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Floquet Topological Insulator in the BHZ Model with the Polarized Optical Field *

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Topological phase of newly found matter has aroused wide interests, especially related with the external periodical modulating. With the help of the Floquet theory, we investigate the possibility of externally manipulating the topological property in a HgTe/CdTe quantum well system with the polarized optical field. We give the phase diagram, showing that by modulating the parameters of the polarized optical field, especially the phase, the topological phase transition can be realized in the QW and lead to the so-called Floquet topological insulator. When the optical field is weak, the driven QSH state of QW is robust with the optical field. However, when the optical field is relatively larger, the group velocity of edge states and the gap between the bulk states exhibit certain oscillations. The implications of our results are discussed.

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Topological insulator (TI) is a new state of matter, with the striking property that the conduction of electrons occurs on their surface[1,2] which is characterized by the insulating gap in the bulk and topologically protected gapless edge states. As the backscattering events of the edge states by disorder and defects are lost, TI has been predicted to have wide ranging applications from fault-tolerant quantum computing to spintronics. A problem arises that it is not easy to change the structure and component of the semiconductor materials to achieve the nontrivial topological phase which will limit their applications. In many previous works, the method of externally periodically modulating materials has been proposed, which can drive the system from the trivial phase to the nontrivial quantum spin Hall (QSH) state,[3−11] especially the strategy of optical field modulating. Such an external periodically modulating system can be well analyzed in the framework of Floquet theory[12,13] and therefore the driven TI can be called the Floquet TI.[8] Recently, Rechtsman *et al.* reported an interesting experiment about the Floquet TI,[14] in which by arranging the helical waveguides in a graphene-like honeycomb lattice, the propagation of the photon in the array is equivalent to the temporal evolution of an electron as it moves through a two-dimensional lattice and thus the Floquet TI can be delicately simulated.

In the HgTe/CdTe quantum well (QW), which owns the fascinating QSH state,[15] the topological order is controlled by the thickness of the HgTe layer:[16,17] below the critical thickness of $d_c \approx 63$ nm, the system is a trivial insulator, whereas above d_c, the system behaves as a TI. In this Letter, we try to investigate the effect of the optical fields with different polarization directions on the QW. There are also some other models that are reported to exhibit the QSH state, such as the graphene Kane–Mele (KM) model[18] and the silicene.[19,20] An important distinction between these models is that only one Dirac point exists at the Γ point in the Brillouin zone of the QW while a pair of Dirac points exist at K and K' points in the KM model and silicene. Thus the optical field will not break the time-reversal symmetry of the QW.

In the work by Lindner *et al.*,[8] it was proposed that, when a resonate optical field acts on the QW where the real processes of single-photon or multiple-photon absorption by electrons occur, the QW can be driven from the trivial phase to the FTI. Here we consider the effect of off-resonate coherent polarized optical field on the QW, especially the role played by the phase. In QSH state, as spin-orbit coupling exists and S_z is not a good number, the spin Chern number as a topological invariant is proposed[21,22] to describe the phase transition between different topological phases, just as the Chern number is used to classify the different quantum Hall states.[23] When the system is modified in the photon-dressed bands, the occupied electronic states will change their topological property and the spin Chern number is modified.

We will give the phase diagram of the spin Chern number showing the influences of the optical field, concentrating on the properties of the optically driven Floquet TI. In the case of the weak optical field, the optically-driven QSH state of QW is robust to the optical field. However, when the optical field is relatively larger, the group velocity of the edge states and the gap between the bulk states exhibit the behaviors of oscillating.

In the low-energy region close to the Γ point, the Bernevig–Hughes–Zhang (BHZ) model[15] has been successfully proposed to describe the HgTe/CdTe QW bulk band structure. The effective 4×4 Hamiltonian

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can be written as

$$H_0(k) = \begin{pmatrix} h_0(\boldsymbol{k}) & 0 \\ 0 & h_0^*(\boldsymbol{k}) \end{pmatrix}. \quad (1)$$

The upper block $h_0(\boldsymbol{k})$ is for upspin ($m_s = 1/2$, $3/2$) and the lower block is for the downspin ($m_s = -1/2$, $-3/2$). To the lowest order in $\boldsymbol{k}$,

$$h_0(\boldsymbol{k}) = -Dk^2 I + A\sigma \cdot \boldsymbol{k} + (M - Bk^2)\sigma_z, \quad (2)$$

where σ denotes the Pauli matrices and $\boldsymbol{k} = (k_x, k_y)$ is the 2D in-plane momentum. The parameters A, B and D are determined by the material structure, M gives the energy offset between s and p orbitals and changes with the width of the QW. When M turns from a positive value to negative, the phase transition from the trivial insulator to the QSH state happens. The second term is similar to the Dresselhauss-type spin-orbit coupling term which acts as a source of violating S_z conservation. As $h_0^*(-\boldsymbol{k})$ is the time reversal partner of $h_0(\boldsymbol{k})$, we focus on the upper block $h_0(\boldsymbol{k})$. In our calculations, we take the parameters of the QW from the experiment[17] as $A = 3645\,\mathrm{meV\cdot nm}$, $B = -686\,\mathrm{meV\cdot nm^2}$, and $D = -512\,\mathrm{meV\cdot nm^2}$ and the lattice constant $a = 5\,\mathrm{nm}$.

We apply a beam of polarized optical field to irradiate on the QW uniformly and perpendicularly as $\boldsymbol{E}(t) = E_0(\cos(\omega t), \cos(\omega t + \varphi))$, where E_0 and ω are the amplitude and the frequency of the optical field while the phase φ controls the polarization: when $\varphi = 0$ or π, the optical field is linearly polarized; when $\varphi = \pm\pi/2$, the optical field is circularly polarized; when φ takes other values, the optical field is ellipsoid polarized. The corresponding vector potential is given by $\boldsymbol{V}(t) = V_0(\sin(\omega t), \sin(\omega t + \varphi))$ with $V_0 = -E_0/\omega$. As the magnetic moment is weak, we do not consider the Zeeman coupling effect of the optical field.[9] The optical field is assumed to irradiate evenly on the QW and its intensity can be characterized by the dimensionless parameter $V = eV_0 a/\hbar$. The interaction of the optical field and the electron system can be captured by the Peierls substitution, which replaces the momentum $P_i = \hbar k i$ with the covariant momentum $P_i \equiv \hbar k i + eVi$. As the vector potential satisfies $\boldsymbol{V}(t + T) = \boldsymbol{V}(t)$ with $T = 2\pi/\omega$, the Hamiltonian becomes time periodic. If the time scale considered in the system is much larger than T, the powerful Floquet theory[12,13] can often be used to deal with the non-equilibrium problem to obtain the static effective Hamiltonian in a period, of which the topological classification about the static system is rather valid.[6−10]

When the optical field is off-resonant, it does not directly excite electron transitions between different energy bands while it will instead effectively modify its band structure through the processes of the virtual photon absorption and emission. The off-resonant condition can be met when the frequency of the optical field is much larger than the bandwidth of the system $\hbar\omega \gg W$, where W is the bandwidth of the system. The influence of an off-resonant optical field can be captured by the static effective Hamiltonian with the help of the evolution operator U,[3,4,11]

$$h_{\rm eff}(\boldsymbol{k}) = \frac{i\hbar}{T}\log U, \quad U = T\exp\Big[\frac{1}{i\hbar}\int_0^T h(\boldsymbol{k}, t)dt\Big], \quad (3)$$

where T is the time-ordering operator. Intuitively, the effective Hamiltonian $h_{\rm eff}$ describes the dynamics of the system on the time scale much longer than a period T, thus the response can be well described by an average over a period of oscillation. In the limit of $V \ll 1$ and around $\varGamma$ point, the effective Hamiltonian is

$$h_{\rm eff} = h_0 + \Delta h, \quad \Delta h = \frac{1}{\hbar\omega}[h_{-1}, h_{+1}], \quad (4)$$

where h_n is the nth Fourier component of $h(t)$ defined as $h_n = \frac{1}{T}\int_0^T h(t)e^{in\Omega t}dt$. The modification term Δh can be regarded as a result of the second-order perturbation theory which consists of two second-order processes, where electrons absorb and then emit a photon and vice versa. After a straightforward calculation, the photon-dressed effective Hamiltonian is given as

$$h_{\rm eff}(\boldsymbol{k}) = -Dk^2 I + A\Big(1 + \frac{2V^2}{\omega}B\sin\phi\Big)(k_x\sigma_x + k_y\sigma_y) + \Big(M + \frac{V^2}{\omega}A^2\sin\phi - Bk^2\Big)\sigma_z. \quad (5)$$

From the above effective Hamiltonian, it is evident that the influence of the off-resonate optical field on the band structure of the system is twofold. First, it renormalizes the coupling strength between s and p orbits $A \to A(1 + 2v^2 B\sin\phi/\omega)$. Second, it changes the mass term $M \to M + v^2A^2\sin\phi/\omega$. In addition to the intensity parameter V, the phase ϕ also plays an important role, which makes the modulation of the topological phase transition become possible. It is notable when the optical field is linearly polarized, its effects tend to vanish, which is consistent with the previous study related to the interaction between the optical field and the matter.[24]

As the topological classification is valid for the effective static system, the phase transition in the QW can be characterized with the help of the topological number. For a pure electronic system which can be simplified by two bands, the Chern number for each band is defined as[25,26]

$$C_\pm = -\frac{1}{2\pi i}\int_{\rm FBZ} dk\nabla_k \times \langle\pm, k|\nabla_k|\pm, k\rangle, \quad (6)$$

where the integral in the present problem is confined within the Floquet Brillouin zone. The Chern number of the system is the sum of the two bands as $C = C_+ + C_-$, which is proportional to the Hall conductance and vanishes due to the time reversal symmetry in the QSH phase. Instead the spin Chern number is proposed which can be constructed as[22] $C_s = (C_+ + C_-)/2$. It is also invariant when the

Hamiltonian experiences small deformations and perturbations, showing the robust property of the topological phase. The system lies in the nontrivial QSH state when $C_{\rm s}$ takes nonvanishing value while it is in the trivial insulating state when $C_{\rm s}$ vanishes.

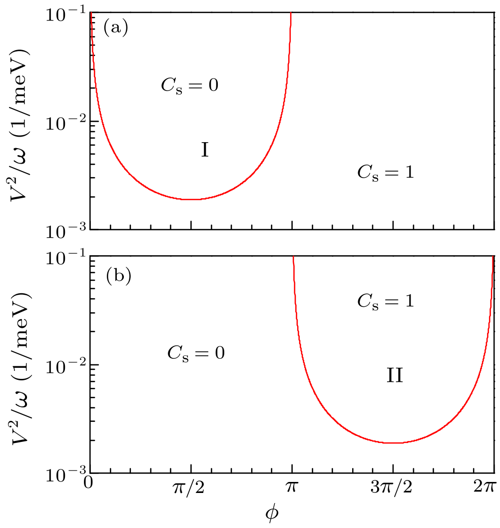

Fig. 1. (Color online) Phase diagram of the spin Chern number of the QW in the space $(\phi, V^2/\omega)$ with (a) $M = -10$ meV, and (b) $M = 10$ meV. The spin Chern number value ($C_{\rm s} = 0, 1$) is shown in the corresponding region, which distinguishes the different topological phases.

We obtain the spin Chern number of the QW under the polarized optical field as

$$C_{\rm s} = \frac{1}{2} + \frac{1}{2}{\rm sgn}\Big(\frac{M + \frac{v^2}{\omega}A^2 \sin\phi}{B}\Big). \tag{7}$$

Equation (7) gives the main result of this work. In Fig. 1, the phase diagrams of the spin Chern number $C_{\rm s}$ of the QW system driven by the polarized optical field are given in the space of $(\phi, V^2/\omega)$ under different situations. We can see the border line between different phases, if the frequency of the optical field is chosen properly, V is much less than 1, which is in good agreement with the assumption of the perturbation theory. When $M < 0$ in Fig. 1(a) and the system is initially in the nontrivial QSH state with $C_{\rm s} = 1$, if the phase ϕ lies between 0 and π and the optical intensity parameter V lies above the U-shape curve, the topological phase transition to the trivial phase with $C_{\rm s} = 0$ driven by the optical field can be realized. If ϕ lies between π and 2π, whatever V takes, the phase transition will not happen. Contrarily, in Fig. 1(b) with $M > 0$ and the system initially lies in the trivial phase, only if ϕ takes the value between π and 2π, modulating the strength of the optical field will induce the phase transition. In addition, if the optical field is circularly polarized, a minor magnitude of V^2/ω can drive the phase transition. Note when ϕ is approaching π or 2π, the phase boundary tends to diverge.

In Fig. 2, we plot the dispersion curves of QW corresponding to the cross points I and II in Fig. 1. In the numerical calculation, we choose the period boundary condition in the x-direction and therefore the wavevector k_x is a good quantum number. In the numerical calculation, the width of the lattice is set to be $80a$ to eliminate the influence of the finite-size as possible.[27] In Fig. 2(a), a large gap exists between the valence band and the conduction band while in Fig. 2(b), there is a pair of helical edges where states carrying opposite spins on different edges are dispersed from the valence bands to the conduction bands. Thus we demonstrate that with the proper conditions, the polarized optical field can drive the system from the QSH state into the insulating phase or from the insulating phase to the QSH state.

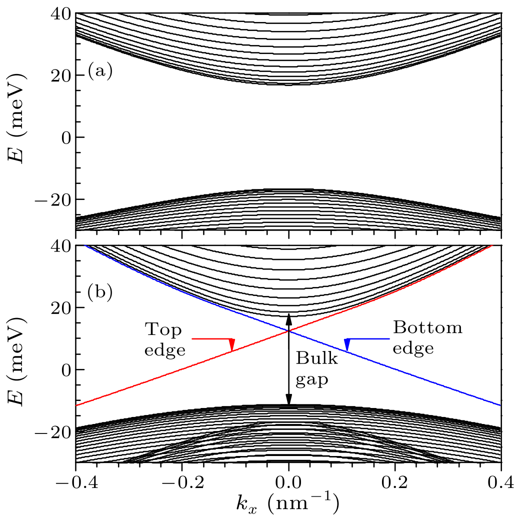

Fig. 2. (Color online) Plot of the dispersion curves of the QW around the Γ point under the optical field with (a) and (b) corresponding with the crossing points I and II in Fig. 1, respectively. (a) The optically-driven insulating phase, and (b) the optically-driven QSH state, where the edge states are shown with arrows. The unit cell of the lattice is set to be $80a$.

The Dirac-type linear dispersion can be used to describe the edge states as $h_\pm(\boldsymbol{k}) = \pm v_{\rm g}k_x + \epsilon D$, where $v_{\rm g}$ is the group velocity in the x–y plane and ϵD is the energy of the Dirac point, precisely the crossing point of the edge state dispersion lines. In the FTI of the QW, the group velocity on the top edge of the system directs to the right and on the bottom edge to the left, corresponding to the clockwise circulation. As there are no edge states circulating in the anti-clockwise direction, the edge states presented in Fig. 2(b) cannot be backscattered and are topologically protected.

To further study the property of the QSH state driven by the optical field, in Fig. 3 we plot the group velocity $v_{\rm g}$ and the gap $\varDelta$ between the bulk states at Γ point (see Fig. 2(b)) when the phase ϕ changes under different conditions, in which the results exhibit nontrivial behaviors. It shows when the parameter of the optical field V^2/ω is relative weak and increases, $v_{\rm g}$ becomes larger and can reach the order of 10^5 m/s. On the other hand, the band size remains unchanged for a large range of phase ϕ despite the range enlarging with V^2/ω. These results show that the QSH state shows a certain robustness to the weak optical field. The group velocity v_g arrives at its maximum when $\phi = 3\pi/2$. However, at the strong optical field when $V^2/\omega > 0.02$, the peak of v_g at $\phi = 3\pi/2$ begins to split and two peaks gradually form while $V^2/\omega = 0.03$ the gap exhibits certain oscillation and will change. In the inset of Fig. 3(b), the size of the gap with V^2/ω is given with $\phi = 3\pi/2$ in which the gap keeps unchanged under the weak optical field, then after a small dip it increases linearly with the strong optical field. When approaching the boundary of the phase transition, v_g and Δ decrease quickly. When the optical field is strong enough, only the second-order perturbation process may not well capture the process, which will be our future work. These results suggest that we can modulate the property of the edge states at our purpose with the controllable polarized optical field.

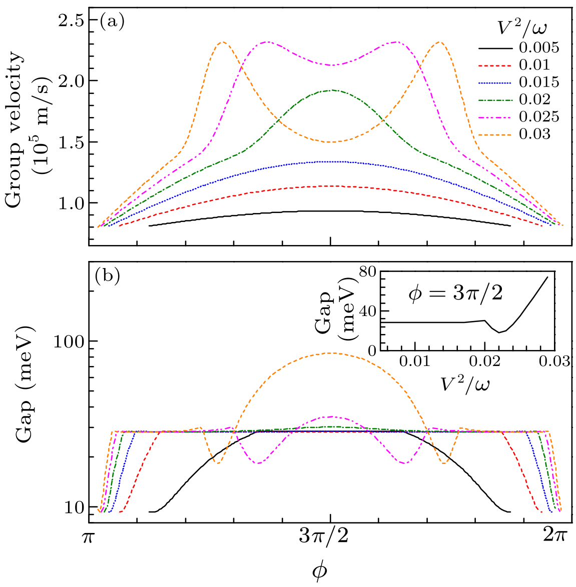

Fig. 3. (Color online) Plot of the group velocity (a) and the bulk band gap of the QW versus the phase ϕ in the optically-driven QSH state. The different curves correspond to the different optical field parameters V^2/ω. The legends are the same as those in both figures. The inset of (b) shows the behavior of the gap versus V^2/ω when the phase is fixed at $3\pi/2$.

Now we discuss the parameter of the optical field suitable for realistic experimental manipulation. The ultraviolet optical field with $\omega = 10\,\text{eV}$ can well meet the off-resonate condition for the HgTe/CdTe QW whose bandwidth is about several hundreds of meV. For example, if the intensity of the optical field takes $1.7 \times 10^9\,\text{W/m}^2$ and the circularly polarized optical field is concerned, the corresponding amplitude is $E_0 = 1.4 \times 10^{10}\,\text{W}\cdot\text{mV}^{-1}\text{m}^{-1}$, which determines $V^2/\omega = 5 \times 10^{-3}$. Thus our proposal is quite feasible in experiment.

In summary, we have investigated the possibility of driving the topological phase transition with the polarized optical field in the BHZ model. We find that when modulating the amplitude, frequency and especially the phase of polarization, the topological phase transition may be driven. The resulting Floquet TI has a rather steady property and may be detected in experiment. We suggest that the ac electric field may play a similar role with the polarized optical field as the electric field has a much larger strength. As the Floquet TI has the potential to provide an entirely new platform for probing and understanding many topological properties, our study may help to show a better understand of such an electron system.

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