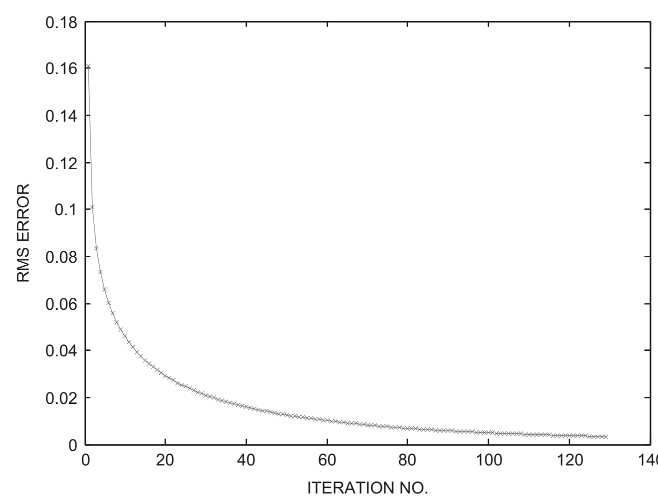

Fig. 7. The RMS error versus the number of iterations at location 2.

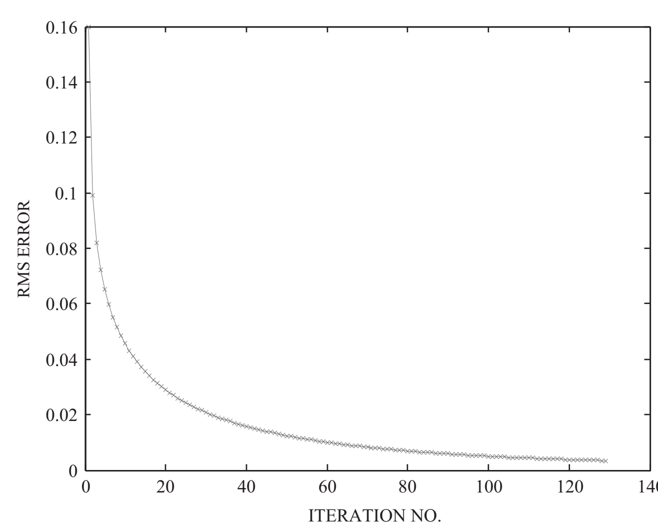

Fig. 8. The RMS error versus the number of iterations at location 4.

beam width is 0.0118 cm, thus demonstrating the reliability of our method.

According to a previous study [17], if the transformation between two wave functions is performed using a unitary operator and one of the wave functions is absolutely square integrable, the phase reconstruction from the known intensity distribution in any two planes of an imaging system can be achieved using the Gerchberg–Saxton algorithm. Because the model does not involve diffraction loss (the radius of the beam is smaller than the radius of the aperture), our choice of the distances of the intensity measurement meets the requirement. Moreover, the results of phase retrieval also demonstrate that the choice of distances is correct. Although the choice of distances is feasible, it is not the optimal solution. Because the goal of our paper is to describe an effective method to indirectly measure the beam size of a focused field based on the intensity distributions measured at the out-of-focus distance, the optimal choice of distances is not the main purpose of this study. For the best choice, other sources explain how to choose the proper out-of-focus distance [18].

5. Conclusion

In this paper, a simple method is presented to measure the intensity distribution of the small beam spot. The method involves several processes. First, an iterative phase retrieval algorithm is employed to calculate the phase distribution from two known intensity distributions at the out-of-focus distance. Then, based on the obtained phase distribution, the intensity distribution near the small beam spot is computed via the Fresnel-like transform. Finally, the obtained intensity and the known intensity near the small beam spot are compared to determine the accuracy of the results. The model is applied to a lens of focal length $f=180$ cm. The results demonstrate the feasibility of our approach. To obtain the phase distribution, the Gerchberg–Saxton algorithm is selected to accomplish the calculation in our current work. Other algorithms, such as the gradient-search algorithm [9], input–output algorithm [10], phase diversity [11,12] and genetic algorithm [13], are also feasible. Moreover, we can utilize the wave-front sensors to obtain the phase distribution in real-time operations [3]. Additionally, we can generalize our approach to the complex optical systems characterized by the ABCD ray-transfer matrix.

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