Improve the accuracy of interaction matrix measurement for liquid-crystal adaptive optics systems

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Abstract: We present a novel method to measure the interaction matrix of liquid-crystal adaptive optics systems, by applying least squares method to mitigate the impact of measurement noise. Experimental results showed a dramatic gain in the accuracy of interaction matrix, and a considerable improvement in image resolution with open loop adaptive optics correction.

OCIS codes: (010.1080) Active or adaptive optics; (230.3720) Liquid-crystal devices; (230.6120) Spatial light modulators.

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1. Introduction

Due to its low cost, reliability, low power consumption, no moving mechanical components and high resolution [1], compared to deformable mirrors (DM), liquid crystal wavefront corrector (LC corrector), or liquid crystal spatial light modulator (LC SLM) has been used in many adaptive optics systems, both for large telescope systems [2–4] and retinal imaging systems [5–7].

The interaction matrix of an AO system describes the control relationship between the wavefront corrector and the wavefront sensor (WFS), is used to derive the wavefront corrector signal from the WFS measurements during AO correction. Hence the accuracy of interaction matrix directly affects the performance of AO systems, and is extremely severe in the liquid-crystal adaptive optics (LC-AO) systems, since we studied an open loop control technique [8] instead of closed loop as it's widely used in DM based AOs, for the consideration of light energy efficiency.

Up to date, there have been many researches on the interaction matrix for DM based AO systems [9–11]. By pushing every actuator of the DM with Hadamard patterns, Kasper [9] evidently improve the quality of the interaction matrix for DM based AO systems. However if we try to apply Zernike modes (we use Zernike modal for wavefront representation) with Hadamard patterns to the LC corrector, the LC corrector would saturate, i.e. the phase modulation rate would outstrip the capability of LC corrector. Therefore, this heuristic method cannot be transferred to LC-AO systems.

Hence we introduce a novel method to measure the interaction matrix of LC-AO systems with high accuracy, by applying least squares method. In Section 2 we give a detailed introduction of our method and the laboratory setup of our experiment. The experimental results and discussions are described in Section 3, and the conclusions are drawn in Section 4.

2. Measurement of interaction matrix for LC-AOs

2.1 Least squares interaction matrix measurement method

Assume the number of Zernike modes used in expansion is m, number of micro-lens of SH-WFS (Shack-Hartmann Wavefront Sensor) is n. Then the $2n*m$ interaction matrix M_I describes the relationship between the Zernike modes coefficients vector V_Z applied to LC corrector, the slopes vector V_S measured by SH-WFS and the measurement noise vector V_N, as follows:

$$V_S = M_I * V_Z + V_N. \tag{1}$$

During AO correction, control matrix M_C is used to reconstruct the required Zernike pattern V_C form the SH-WFS measurements V_S:

$$V_Z = M_C * V_S. \tag{2}$$

Where,

$$M_C = (M_I^T * M_I)^{-1} * M_I^T. \tag{3}$$

M_C is the pseudo-inverse of M_I, for M_I may not be squares or reversible, M_I^T is the transposition of M_I.

Previously, we measure the interaction matrix M_I with what we call identity matrix (IM) method: simply applying normalized Zernike modes to LC corrector respectively, then measure the respondent slopes V_S with SH-WFS for each Zernike modes [12]:

$$\begin{cases} V_{S1} = M_I * \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T + V_{N1} \\ V_{S2} = M_I * \begin{bmatrix} 0 & 1 & \cdots & 0 \end{bmatrix}^T + V_{N2} \\ \vdots \\ V_{Sm} = M_I * \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix}^T + V_{Nm} \end{cases}. \tag{4}$$

$$M_S = M_I + M_N. \tag{5}$$

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Where $M_S = \begin{bmatrix} V_{S1} & V_{S2} & \cdots & V_{Sm} \end{bmatrix}$ is the slope matrix measured from SH-WFS, $M_N = \begin{bmatrix} V_{N1} & V_{N2} & \cdots & V_{Nm} \end{bmatrix}$ is the measurement noise matrix. The identity matrix method simply assumes M_S is a good estimation $\hat{M}_I$ of M_I, i.e. $\hat{M}_I = M_S$. While, apparently, from Eq. (5) we know the estimation $\hat{M}_I$ embodies all the impacts of M_N.

In order to reduce the impacts of M_N in M_I measurement procedure, we apply $K(K \gg m)$ groups of random Zernike coefficients V_Z (with value ranging from −0.5 to 0.5) to LC corrector, instead of m groups of normalized Zernike modes, and measure the corresponding K groups of slopes V_S from SH-WFS. Then calculate M_I using least squares method:

$$\begin{cases} V_{S1} = M_I * V_{Z1} + V_{N1} \\ V_{S2} = M_I * V_{Z2} + V_{N2} \\ \vdots \\ V_{SK} = M_I * V_{ZK} + V_{NK} \end{cases}. \tag{6}$$

$$M_S = M_I * M_Z + M_N. \tag{7}$$

$$\begin{aligned} \tilde{M}_I &= M_S * M_Z^T * (M_Z * M_Z^T)^{-1} \\ &= M_I + M_N * M_Z^T * (M_Z * M_Z^T)^{-1}. \end{aligned} \tag{8}$$

where $M_S = \begin{bmatrix} V_{S1} & V_{S2} & \cdots & V_{SK} \end{bmatrix}$, $M_N = \begin{bmatrix} V_{N1} & V_{N2} & \cdots & V_{NK} \end{bmatrix}$. And $M_Z = \begin{bmatrix} V_{Z1} & V_{Z2} & \cdots & V_{ZK} \end{bmatrix}$ is the random Zernike coefficients matrix (K groups of V_Z s) applied to LC corrector.

From the minimum-variance estimation property [13] of least-squares problem, we know that $\widetilde{M}_I$ is the minimum-variance unbiased linear estimator of M_I.

2.2 Laboratory setup to verify least squares interaction matrix measurement method

To verify our least squares interaction matrix measurement method, we set up a laboratory experiment as shown in Fig. 1, with parameter n = 225 (SH-WFS has 15*15 micro-lens), m = 35 (35 modes for Zernike modal wavefront representation), K = 10000. MS is a monochromatic slice around 780 nm, the working waveband of LC corrector. LC corrector and SH-WFS are well placed such that they are conjugated to each other. With a turbulence phase plate placed between Lens L1 and L2, conjugated with LC corrector, we can evaluate the performance of this method while correcting atmospheric turbulence.

The system must be convenient to switch form open loop to closed loop, for it has to be closed loop while measuring interaction matrix. This could be done simply by removing the ½ wave plate from the optical path, for only the S polarized light with polarization direction parallel to the alignment direction of LC molecules could be modulated by LC corrector. With the ½ wave plate in the optical path, the S polarized light modulated by LC corrector passes through ½ wave plate and becomes P polarized light, which is transmitted by PBS and, hence, cannot be received by the SH-WFS. Therefore, only the non-modulated light reaches the SH-WFS, hence the system is open loop. Without the ½ wave plate, the S polarized light reflected from the PBS is received by SH-WFS, so the system becomes closed loop.

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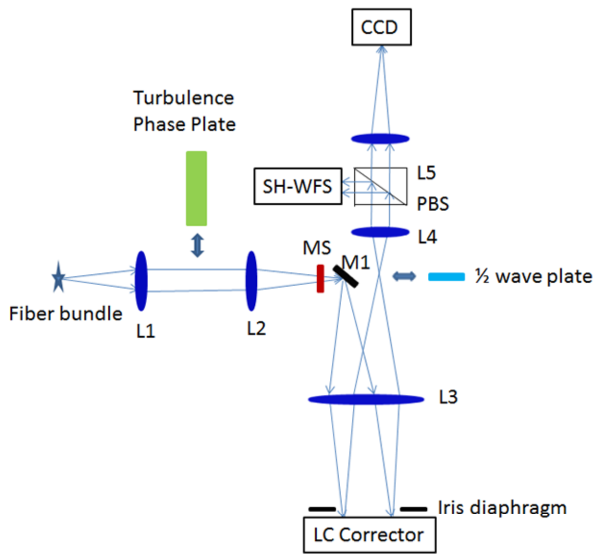

Fig. 1. LC adaptive optics compensation setup in lab.

3. Experimental results

3.1 Accuracy of the least squares interaction matrix measurement method

After the interaction matrix measurement procedure, apply another random Zernike coefficients V_Z (differ from those used in measuring interaction matrix with least squares method, with value also ranging from –0.5 to 0.5) to LC corrector, then measure slopes V_S with SH-WFS in closed loop. From Eq. (1) and Eq. (2), we define slopes reconstruct error E_S and Zernike coefficients reconstruct error E_Z to evaluate the accuracy of M_I:

$$E_S = V_S - M_I * V_Z. \tag{9}$$

$$E_Z = V_Z - M_C * V_S = V_Z - (M_I^T * M_I)^{-1} * M_I^T * V_S. \tag{10}$$

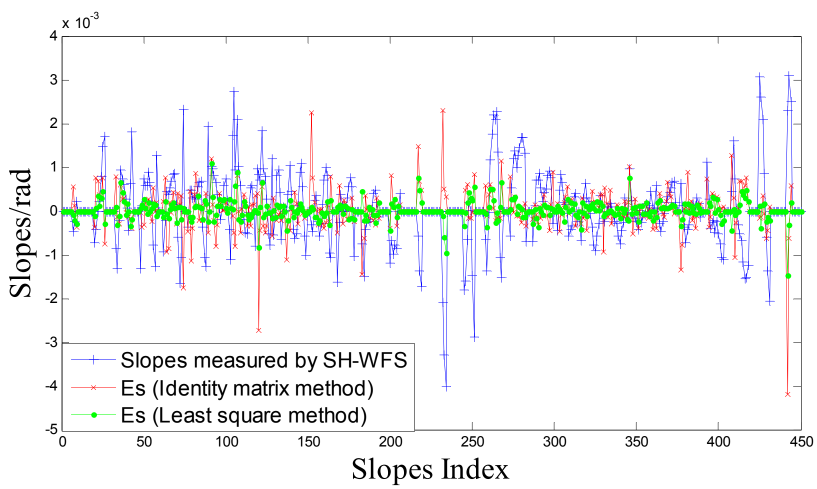

Fig. 2. Slopes reconstruct error of identity matrix method and least squares method.

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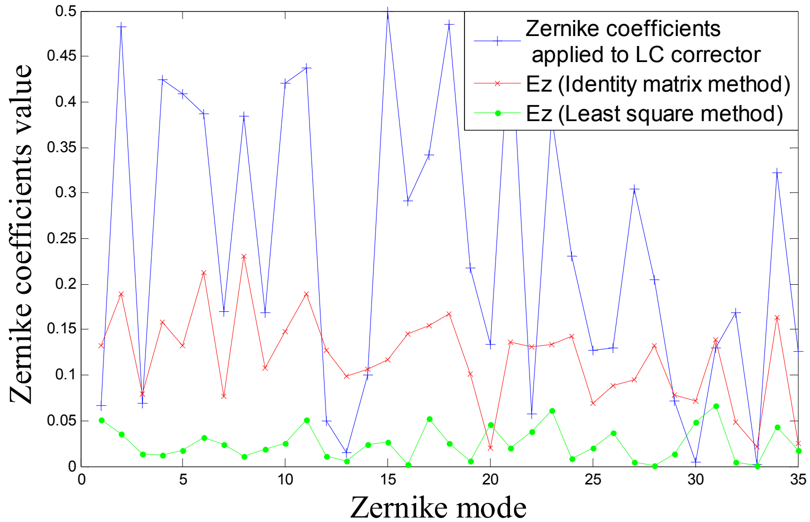


Fig. 3. Zernike coefficients reconstruct error of identity matrix method and least squares method.

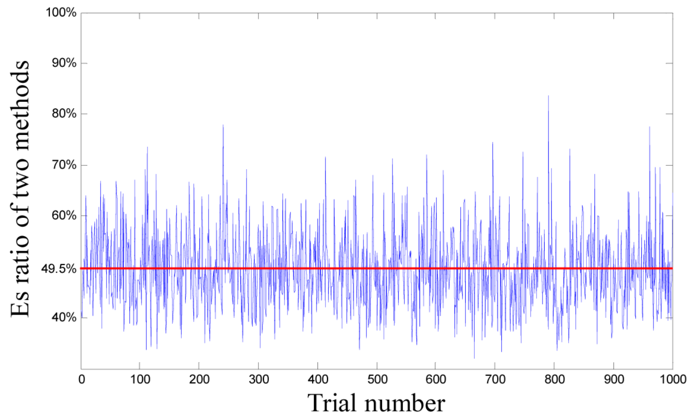
Figure 2 shows the slopes reconstruct error, while Fig. 3 shows the Zernike coefficients reconstruct error of the identity matrix method and the least squares method. In order to describe the improvement of our new method quantitatively, we define:

$$Ratio_S = \sqrt{E_{S2}^T * E_{S2} / (E_{S1}^T * E_{S1})}. \tag{11}$$

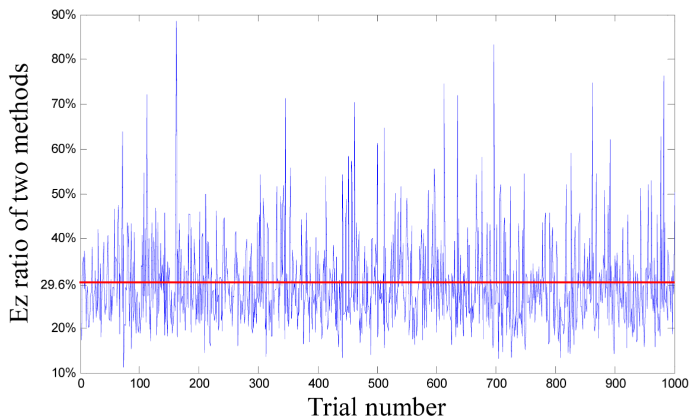
$$Ratio_Z = \sqrt{E_{Z2}^T * E_{Z2} / (E_{Z1}^T * E_{Z1})}. \tag{12}$$

To evaluate the efficiency of the least squares method more precisely, we repeat the previous procedure 1000 times, by applying 1000 different Zernike coefficients V_Z to LC corrector, then calculate $Ratio_S$ and $Ratio_Z$ of each time, the result is shown in Figs. 4 (a) and 4(b). The average $Ratio_S$ is about 49.5% and the average $Ratio_Z$ is about 29.6%. This means the slopes reconstruct error could drop to 49.5%, and the Zernike coefficients reconstruct error could drop to 29.6%, by using the least squares method compared to the identity matrix method.

Condition number is widely used to evaluate the quality of interaction matrix [11]. The condition number of the identity matrix method is $\kappa_{IM} = 5.3$, while the condition number of the least squares method is $\kappa_{LS} = 6.7$. κ_{LS} is only slightly larger than κ_{IM}, therefor the least squares method could dramatically improve the accuracy with hardly any impact on the quality of the interaction matrix.



(a) Ratio of slopes reconstruct error of two methods



(b) Ratios of Zernike coefficients reconstruct error of two methods

Fig. 4. (a) $Ratio_S$ and (b) $Ratio_C$ in 1000 trials with same conditions, only differ in the Zernike coefficients applied to LC corrector.

3.2 performance of open loop AO correction

In order to carry out open loop AO correction with the interaction matrix measured by these two methods, the turbulence phase plate and the ½ wave plate are placed in the optical path, see in Fig. 1. The turbulence phase plate is a commercial product from Lexitek Inc., with an atmospheric coherence length of 0.24 mm (diameter of the light spot on the turbulence phase plate is 2.25 mm). The results in Fig. 5 show a considerable improvement in image resolution.

(a) (b) (c)

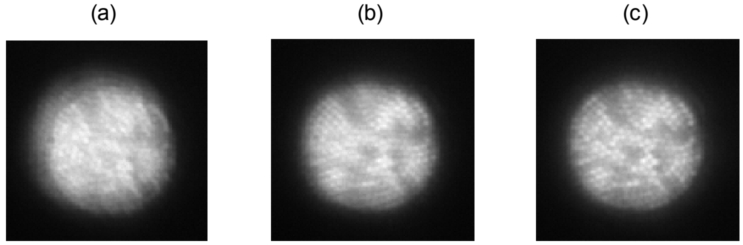


Fig. 5. Fiber bundle images: (a) before AO correction; (b) after correction with interaction matrix measured with identity matrix method; (c) after correction with interaction matrix measured with least squares method.

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To quantify the improvement in image quality we calculate the radially average power spectrum of the images, see in Fig. 6. Figure 7 shows a maximum gain of almost a factor of three in the improvement of the image radially average power spectrum (ratio of the image power spectrum corrected with least squares method to the image power spectrum corrected with identity matrix method).

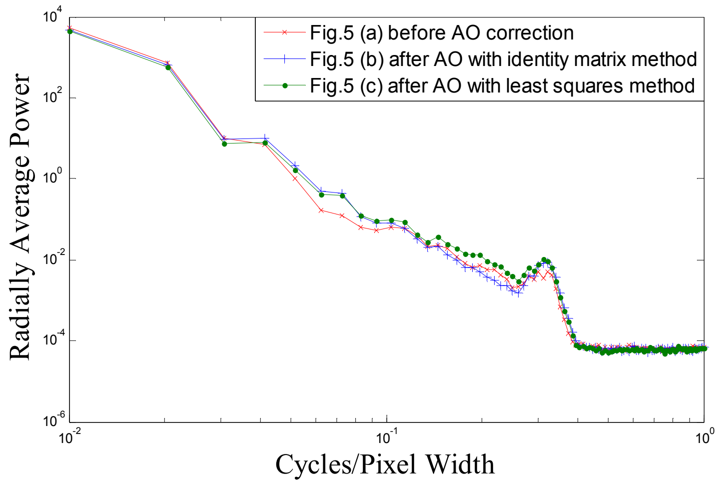

Fig. 6. Radially averaged power spectrum of images shown in Fig. 5: (a) before AO correction; (b) after correction with interaction matrix measured with identity matrix method; (c) after correction with interaction matrix measured with least squares method.

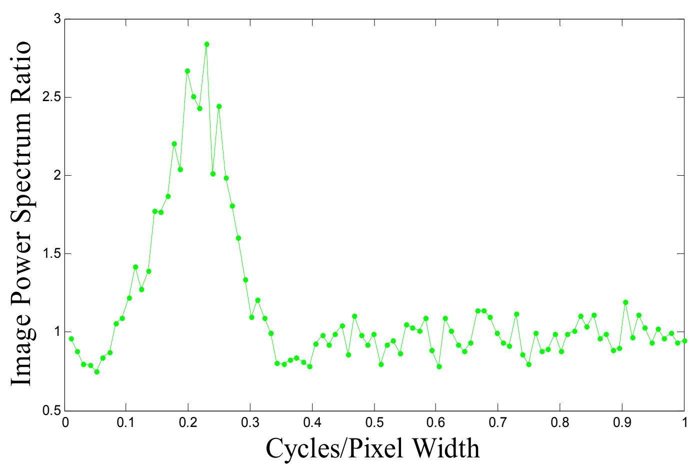

Fig. 7. Radially averaged power spectrum ratio between Fig. 5(c) (image after correction with interaction matrix measured with least squares method) and Fig. 5(b) (image after correction with interaction matrix measured with identity matrix method).

4. Conclusion

This paper presented a least squares method to tremendously improve the accuracy of interaction matrix of liquid-crystal adaptive optics systems. This is especially important for open loop LC-AO systems. Experimental results showed that averagely the slope reconstruction error and the Zernike coefficients reconstruction error could reduce to 49.5% and 29.6% with this new method, respectively. Open loop AO correction also showed considerable improvement in image resolution, a comparison of radially average power spectrum showed a maximum gain of almost a factor of three.

This method can sufficiently mitigate the impact of measurement noise to the interaction matrix measurement procedure. It will be applied to a LC-AO system designed for a 2 meter diameter telescope, currently under development at Changchun Institute of Optics, Fine Mechanics and Physics (CIOMP), Chinese Academy of Sciences (CAS). And it can be transferred to a DM based AO system with hardly any change.

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